

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

BMT5B05 - Abstract Algebra

(2022 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

SECTION A: Answer the following questions. Each carries two marks

(Ceiling 25)

1. $*$ defined on $\mathbb{Z}$ by letting $a * b = a - b$. Whether the operation $*$ is associative?
2. Define $*$ on $\mathbb{Z}$ by $a * b = \max\{a, b\}$. Determine whether or not $*$ gives a group structure on $\mathbb{Z}$. If it is not a group, say which axiom fail to hold.
3. Make addition and multiplication tables of $\mathbb{Z}_5$.
4. If G is a group with binary operation $*$, then for all $a, b, c \in G$, prove that $b * a = c * a$ implies $b = a$.
5. Describe Klein 4-group.
6. Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 5 & 2 & 9 & 3 & 1 & 8 & 4 & 6 & 10 \end{pmatrix}$ as a product of disjoint cycles.
7. Let $\sigma = (2 \ 5 \ 6 \ 8 \ 10 \ 4 \ 7 \ 9 \ 3 \ 11)$ be a cycle in S_{14} . Is σ an even transposition? Why or why not?
8. Give an example for a group of order 7 which is not cyclic.
9. Find all cyclic subgroups $\mathbb{Z}_{30}$.
10. Let $(\mathbb{R}^\times, \cdot)$ be set of nonzero real numbers under multiplication and $(\mathbb{R}^+, \cdot)$ be set of positive real numbers under multiplication. Define $\phi : \mathbb{R}^\times \rightarrow \mathbb{R}^+$ by $\phi(x) = |x|$. Prove that ϕ is a homomorphism.
11. Let $G = \mathbb{Z}_3 \times \mathbb{Z}_6$ and let $H = \langle (1, 2) \rangle$. List all cosets of H .
12. Let $\phi: G \rightarrow G'$ be a group homomorphism. If ϕ is one-one, then prove that $\ker \phi = \{e\}$.
13. Find the multiplicative inverse of $(1 - \sqrt{2})$ in the field $\mathbb{Q}(\sqrt{2})$.
14. Find all divisors of zeros in $\mathbb{Z}_{12}$.
15. Is $A = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z} \text{ and } m \text{ is odd}\}$ subring of the field $\mathbb{R}$ of real numbers? Why or why not?

SECTION B: Answer the following questions. Each carries five marks

(Ceiling 35)

16. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Verify whether
- (i) $\sigma\tau = \tau\sigma$ (ii) $(\sigma\tau)^{-1} = \tau^{-1}\sigma^{-1}$.
17. Let S be any set and let σ be a permutation on S . Prove that σ can be written as product of disjoint cycles.
18. Prove that every cyclic group is abelian. Prove that S_3 is not a cyclic group.
19. Let H be a subgroup of G . Then for any $a \in G$, prove that the coset aH has the same cardinality as H .
20. Let G be a group, and suppose that a and b are any elements of G . Prove that $(ab)^2 = a^2b^2$ if and only if $ab = ba$.
21. Let G be a cyclic group and let H be a nonempty subgroup of G . Prove that H is cyclic.
22. Let G_1 and G_2 be groups and let $\phi: G_1 \rightarrow G_2$ be a homomorphism. Prove that
- (i) If e is the identity element of G , then $\phi(e) = e'$ is the identity element of G' .
- (ii) If $a \in G$, then $\phi(a^{-1}) = [\phi(a)]^{-1}$.
- (iii) If $a \in \ker \phi$, then $a^{-1} \in \ker \phi$.
23. Prove that in the ring $\mathbb{Z}_n$, the divisors of zero are precisely those nonzero elements that are not relatively prime to n .

SECTION C: Answer any two questions ($2 \times 10 = 20$ Marks)

24. (i) Let G be a group, let $a \in G$ and let $H = \{a^n: n \in \mathbb{Z}\}$. Prove that H is a subgroup of G .
- (ii) If a is a generator of a finite cyclic group G of order n , then prove that the generators of G are the elements of the form a^r , where r is relatively prime to n .
25. (i) Let G be a group and let H be a subgroup of G . For $a, b \in G$ define $a \sim b$ if $ab^{-1} \in H$. Prove that $\sim$ is an equivalence relation.
- (ii) Let G be a finite group and let H be a subgroup of G . Prove that the order of H is a divisor of the order of G .
26. (i) Define permutation group.
- (ii) Prove that every group is isomorphic to a permutation group.
27. (i) Define integral domain. Give an example.
- (ii) Prove that every finite integral domain is a field.

