(37)

1M3N25420 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2025

MMT3C11 – Multivariable Calculus & Geometry

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer *all* questions. Each question carries *1 weightage*

1. Define dimension of a vector space.Show that dim $R^n = n$.
2. Prove that $d(a, b) = ||a - b||$ is a metric on the set of all linear transformations from $R^n \to R^m$.
3. Calculate the tangent vector at each point of the curve $\gamma(t) = (e^{2t}, 2t^2)$.
4. Compute the curvature of the curve $\gamma(t) = \left(\frac{4}{5}cost, 1 - sint, \frac{-3}{5}cost\right)$.
5. Is the surface $x^2 + y^2 + z^4 = 1$ smooth? Justify your answer.
6. Calculate the first fundamental form of $\sigma(u, v) = (\cos h\, u, \sinh u, v)$.
7. Calculate the Gauss map G of the paraboloid S with equation $z = x^2 + y^2$. What is the image of G.
8. Show that the Weingarten map changes sign when the orientation of the surface changes.

(8 x1=8 weightage)

Part B

Answer any *two* questions from each unit. Each carries *two weightage*

Unit 1

9. Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.
10. Prove that BA is linear if A and B are linear transformations. Prove also that A^{-1} is linear and invertible.
11. Prove that $\det([B][A]) = \det[B] \det[A]$ if A and B are n by n matrices.

Unit 2

12. Show that a parametrized curve has a unit- speed reparametrization if and only if it is regular.
13. Prove that any regular plane curve whose curvature is a positive constsnt is part of a circle.
14. Let $f : S_1 \rightarrow S_2$ be a diffeomorphism, then prove that, for all $p \in S_1$ the linear map $D_p f: T_p S_1 \rightarrow T_{f(p)} S_2$ is invertible.

Unit 3

15. State and prove Meusnier's Theorem.
16. Show that the Gaussian and mean curvatures of a surface S are smooth functions on S.
17. Calculate the principal curvatures of the helicoid $\sigma(u, v) = (v \cos u,\ v \sin u,\ \lambda u)$.

(6 X2=12 weightage)

Part C

Answer any *two* questions. Each carries *5 weightage*

18. State and prove inverse function theorem.
19. Suppose f maps an open set $E \subset R^n$ into R^m, and f is differentiable at a point $x \in E$. Then prove that the partial derivatives $(D_j f_i)(x)$ exist, and

$$f'(x)e_j = \sum_{i=1}^{m} (D_j f_i)(x) u_i, \quad (1 \leq j \leq n).$$

20. Let $\gamma(t)$ be a regular curve in R^3 with nowhere vanishing curvature. Prove that its torsion is given by $\tau = \frac{(\dot{\gamma}\, X\, \ddot{\gamma}).\dddot{\gamma}}{\|\dot{\gamma} X \ddot{\gamma}\|^2}$, where the X indicate the vector product and the dot denotes d/dt.
21. Let S be a connected surface of which every point is an umbilic. Then, prove that S is an open subset of a plane or a sphere.

(2 X 5= 10 weightage)

38

1B3N25421 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2025

MMT3C12 – Complex Analysis

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer all the questions. Each carries 1 weightage.

1. Prove that if $f : G \to C$ is differentiable at a point a in G then f is continuous at a.
2. Give the principal branch of $\sqrt{1-z}$.
3. Prove that a Mobius transformation is a composition of translation, dilation and the inversion.
4. State and prove Cauchy's theorem|
5. Define a star-shaped set and give an example.
6. What do you mean by fixed end point homotopic curves.
7. A differentiable function f on $[a,b]$ is convex iff it is increasing.
8. Find the residues of the function $f(z) = zsin\frac{1}{z}$

Part B

Answer any two questions from each unit.
Each question carries 2 weightage.

