1M1N21109 (Pages : 2) Reg. No:…………………...

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2021

MST1C01 - Analytical Tools for Statistics – I

(2019Admission onwards)

Time: 3 hours Max. weightage : 30

PART A

Answer any four(2 weightages each)

1. Define the limit of multivariable function. Give an example.
2. Define absolute maxima and minima of multivariable function.
3. Find the real and imaginary part of the complex function: w= 3z; where z=4+2i.
4. State Morera's Theorem.
5. Define analyticity of a complex function. Is $f(z) = |z|$ analytic? Justify
6. What is singularity. Give one example.
7. Define Laplace transform. Find Laplace transform of $f(t) = e^{at}$

(2 x 4=8 weightages)

PART B

Answer any four(3 weightages each)

8. Check whether the function $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2}, (x,y) \neq (0,0) \\ 0 \qquad , (x,y) = (0,0) \end{cases}$ is continuous at origin.
9. Find partial derivatives of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
10. Obtain series expansion of the function $f(z) = \frac{1}{z-a}$ at the point $z = a$.
11. State and prove Cauchy's residue theorem.
12. State and prove Cauchy's integral theorem.
13. Prove the Taylor's theorem for multivariable function.
14. Solve the initial value problem using Laplace transformation

    $y'' - y = t$, y'(0)=1, y(0)=1

(3x 4=12 weightages)

PART C

Answer any two(5 weightages each)

15. Suppose that D is a closed and bounded set in R^n. If $f: D \to R^n$ is continuous, then show that it is uniformly continuous in D.

16. (a) Check whether Cauchy-Riemann equations satisfy for the function f(z)=z+1

    (b) f(z) = u+iv is an analytic function with u=2x(1-y). Determine the function completely.

17. Obtain the inverse Laplace transform for the function $\frac{20}{s^3-2s^2}$.

18. Find Laplace transforms of following functions

    (a) $Cos^2\theta t$

    (b) $Sinh4t.e^{-t}$

    (c) $te^t + \cos(ht + a)$

(5x2=10 weightages)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2021

MST1C02 - Analytical Tools for Statistics – II

(2019Admission onwards)

Time: 3 hours Max. weightage : 30

Part A
Short Answer Type questions
(Answer any four questions. Weightage 2 for each question)

1. Define quotient space, sub spaces and orthogonality.
2. Distinguish between Hermitian and skew Hermitian matrices.
3. Explain the working procedure of Inverse of a partitioned matrix.
4. Define minimal polynomial of a matrix.
5. Find the eigen values and eigen vectors of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

6. Write the properties of g-inverse.
7. What do you mean by extreme of quadratic forms.

(4 x 2= 8 weightage)

Part B
Short Essay Type/ problem solving type questions
(Answer any four questions. Weightage 3 for each question)

8. a) Define linear dependence

   b) Show that all subsets of a linear independent sets are linear independent.

9. a) Explain the uses of rank factorization of a matrix.

   b)Construct the specific rank factorization of a matrix for the following matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 7 & 3 & 9 \\ 1 & 5 & 3 & 1 \\ 1 & 2 & 0 & 8 \end{bmatrix}$$

10. a) Show that all eigen values of a Hermitian matrix are real.

    b) Show that a matrix A is idempotent if and only if all its eigenvalues are either 0 or 1.

11. a) What is spectral representation of a real symmetric.

    b) Show that geometric multiplicity can never exceed the algebraic multiplicity.

12. a) Explain Jordan canonical form.

b) Find Jordan canonical form of

$$A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

13. a) Explain rank and signature of quadratic form.

b) Classify the quadratic form $Q(x1, x2, x3) = x^2{}_1 + 2x^2 - 7x^2{}_3 - 4x1x2 + 8x1x3$.Then find its rank and the signature.

14. a) Solve the system of linear equation

$x + y + z + w = 13, 2x + 3y - w = -1, -3x + 4y + z + 2w = 10$ and $x + 2y - z + w = 1$.

b) Show that a homogeneous system always has at least one solution.

(4 x 3= 12 weightage)

Part C
Long Essay Type questions
(Answer any two questions. Weightage 5 for each question)

15. a) Define vector space

b) Check the set of vectors are linear independent or not,

$$S = \{(1,2,3), (2,3,4), (3,4,6)\}$$

c) Show that two vectors are linearly dependent if and only if one of them is a scalar multiple of the other

16. a) Define triangular matrix. State its properties.

b) If A is an $m \times n$ matrix and $B$is $n \times k$, then show that

rank $(A) +$ rank $(B) - n \leq$ rank (AB).

c) Define projection matrix. Also write the properties of a projection matrix.