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(Pages : 2)

Reg. No:.....

Name:

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

BMT5B06 - Basic Analysis

(2022 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A

All Questions can be attended. Each question carries 2 marks. Ceiling 25 marks

1. Show that there does not exist a rational number r such that $r^2 = 3$
2. If $a, b = 0$ prove that *either* $a = 0$ or $b = 0$.
3. If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$, then prove that $a = 0$.
4. State and Prove Bernoulli's Inequality.
5. Prove that if $y > 0$, there exists $n_y \in \mathbb{N}$ such that $n_y - 1 < y \leq n_y$
6. If $a < 0$ show that $\sup(aS) = a \inf S$
7. Show that $\lim \left(\frac{3n+2}{n+1} \right) = 3$
8. If $X = (x_n)$ and $Y = (y_n)$ are convergent sequences of real numbers and if $x_n \leq y_n$ for all $n \in \mathbb{N}$, then prove that $\lim x_n \leq \lim y_n$.
9. If $X = (x_n)$ is a sequence of real numbers that converges to $x \in \mathbb{R}$, then prove that any subsequence $X' = (x_{n_k})$ also converges to x .
10. If $c > 1$, show that $\lim c^n = +\infty$
11. Show that the set $I = [0, 1]$ is closed.
12. Find the reciprocal of $z = 2 - 3i$
13. Describe the set of points z in the complex plane that satisfy $|z| = |z - i|$
14. Find the real and imaginary parts of $f(z) = \frac{\bar{z}}{z+1}$
15. Find the principal square root of $z = 2 + i$.

Section B

All Questions can be attended. Each question carries 5 marks. Ceiling 35 marks

16. State and Prove Cantor's Theorem.

17. If $a, b \in \mathbb{R}$, then prove that:

a) $||a| - |b|| < |a - b|$

b) $|a - b| < |a| + |b|$

18. State and Prove Characterization of Intervals Theorem

19. Show that the set $\mathbb{R}$ of real numbers is uncountable.

20. State and prove Monotone subsequence Theorem

21. Prove that,

a) The union of an arbitrary collection of open sets in $\mathbb{R}$ is open

b) The intersection of any finite collection of open sets in $\mathbb{R}$ is open

22. Find the image of the vertical line $x = 1$ under the complex mapping $w = z^2$

23. Sketch the set S of points in the complex plane satisfying the inequality $2 < |z - i| < 3$.

Determine whether the set is a) open b) closed c) a domain, d) bounded or, e) connected

SECTION C

Answer any Two Questions. Each question carries 10 Marks. 20 Marks from this section

24. a) Prove that the following statements are equivalent;

i) S is a countable set

ii) There exists a surjection of N onto S

iii) There exists an injection of S into N

b) Describe the set A of all $x \in \mathbb{R}$ such that $|x - 1| < |x|$

25. a) State and Prove Squeeze Theorem

b) If $X = (x_n)$ is a sequence of real numbers that converges to x and if $x_n \geq 0$, then prove that the sequence $(\sqrt{x_n})$ converges to $\sqrt{x}$.

26. State and Prove Cauchy Convergence Criterion

27. Find a complex linear function that maps the equilateral triangle with vertices $1 + i, 2 + i$

and $\frac{3}{2} + \left(1 + \frac{1}{2}\sqrt{3}\right)i$ onto the equilateral triangle with vertices $i, \sqrt{3} + 2i$ and $3i$.

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

BMT5B07 - Numerical Analysis

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks.

- How many bisection iterations are necessary to approximate a root of $f(x) = x^2 - 3$ in $[1,2]$ with accuracy 10^{-4} ?
- Show that $g(x) = \frac{x+2}{4}$ has a unique fixed point on the interval $[0,1]$.
- Apply Newton-Raphson method to find p_2 for $f(x) = x^2 - 5$, with $p_0 = 2$.
- Let $f(x) = (x - 1)^2(x + 2)$. Show that f has a zero of multiplicity 2 at $x = 1$.
- Find the Lagrange interpolating polynomial through the points $(0,1)$, $(1,3)$, $(2,12)$.
- Construct a forward difference table if $f(0.0) = 1.00000$, $f(0.2) = 1.22140$, $f(0.4) = 1.49182$, $f(0.6) = 1.82212$, $f(0.8) = 2.22554$.
- Using the three-point midpoint formula, approximate the first derivative of the function $f(x) = xe^x$ at $x = 2$ with $h = 0.2$.
- Evaluate $\int_0^4 \frac{dx}{(1+x^2)^{3/2}}$ using Trapezoidal rule, with $n = 6$.
- Evaluate $\int_0^1 (x^2 + 1)dx$ using Simpson's rule.
- Approximate $\int_0^{\pi/2} \sin x \, dx$ with $h = \pi/8$ using Composite Midpoint rule.
- Show that $f(t, y) = t + y$ satisfies the Lipschitz condition on the interval $D = \{(t, y) / 0 \leq t \leq 1 \text{ and } -\infty < y < \infty\}$.
- Write the conditions for the well-posedness of an IVP $y' = f(t, y)$, $a \leq t \leq b$, $y(a) = \alpha$.