Unit I

9. Let f and g be analytic on G and Ω respectively and $f(G) \subset \Omega$ then prove that $g \circ f$ is analytic and $(g \circ f)' = g'(f(z))f'(z) \quad \forall z \in G$.
10. Let G be either the whole plane C or some open disk. If $u : G \to R$ is a harmonic function then prove that u has a harmonic conjugate.

11. If G is open and connected and $f : G \to C$ is differentiable with $f'(z) = 0$ for all z in G, then prove that f is constant.

Unit II

12. Let f be analytic in $B(a, r)$ then $f(z) = \Sigma_{n=0}^{\infty} a_n(z-a)^n$ for $| z - a |< R$ where $a_n = \frac{1}{n!} f^{(n)}(a)$ and this series has radius of convergence greater than or equal to R.

13. Let γ be a closed rectifiable curve in C. Then prove that $n(\gamma; a)$ is a constant for a belonging to a component of $G = C - \{\gamma\}$. Also prove that $n(\gamma, a) = 0$ for a in the unbounded component of G.

14. Show that if $f : C \to C$ is a continuous function such that f is analytic on $[-1, 1]$ then f is an entire function.

Unit III

15 If f has an isolated singularity at a then prove that the point $z = a$ is a removable singularity iff $lim_{z \to a}(z-a)f(z) = 0$.

16 State and prove the Casorati-Weierstrass Theorem

17 Let $f : D \to D$ be a one-one analytic map of D onto itself and suppose that $f(a) = 0$. Then prove that there is complex number c with $| c |= 1$ such that $f = c\varphi_a$.

Part C

Answer any two questions.

Each question carries 5 weightage.

18. (a) Let $\gamma : [a, b] \to C$ is piecewise smooth then prove thatγ is of bounded variation and $V(\gamma) = \int_a^b | \gamma'(t) | \, dt$.

    (b) Define $\gamma(t) = e^{it}$ for $0 \le t \le 2\pi$ and find $\int_\gamma z^n dz$ for every integer n.

19. Let G be open in C and let γ be a rectifiable path in G with initial and end points α and β respectively. If $f : G \to C$ is a continuous function with primitive $F : G \to C$ then prove that $\int f = F(\beta) - F(\alpha)$ (Where F is a primitive of$f$ when $F' = f$).

20. (a) Let $f : G \to C$ be analytic and suppose $B(a, r) \subset G, r > 0$ if $\gamma(t) = a + re^{it}$ then prove that $f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w-z} dz$ for $| z - a |$

    (b) Evaluate $\int_\gamma \frac{dz}{z^2+1}$ where $\gamma(t) = 2e^{it}, 0 \le t \le 2\pi$

21. (a)State and prove Residue Theorem

    (b) Show that for $a > 1$, $\int_0^\pi \frac{d\theta}{a+cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$

1B3N25422 (Pages : 2) Reg. No:.......................39

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2025

MMT3C13 – Functional Analysis

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer all questions. Each question has weightage 1

1. State Minkowski's inequality for sequence
2. Define a Normed space and give an example
3. Let X be a normed space, if E_1 is open in X and $E_2 \subset X$, then show that $E_1 + E_2$ is open in X
4. Define a Support functional and give an example
5. Define a Closed map and give an example
6. State the Banach – Steinhaus theorem
7. Define a Canonical embedding on a Normed space
8. Show that C_{00} is not a Banach Space

(8 x 1 = 8 weightage)

PART B

Answer any six questions, choosing two questions from each unit.
Each question has weightage 2

Unit I

9. Prove that l^p, $1 \le p < \infty$ is separable
10. State and prove the Holder's inequality for functions
11. Let Y be a closed subspace of a normed space X. For x + Y in the quotient space X/Y, $|||x+Y||| = \inf\{||x+y|| : y \in Y\}$. Then prove that $|||\ \ |||$ is a norm on X/Y

