

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
Second Semester B.Sc Degree Examination, March/April 2021

BMT2C02 – Mathematics – 2

(2020 Admission onwards)

Time: 2 hours

Max. Marks : 60

PART A

Answer all questions. Each question carries 2 marks.

Maximum marks from this section is 20.

1. Convert the polar coordinate $(4, -\pi)$ into cartesian coordinate.
2. Differentiate $\cosh^{-1} x^2$
3. Consider the curve $x = 3 \cos t, y = \sin t$. Find the points where the tangent is horizontal.
4. Find $\lim_{n \rightarrow \infty} \frac{3n^2+1}{n^2+n}$
5. Evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$ by using Trapezoidal rule with $n = 4$.
6. Show that the series $\sum_{i=1}^{\infty} 1 + \frac{1}{2^i}$ diverges.
7. Test the convergence of $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$.
8. Give an example of a vector space. Explain your answer
9. Check whether the set of vectors $(3,5), (2,10)$ are linearly independent or not.
10. Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 4 & 10 \end{bmatrix}$
11. Verify that the matrix $A = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ is orthogonal.
12. Evaluate the determinant of the matrix $A = \begin{bmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{bmatrix}$.

PART B

Answer all questions. Each question carries 5 marks

Maximum mark from this section is 30

13. Show that $\sinh^2 x = \frac{\cosh 2x - 1}{2}$
14. Find the length of the curve $f(x) = (x-1)^{\frac{3}{2}} + 2$ on $[1, 2]$.

15. (a) Write Alternating series test.

(b) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ converges.

16. Find the Taylor series generated by $f(x) = e^x$ at $x_0 = 0$.

17. Let $B = \{u_1, u_2\}$, where $u_1 = (3, 1)$, $u_2 = (1, 1)$ is a basis for \mathbb{R}^2 . Find an orthogonal basis for \mathbb{R}^2 using the Gram - Schmidt orthonormalization process.

18. Solve the linear system

$$x_1 + 2x_2 - x_3 = 0$$

$$2x_1 + x_2 + 2x_3 = 9$$

$$x_1 - x_2 + x_3 = 3$$

Using Gaussian elimination.

19. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{bmatrix}$

PART C

Answer any ONE question. One question carries 10 marks

20. (a) Graph the polar curve $r = \cos 2\theta$.

(b) Find the area enclosed by the cardioids, $r = 1 + \cos \theta$, $0 \leq \theta \leq 2\pi$.

21. Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & 7 \end{bmatrix}$

Also verify Cayley Hamilton theorem.

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
 Second Semester B.Sc Degree Examination, March/April 2021
BAS2C02 – Life Contingencies
 (2020 Admission onwards)

Time: 2 hours

Max. Marks : 60

PART –A(Short Answers)**(Each question carries two marks .Maximum 20 Marks)**

1. Write down an expression for t^q_x in terms of the function l_x .
2. In a certain population, the force of mortality equals 0.025 at all ages
Calculate the probability that a life aged exactly 10 will die before age 12.
3. Calculate the following probabilities using AM92 mortality $2P_{[42]}$ and $2P_{42}$.
4. Define n-year term assurance contract.
5. Explain critical illness assurance contracts.
6. Claire aged exactly 30, buys a whole life assurance with a sum assured of £50,000 payable at the end of the year of her death. Calculate the expected present value of this benefit using AM92 Ultimate mortality and 6% *pa* interest.
7. Explain temporary life annuity contracts payable in advance.
8. Calculate the value of $\ddot{a}_{[40]:20}$ using AM92 mortality and 4% *pa* interest.
9. Assuming that both lives are independently subject to AM92 mortality calculate ${}_3P_{45:41}$ and $\mu_{38:30}$.
10. Define contingent assurances and reversionary annuities.
11. Describe ${}^P_{xy}$ and t^q_{xy}
12. Express the probability $n^2_{q_{xy}}$ as an integral.

PART B**(Each question carries five marks. Maximum 30 Marks)**

13. Using ELT15 (Males) mortality, calculate the probability of a 37-year old dying between age 65 and age 75.
14. Calculate the value of $0.5 P_{58}$ using ELT15 (Females) mortality, assuming a uniform distribution of deaths between integer ages.

15. Derive expected present value and variance of whole life assurance contract if payment are made at end of years.
16. Derive the formula for the variance of an immediate whole life annuity annually in advance.
17. Calculate $A_{30:\overline{25}}$ and $\bar{a}_{30:\overline{25}}$ independently, assuming AM92 mortality and interest.
- (ii) Calculate the expected present value of a payment of £2,000 made 6 months after the death of a life now aged exactly 60, assuming AM92 Select mortality and interest.
18. Calculate:
- $P_{62:65}$
 - ${}_3q_{50:50}$

Assuming that the two lives are both independently subject to AM92 Ultimate mortality

19. Prove that ${}_nq_x^2 = \frac{1}{2} {}_nq_{\overline{x}}$

PART -C

Answer any one question and carries 10 Marks.