(Ceiling ... 20 Marks)

Section B

All questions can be attended. Each question carries 5 marks.

13. Use the False Position method to approximate a root of $f(x) = x^3 - x - 2 = 0$ on the interval $[1, 2]$. Stop when successive approximations differ by less than 10^{-2} .
14. Construct the Lagrange interpolating polynomial for the function $f(x) = e^{2x} \cos 3x$ using $x_0 = 0, x_1 = 0.3, x_2 = 0.6$ and $n = 2$.
15. Use Stirling's formula to estimate $f(2.5)$ from the data $f(2) = 7.389, f(3) = 20.085, f(4) = 54.598$.
16. Let $f(x) = 3xe^x - \cos x$. Use second derivative midpoint formula to approximate $f''(1.3)$ by using the following table with $h = 0.1$ and with $h = 0.01$. Also compare the result with the exact value of $f''(1.3)$.

x	1.20	1.29	1.30	1.31	1.40
$f(x)$	11.59006	13.78176	14.04276	14.30741	16.86187

17. Use the Composite Simpson's rule to approximate the integral $\int_1^2 x \ln x \, dx, n = 4$.
18. Use Taylor's method of order four to approximate solution of $y' = y + t, y(0) = 1$, at $t = 0.2$ with $h = 0.2$.
19. Use the Modified Euler method to approximate the solution to initial-value problem $y' = 1 + \frac{y}{t}, 1 \leq t \leq 2, y(1) = 2$, with $h = 0.25$.

(Ceiling ... 30 Marks)

Section C

Answer any ONE question.

20. (a) Use Secant method to approximate the root of $f(x) = x^3 - 2x - 5$ with initial guesses 2 and 3. Perform 4 iterations.
- (b) Find $\int_0^1 x^2 e^x \, dx$ using Composite Trapezoidal rule with $n = 4$ and compare with the exact value.
21. (a) Use Euler's method with step size $h = 0.1$ to approximate $y(0.4)$ for the initial-value problem $y' = y - t^2, y(0) = 1$.
- (b) Use fourth order Runge-Kutta method to approximate $y(0.4)$ for $y' = t + y$ with $y(0) = 1, h = 0.2$.

(1 X 10 = 10 Marks)

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Reg. No:.....

Name:

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

BMT5B08 - Linear Programming

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

Session A

All questions can be attended. Each question carries 2 marks.

- 1) Define a hyper plane in $\mathbb{R}^n$. Sketch a hyperplane in $\mathbb{R}^2$.
- 2) Let $S = \{(x, y) \in \mathbb{R}^2; |x| \leq 1 \text{ \& } |y| \leq 1\}$. Find the extreme points of S .
- 3) Check whether the solution $x = 5, y = 1, z = 1$ is a feasible solution of the following LPP

$$\text{Maximize } f(x, y, z) = 2x + y - 2z$$

$$\text{subject to } y + 4z \leq 8; x + 2y + 3z \leq 8; x, y, z \geq 0$$

- 4) Find the basic solution of the following tableau

x_1	t_2	-1	
5	-1	10	$= -t_1$
-2	3	15	$= -x_2$
1	0	-5	$= -t_3$
350	200	32	$= f$

- 5) Take the negative transpose of the following tableau

x_1	1	3	2
x_2	-4	1	9
x_3	-2	2	1
-1	5	-4	0
$= t_1 = t_2 = g$			

- 6) Sketch the constraint set of the following LPP

$$\text{Maximize } f(x, y) = x + 3y$$

$$\text{subject to } x + 2y \leq 10$$

$$-3x - y \leq -15$$

- 7) Write down the non-canonical problem associated with the tableau

x_1	x_2	-1	
20	10	5	$= -t_1$
18	-6	10	$= 0$
15	8	6	$= 0$
12	18	0	$= f$

- 8) State the dual maximization problem of the following minimization problem

$$\text{Minimize } f(x_1, x_2, x_3) = 2x_1 - x_2 - 2x_3$$

$$\text{Subject to } 2x_1 + 3x_3 \geq 5$$

$$x_1 - x_2 + 4x_3 = 2$$

$$x_3 \geq 0$$

- 9) Prove that, if a canonical maximization linear programming problem is unbounded, then the dual canonical minimization linear programming problem is infeasible.