Unit II

12. State and Prove Hahn – Banach Separation theorem
13. Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X
14. Let X be a normed space and Y be a closed subspace of X. Then prove that X is a Banach space if and only if Y and X/Y are Banach spaces in the induced norm and the quotient norm, respectively

Unit III

15. Let X and Y be Normed spaces and F: X $\longrightarrow$ Y be linear. Then prove that F is an open map if and only if there exists some $\gamma > 0$ such that for every $y \in Y$, there is some $x \in X$ with $F(x) = y$ and $||x|| \leq \gamma\ ||y||$
16. State and prove open mapping theorem
17. State and prove the Resonance theorem

(6x2= 12 weightage)

PART C

Answer any two questions. Each question has weightage 5

18. Prove that the metric space L^p (E) is complete for $1 \leq p \leq \infty$
19. Let X be a Normed space over K, and let E be a nonempty open convex subset of X and Y be a subspace of X such that $E \cap Y = \emptyset$. Then prove that there is a closed hyperspace Z in X such that $Y \subset Z$ and $E \cap Z = \emptyset$. Consequently, there is some $f \in X'$ such that $f(x) = 0$ for every $x \in Y$, but Re $f(x) \neq 0$ for every $x \in E$
20. State and prove Closed graph Theorem
21. (a) Let X be a linear space over C. Regarding X as a linear space over R, consider a real-linear function u: X $\longrightarrow$ R. Define $f(x) = u(x) - i\,u(ix)$, $x \in X$. Then prove that f is a complex-linear functional on X

    (b) Let X be a linear space over K and Y be a subspace of X which is not hyperspace in X.

    If x_1 and x_2 are in X but not in Y, then prove that there is some x in X such that for all $t \in [0, 1]$, $tx_1 + (1\text{-}t)\,x \notin Y$ and $tx_2 + (1\text{-}\,t)\,x \notin Y$. If X is a normed space, then the complement Y^c is connected

(2x5 = 10 weightage)

(40)

1B3N25423 (Pages : 3) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2025

MMT3C14 – PDE & Integral Equations

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer all questions. Each question carries 1 weightage

1. Classify First order PDEs with examples.
2. Find the domain where the equation $u_{xx} + xu_{yy} + \left(3u_x + \frac{1}{2}u_y\right) = 0$ is elliptic, and the domain where it is hyperbolic.
3. Evaluate $u\left(\frac{1}{2}, \frac{1}{2}\right)$, if $u(x,t)$ is the solution of the Cauchy problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad 0 < x < \infty, \quad t > 0,$$

$$u(x,0) = \cos\left(\frac{\pi}{2}x\right); \quad 0 \le x < \infty,$$

$$u_t(x,0) = 0; \quad 0 \le x < \infty$$

4. Show that the Dirichlet problem in a bounded domain:

$$\Delta u = f(x,y); \quad (x,y) \in D$$

$$u(x,y) = g(x,y); \quad (x,y) \in \partial D$$

has at most one solution in $C^2(D) \cap C(\bar{D})$.

5. Describe Separated and Periodic boundary conditions in heat conduction problems.
6. If $f(x) = ax$ is the solution of the Fredholm integral equation $f(x) = x + \int_0^1 xt\, f(t)\, dt$, find the value of a.
7. Write the properties of Green's function.
8. Give two functions one of which is a separable kernel and the other is not. Explain why.

(8 x 1 = 8)

Part B

Answer any two questions from each unit. Each question carries 2 weightage

Unit I

9. Show that the Cauchy problem $uu_x + u_y = 1,\ u(3x, 2) = 4 - 3x$ has a unique solution. Also find the solution.

10. Suppose that the equation $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$, where $a, b, \dots, f, g$ are given functions of x and y, is elliptic in a planar domain D. Assume further that a, b, c are real analytic functions in D. Show that there exists a coordinate system (ξ, η) in which the equation has the canonical form

$$w_{\xi\xi} + w_{\eta\eta} + l_1[w] = G(\xi, \eta),$$

where l_1 is a first-order linear differential operator, and G is a function which depends on the given PDE.