20. Calculate the value of ${}_{1.75}P_{45.5}$ using AM92 Ultimate mortality and assuming that
- Deaths are uniformly distributed between integer ages.
 - The force of mortality is constant between integer ages.
21. Derive the relationship between insurance payable at moment of death and the year of death.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
Second Semester B.Sc Degree Examination, March/April 2021
BST2B02 – Bivariate Random Variables & Probability Distributions
(2020 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

SECTION-A**Each question carries 2 Marks.****Maximum Marks that can be scored in this section is 25.**

1. Define marginal probability density function
2. For a random variable X , $P(X = 2) = 2P(X = 3) = P(X = 4)$. Find $V(x)$
3. Obtain the expected number of heads obtained in three tosses of a fair coin
4. The mean and variance of the binomial distribution is 8 and 3 respectively. Find the value of probability of failure
5. Define conditional variance of a random variable X given Y
6. Examine the effect of the shifting of the origin and change of scale on the m.g.f of a random variable
7. Define a degenerate random variable
8. Obtain mean deviation about mean of binomial distribution
9. Obtain second raw moment of discrete uniform distribution
10. State and prove multiplication theorem of expectation
11. First three raw moments of X are -2, 60 and -65. Obtain coefficient of skewness based on moments
12. Explain the term convergence in probability
13. If X is a Poisson variate and $P(X = 0) = P(X = 1) = k$. Find k
14. If X and Y are two independent random variables with mean 10 and 20 and variance 2 and 3 respectively. Find the variance of $3x + 4y$
15. If $f(x, y) = \frac{1}{252} x^2(y + 2)$, $x = 1, 2, 3$ and $y = 1, 2, 3, 4$ is the joint p.m.f of (X, Y) .
Find the marginal p.m.f of X

SECTION-B**Each question carries 5 Marks.****Maximum Marks that can be scored in this section is 35.**

16. If X is a random variable with the following p.m.f,

| | | | |
|--------|-----|-----|-----|
| X | -2 | 0 | 2 |
| $f(x)$ | 1/4 | 1/2 | 1/4 |

Obtain β_1 and β_2 , the measure of skewness and kurtosis

17. State Cauchy- Schwartz inequality. Use it to prove that $-1 \leq r_{xy} \leq 1$, where r_{xy} is the Karl Pearson coefficient of correlation between any two random variables.
18. Derive the m.g.f of a Poisson variate. Hence obtain its first four central moments.
19. Define raw moments and central moments. State the interrelationship between raw moments and central moments.
20. For two random variables X and Y , the joint p.d.f is given by

$$f(x, y) = 2 - x - y, 0 \leq x \leq 1, 0 \leq y \leq 1.$$
 Find $\text{cov}(x, y)$.
21. Two unbiased dice are thrown. If X is the sum of the numbers showing up on the two dice, find
 that $P\{|X - 7| \geq 3\} \leq \frac{35}{54}$. Compare this with the actual probability.
22. Let X and Y be two random variables with joint p.d.f

$$f(x, y) = 8xy, 0 \leq x \leq y \leq 1.$$
 Obtain the marginal p.d.f of X and Y .
23. Derive Poisson distribution as a limiting case of negative binomial distribution.

SECTION-C

(Answer any two Questions and each carries 10 marks)

24. (a) Explain the lack of memory property of geometric distribution.
 (b) 10% of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 100 tools chosen at random, 2 will be defective by using (i) the binomial distribution (ii) the normal approximation to the binomial.
25. Let X and Y are two random variables with joint p.d.f

$$f(x, y) = 2, 0 < x < y < 1.$$
 Find (a) correlation between X and Y

$$V\left(\frac{X}{Y} = y\right)$$
26. (a) State and prove Bernoulli's law of large numbers.
 (b) Examine whether the weak law of large numbers holds good for the sequence $\{X_n\}$ of independent random variables where $P\{X_n = \frac{1}{\sqrt{n}}\} = \frac{2}{3}$ and $P\{X_n = -\frac{1}{\sqrt{n}}\} = \frac{1}{3}$.
27. (a) If X and Y are independent Poisson variates. Obtain the conditional distribution of X given $X + Y = n$.
 (b) Let $f(x, y) = \frac{1}{8}(6 - x - y), 0 \leq x < 2, 2 \leq y < 4$. Find $P(X < 1/Y < 3)$.