- 10) Define assignment problem.

- 11) Give an example of unbalanced transportation problem

- 12) A basic feasible solution of a transportation problem

8	7	3	60
3	8	9	70
11	3	5	80
50	80	80	

$$\text{is } x_{11} = 20, x_{13} = 40, x_{21} = 30, x_{23} = 40, x_{32} = 80.$$

Check for the optimality of this solution

Session B

All questions can be attended. Each question carries 5 marks.

- 13) Solve graphically

$$\text{Maximize } P = 2x_1 - 3x_2$$

$$\text{Subject to } x_1 - x_2 \geq -3$$

$$2x_1 + 3x_2 \leq 5$$

$$x_1 + 4x_2 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

14) Prove that the following LPP is unbounded.

$$\text{Minimize } g(x, y) = y - 5x$$

$$\text{Subject } x - y \geq 1$$

$$y \leq 8$$

$$x, y \geq 0$$

15) Solve the LPP

x_1	x_2	-1	
-1	2	4	$= -t_1$
1	1	6	$= -t_2$
1	3	9	$= -t_3$
2	3	0	$= f$

16) Consider the dual canonical tableau

	t_3	t_2	-1	
y_1	5	-1	4	$= -t_1$
s_2	-2	3	8	$= -x_1$
s_1	1	0	7	$= -x_2$
-1	-35	-17	-210	$= f$
	y_3	y_2	$= g$	

a) Find the basic solutions of the maximization and minimization problems associated with the above tableau.

b) Do these solutions exhibit complementary slackness? Explain.

17) State and prove the duality equation.

18) Solve the transportation problem

23	27	16	18	30
12	17	20	51	40
22	28	12	32	53
22	35	25	41	

19) Four operators are to be assigned to four machines with effective cost given in the following table. Assign each operator to each machine so as to minimize the cost.

5	3	1	8
7	9	2	6
6	4	5	7
5	7	7	6

Session C
Answer any one. Question carries 10 marks

20) Solve the non-canonical LPP

Maximize: $f(x, y, z) = x + y + z$

Subject to $x + y + z \leq 3$

$$x + y \leq 1$$

$$y + 2z = 2$$

$$x, y \geq 0$$

21) a) write simplex algorithm for maximum basic feasible tableau

b) Convert the following tableau into maximum basic feasible

x_1	x_2	-1	
-1	-2	-8	$= -t_1$
-3	-2	-12	$= -t_2$
5	6	60	$= -t_3$
-3	-5	0	$= f$

c) Solve the above LPP.

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Reg. No:.....

Name:

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

BMT5B09 - Calculus of Multivariable - 1

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended

Each question carries 2 marks

1. Convert the rectangular equation $x^2 + y^2 = 9$ to a polar equation
2. Write the integral formula for finding the area of the region R
3. Plot the point $(4, \frac{\pi}{4})$ with the polar coordinates and (0,5) with the rectangular coordinates.
4. Find a rectangular equation whose graph contains the curve given by $x = 5 \cos t$ and $y = 5 \sin t$. Sketch the curve and find its orientation.
5. Write the equations used to convert a point from spherical coordinates to rectangular coordinates.
6. Find an equation in rectangular coordinates for the surface $r^2 \cos 2\theta - z^2 = 4$.
7. Find parametric equations and symmetric equations for the line passing through the points P(-3,3,-2) and Q(2,-1,4). At what point the line does intersect the xy-plane.
8. Identify and sketch the surface $12x^2 - 3y^2 + 4z^2 + 12 = 0$
9. Find the derivative of $\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + e^{-t}\mathbf{j} - \sin 2t\mathbf{k}$.
10. Find the level surfaces of the function f defined by $f(x, y, z) = x^2 + y^2 + z^2$.
11. State the properties of continuous functions of two variables.
12. Find f_x and f_y if $f(x, y) = x \cos xy^2$.