11. Find a compatibility condition for the Cauchy problem

$$u_x{}^2 + u_y{}^2 = 1, \qquad u(\cos s, \sin s) = 0,\ \ 0 \le s \le 2\pi$$

Also solve the problem.

Unit II

12. Solve the problem

$$u_t = 17u_{xx}\,; \qquad 0 < x < \pi, \qquad t > 0,$$

$$u(0, t) = u(\pi, t) = 0\,; \quad t \ge 0$$

$$u(x, 0) = \begin{cases} 0\ ; & 0 \le x \le \pi/2 \\ 2\ ; & \pi/2 < x \le \pi \end{cases}$$

13. Solve the Laplace equation in the rectangle $0 < x < b,\ 0 < y < d$, subject to the Dirichlet boundary conditions $u(0, y) = f(y),\ u(b, y) = g(y),\ u(x, 0) = 0, u(x, d) = 0$.

14. Outline the energy method. Demonstrate the energy method for the Neumann problem for the vibrating string.

Unit III

15. Determine the characteristic values and corresponding characteristic functions of the integral equation

$$y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + \xi)\, y(\xi)\, d\xi$$

16. Derive the formula $\underbrace{\int_a^x \cdots \int_a^x}_{n\ times} f(x)\, dx \ldots dx = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi) d\xi$

17. Consider the homogeneous Fredholm integral equation $y(x) = \lambda \int_a^b K(x,\xi)\, y(\xi)\, d\xi$, where the kernel $K(x,\xi)$ is symmetric. Show that the eigenfunctions corresponding to distinct eigenvalues are orthogonal over (a, b).

(6 x 2 = 12)

Part C

Answer any two questions. Each question carries ***5 weightage***

18. State and prove the conditions for the uniqueness of the solutions for the Cauchy problem of general first order nonlinear PDE.

19. (a) Apply the method of separation of variables to solve the problem:

$$u_{tt} - c^2 u_{xx} = 0; \quad 0 < x < L, \qquad t > 0,$$
$$u_x(0,t) = u_x(L,t) = 0; \quad t \geq 0,$$
$$u(x,0) = f(x); \quad 0 \leq x \leq L,$$
$$u_t(x,0) = g(x); \ 0 \leq x \leq L.$$

(b) State and prove The weak maximum principle and The strong maximum principle.

20. (a) Show that, if $y(x)$ satisfies the DE $y'' + xy = 1$ and the conditions $y(0) = 0$, $y'(0) = 0$, then y also satisfies the Volterra equation $y(x) = \int_0^x \xi(\xi - x) y(\xi)\, d\xi + \frac{1}{2}x^2$.

(b) Prove that the converse of the preceding statement is also true.

21. (a) Find the canonical form of one dimensional wave equation and hence solve the equation.

(b) Determine the resolvent kernel associated with the kernel $K(x, \xi) = \cos(x + \xi)$ in the interval $(0, 2\pi)$ in the form of a power series in λ.

(2 x 5 = 10)

1B3N25424 (Pages : 2) Reg. No:............................ 41

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2025

MMT3E03 – Measure & Integration

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer all questions. Each question carries 1 weightage.