17. a) Explain singular value decomposition.

b) What is meant by diagonalization of a matrix?

c) Show that every symmetric matrices are orthogonally diagonalizable.

18. a) Find the Moore-Penrose inverse for the following matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$.

b) Show that original matrix is the generalized inverse of its generalized inverse.

c) If A is an invertible matrix, then show that for each and every b in $\mathbb{R}^n$, the equation $Ax = b$ has the unique solution $x = A^{-1}b$.

(2 x 5= 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2021

MST1C03 - Distribution Theory

(2019 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. For any integer valued random variable X, show that $\sum_{n=0}^{\infty} t^n\ P(X \leq n) = \frac{P(t)}{1-t}$ where P(t) is the p.g.f of X.
2. If X is a non-negative continuous r.v with distribution function F(x), whose E(X) =μ exists.Show that μ = $\int_0^{\infty}[1 - F(x)]dx$.
3. Let X be an exponential random variable with parameter λ and Y=[X], integer part of X. Find the distribution of Y and identify the distribution.
4. Obtain the characteristic function of the standard Laplace distribution.
5. If E(X) exist, show that E(X)=E{E(X/Y)}.
6. Establish additive property of Chi-square distribution.
7. Find the expectation of the n^{th} order statistic taken from a random sample of size n, from the uniform distribution over (0, θ).

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer **any four** questions. Weightage 3 for each question)

8. Show that under certain conditions (to be specified) Hypergeometric distribution tends to Binomial distribution.
9. What is Pearson system of distributions? Obtain the distribution when the roots of the quadratic equation involved in the system are real and of opposite signs.
10. If (X, Y) follows Bivariate Normal with parameters (0,0,1,1, ρ) show that X+Y and X-Y are independently distributed.
11. Let X, Y be random samples from exponential distribution f(x)= exp(-x), 0<x<∞. Show that X + Y and X/Y are independently distributed. Identify their distributions.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2021

MST1C04 - Probability Theory

(2019Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

[Answer any 4 questions. Weightage 2 for each questions]

1. Define limsup and liminf of a sequence of set. Show by an example that limsup$A_n \supseteq$ liminf A_n.
2. Define a random variable and probability space induced by a random variable. Explain it with the help of an example.
3. Define distribution function of a random variable. Show that a distribution function is right continuous.
4. If X is a non-negative integer valued random variable, show that E(x) = $\sum_{k=1}^{\infty} P(X \geq k)$
5. Define independence of events. Show by an example that pair wise independence need not imply mutual independence.
6. Define convergence in law and convergence in probability. Show by an example that convergence in law need not imply convergence in probability.
7. Define a characteristic function. Show that a characteristic function is uniformly continuous.

Part B

[Answer any 4 questions. Weightage 3 for each questions]

8. Define Borel field in two ways and establish their equivalence.
9. Establish continuity property of a probability measure.
10. State and prove Liapunov inequality.
11. Let $X_1, X_2, \ldots$ be iid random variables with finite second moments.

    Let $Y_n = \frac{2}{n(n+1)} \sum_{i=1}^{n} i\, X_i$ show that $Y_n \to E(X_1)$ in probability.
12. State and prove Kolmogorov 0-1 law.
13. Show that convergence in probability implies convergence in law.
14. State and prove Lindberg-Levy Central limit theorem.

Part C

[Answer any 2 questions. Weightage 5 for each questions]

15. State and establish Jordan decomposition of a distribution function.

Obtain the Jordan decomposition of

$$F(x) = 0, x < 0$$

$$= \frac{1}{4}, 0 \leq x < 1$$

$$= \frac{3}{8}, 1 \leq x < 2$$

$$= \frac{3}{8} + \frac{5}{8} * e^{x-2}, x > 2$$

16. State and prove Khinchin's theorem.

Determine whether the WLLN holds for the independent sequence of random variables

$$P\left(x_n = \frac{n}{\ln(n)}\right) = \frac{\ln(n)}{2n} = P\left(X_n = -\frac{n}{\ln(n)}\right)$$

$$And\ P(X_n = 0) = 1 - \frac{\ln(n)}{n}, n = 2,3, ...$$

17. State and prove Kolmogrov's strong law of large numbers for iid sequence of random variables.

18. State and prove Levy continuity theorem for a sequence of characteristic functions.