(Ceiling 20 Marks)

Section B

All questions can be attended
Each question carries 5 marks

13. Show that the surface area of a sphere of radius r is $4\pi r^2$.
14. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.
15. Find the length of the curve defined by the vector function $\mathbf{r}(t) = \frac{1}{2}t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + \ln t\mathbf{k}$ over the interval $1 \leq t \leq 2$.
16. Sketch the curve defined by the vector function $\mathbf{r}(t) = 3t\mathbf{i} + t\mathbf{j} + (4 - t^2)\mathbf{k}, -2 \leq t \leq 2$.
17. Let $\mathbf{r}(s) = 2 \cos 2s\mathbf{i} + 3 \sin 2s\mathbf{j} + 4s\mathbf{k}$, where $s = f(t) = t^2$. Find $\frac{d\mathbf{r}}{dt}$.
18. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ does not exist.
19. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^2 - z + 2yz^2 = 5$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(Ceiling 30 Marks)

Section C

Answer any one question

20. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid $r = 2 + 2 \cos \theta$.
21. (a) A particle moves along a curve described by the vector function $\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j} + \ln t\mathbf{k}$. Find the tangential scalar and normal scalar components of acceleration of the particle at any time t .
- (b) Show that the function f defined by $f(x, y) = 2x^2 - xy$ is differentiable in the plane.

(1 X 10 = 10 Marks)

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2025

(Open course)

BMT5D03 - Linear Mathematical Models

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended.

Each questions carries 2 marks.

- Find the slope of the line through the points (7,6) and (-4,5).
- Find the intercept of the graph $3x + 2y = 12$.
- Let $g(x) = -4x + 5$. Find $g(-2)$ and $g(0)$.
- Write the augmented matrix for the system

$$\begin{aligned} 3x + y &= 6 \\ 2x + 5y &= 15 \end{aligned}$$

- Let $A = \begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$. Find $A + B$
- Find the value of the variable in the equations $\begin{bmatrix} 3 & 4 \\ -8 & 1 \end{bmatrix} = \begin{bmatrix} 3 & x \\ y & z \end{bmatrix}$
- Find A^{-1} if $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$.
- Graph $y = -3x$.
- Define slack variable.
- Find the initial simplex tableau for the linear programming problem given below

$$\begin{aligned} \text{Maximize } z &= 7x + y \\ \text{subject to: } 4x + 2y &\leq 5 \\ x + 2y &\leq 4 \\ \text{with } x \geq 0, y &\geq 0 \end{aligned}$$
- Define Basic feasible solution in a linear programming problem.
- Find the transpose of the matrix $\begin{bmatrix} 2 & -1 & 5 \\ 6 & 8 & 0 \\ -3 & 7 & -1 \end{bmatrix}$.

(Ceiling 20 Marks)

Section B

All questions can be attended.

Each questions carries 5 marks

- Find the equation of the line through (5,4) and (-10, -2).
- Find the equation of the line that passes through the point (3,5) and is parallel to the line $2x + 5y = 4$

15. Use Gauss-Jordan method to solve the system

$$\begin{aligned}x - 2y &= 2 \\ 3x - 6y &= 5\end{aligned}$$

Find the product AB of matrices

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix}$$

16. Verify that the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$ are inverses of each other.

17. Use the inverse of the coefficient matrix to solve the linear system

$$\begin{aligned}2x - 3y &= 4 \\ x + 5y &= 2\end{aligned}$$

18. Graph the feasible region for the system

$$\begin{aligned}y &\leq -2x + 8 \\ -2 &\leq x \leq 1\end{aligned}$$

19. Write the dual of the standard linear programming problem

$$\text{Maximize } z = 2x + 5y$$

Subject to:

$$\begin{aligned}x + y &\leq 10 \\ 2x + y &\leq 8\end{aligned}$$

$$\text{With } x \geq 0, y \geq 0$$

(Ceiling 30 Marks)

Section C

Answer any one question

20. Use graphical method to solve the linear programming problem

$$\text{Maximize } z = 3x + 4y$$

$$\text{Subject to: } 2x + y \leq 4$$

$$-x + 2y \leq 4$$

$$\text{With } x \geq 0, y \geq 0$$

21. Use simplex method to solve the linear programming problem

$$\text{Maximize } z = 25x + 30y$$

$$\text{Subject to: } x + y \leq 65$$

$$4x + 5y \leq 300$$

$$\text{With } x \geq 0, y \geq 0$$

(1 × 10 = 10 Marks)