1. Let A be a subset of a set X. Determine σ-algebra generated by the set $\{A\}$.
2. Let X be a measurable space and f be a complex measurable function on X. Then prove that there exist a complex measurable function α such that $|\alpha| = 1$ and $f = \alpha|f|$.
3. Suppose $\{E_k\}$ be a sequence of measurable sets in X, such that $\sum_{k=1}^{\infty} E_k < \infty$. Then prove that almost all $x \in X$ lie in at most finitely many of the sets E_k.
4. Prove that characteristic function on open sets are lower semicontinuous.
5. Prove that every complex measure μ on any σ-algebra is bounded.
6. If μ is a positive σ-finite measure on a σ-algebra $\mathfrak{M}$ in a set X, then prove that there is a function $w \in L^1(\mu)$ such that $0 < w(x) < 1$ for every $x \in X$.
7. If $(X, \mathcal{S}, \mu)$ and $(Y, \mathcal{T}, \lambda)$ are complete measure spaces, then give an example to prove that $(X \times Y, \mathcal{S} \times \mathcal{T}, \mu \times \lambda)$ need not be complete.
8. Let $(X, \mathcal{S}, \mu)$ and $(Y, \mathcal{T}, \lambda)$ be measure spaces. Let f be $\mathcal{S} \times \mathcal{T}$ measurable function on $X \times Y$, then prove that f_x is a $\mathcal{T}$-measurable function.

Part B

Answer two questions from each unit. Each question carries 2 weightage.

Unit I

9. State and prove Lebesgue's dominated convergence theorem.
10. If $f \in L^1(\mu)$, then
    (i) prove that $\left|\int_X f d\mu\right| \leq \int_X |f| d\mu$.
    (ii) if $\left|\int_X f d\mu\right| = \int_X |f| d\mu$ then prove that there is a constant α such that $\alpha f = |f|$ $a.e$ on X.
11. State and prove Urysohn's Lemma.

Unit II

12. If $A \subset \mathbb{R}$ and every subset of A is Lebesgue measurable, then prove that $m(A) = 0$.
13. If μ is complex measure on X, then prove that $|\mu|(X) < \infty$.
14. Let μ be a complex measure on σ-algebra $\mathfrak{M}$ in X. Then prove that there is a measurable function h such that $|h(x)| = 1$for all $x \in X$and $d\mu = hd|\mu|$.

Unit III

15. If X is locally compact Hausdorff space and Φ is a bounded linear functional on $C_0(X)$ with $\|\Phi\|=1$ then prove that there exists positive linear functional Λ on $C_c(X)$ such that $|\Phi(f)| \leq \Lambda(|f|) \leq \|f\|$ for all $f \in C_c(X)$.
16. If $E \in \mathcal{S} \times \mathcal{T}$, then prove that $E_x \in \mathcal{T}$ for every $x \in$X.
17. Let $X = Y = [0,1]$ $\mu =$ Lebesgue measure on $[0,1]$and $\lambda =$ counting measure on Y.

    Define $f: X \times Y \to \mathbb{R}$ by $f(x,y) = \begin{cases} 1, & if\ x = y \\ 0, & if x \neq y \end{cases}$. Then prove that f is measurable and

    $\int_Y d\lambda(y) \int_X f(x,y) d\mu(x) \neq \int_X d\mu(x) \int_Y f(x,y) d\lambda(y)$.

Part C

Answer any two questions. Each question carries 5 weightage.

18. (i) Define complete measure.

    (ii) Define μ-completion of a σ-algebra $\mathfrak{M}$ of a measure space $(X, \mathfrak{M}, \mu)$

    (iii) Prove that every measure μ on a σ-algebra can be completed.
19. State and prove the Vitali-Caratheodory Theorem.
20. Let μ be a positive σ-finite measure on a σ-algebra $\mathfrak{M}$ in a set X, and let λ be a complex measure on $\mathfrak{M}$. Then prove that there is a unique pair of complex measures λ_a and λ_s on $\mathfrak{M}$ such that $\lambda = \lambda_a + \lambda_s$, $\lambda_a \ll \mu, \lambda_s \perp \mu$. Further prove that there is a unique $h \in L^1(\mu)$ such that $\lambda_a = \int_E hd\mu$ for every set $E \in \mathfrak{M}$.
21. Let $(X, \mathcal{S}, \mu)$ and $(Y, \mathcal{T}, \lambda)$ be measure spaces. Then prove that $\mathcal{S} \times \mathcal{T}$is the smallest monotone class which contains all elementary sets.