2M3N21312 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2021

MMT3C11 – Multivariable Calculus & Geometry

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer ALL questions. Each question has *1weightage*

1. Prove that if A is a one to one linear operator on a finite dimensional vector space X, then range of A is all of X.
2. Let $A : R^2 \rightarrow R^2$ be defined by $A(x,y) = (x+y, x-y)$. Find the derivative of A at any point (x,y) of R^2.
3. Prove that $\varphi: R \rightarrow R$ defined by $\varphi(x) = \frac{1}{2}x + 5$ is a contraction on R.
4. Find the cartesian equation of the parametrized curve $\gamma(t) = (cos^2 t, sin^2 t)$.
5. Find the curvature of the parametrized curve $\gamma(t) = \left(\frac{4}{5}cost, 1 - sint, -\frac{3}{5}cost\right)$ at any point $\gamma(t)$.
6. Define a surface in R^3 . Give an example.
7. Show that first fundamental form for the plane $\sigma(u,v) = a + u\boldsymbol{p} + v\boldsymbol{q}$ in R^3 is $du^2 + dv^2$.
8. Define the mean curvature H and Gaussian curvature K of a surface at any point p.

$(8\times 1 = 8\, weightage)$

PART B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dimension of X is less than or equal to r.
10. Suppose f maps an open set $E \subset R^n$ into R^m, and f is differentiable at a point $x \in E$. Then prove that the partial derivatives $D_j f_i(x) (1 \le j \le n, 1 \le i \le m)$ exist at all points of E.
11. Prove that if X is a complete metric space and if φ is a contraction of X into X, then there exist one and only x∈X such that $\varphi(x) = x$

Unit II

12. Calculate the torsion of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta)\ \theta \in R$.
13. Prove that any regular plane curve γ whose curvature is a positive constant is part of a circle.
14. Let $\sigma: U \to R^3$ be a patch of a surface S containing a point $p \in S$,and let (u, v) be coordinates in U.Prove that the tangent space of S at p is the vector subspace of R^3spanned by the vectors σ_u and σ_v.

Unit III

15. If γ is a unit- speed curve on an oriented surface S, then prove that its normal curvature is $k_n = \langle\langle \dot{\gamma}, \dot{\gamma} \rangle\rangle$.
16. Let $\sigma(u, v)$ be a surface patch with first and second fundamental forms $Edu^2 + 2Fdudv + Gdv^2$ and$Ldu^2 + 2Mdudv + Ndv^2$respectively. Prove that the Gaussian curvature is $\frac{LN-M^2}{EG-F^2}$
17. Let S be a connected surface of which every point is an umbilic. Prove that S is an open subset of a plane or a sphere.

($6\times 2 = 12$ *weightage*)

PART C

Answer any Two questions. Each question has weightage5.

18. a)Prove that $L(R^n, R^m)$ is a metric space.

    b)Let Ω be the set of all invertible linear operators on R^n, then prove that Ω is an open subset of $L(R^n)$.
19. Suppose f is acontinuously differentiable mapping of an open set $E \subset R^n$ *into* R^n,$f\,'(A)$ is invertible for some $a \in E$ and $b = f(a)$. Then prove that
    a) There exist open sets U and Vsuch that $a \in U, b \in U$, f is one to one on U and $f(U) = V$
    b) If g is the inverse of f defined on V by $g(f(x))$ =x $(x \in U)$, then g is continuously differentiable on V.
20. a) Prove that any reparametrisation of a regular curve is regular.

    b) Prove that a parametrized curve has a unit speed reparametrization if and only if it regular
21. a) Compute the second fundamental form of the surface $\sigma(u, v) = (u, v, u^2 + v^2)$

    b) Define Weingarten map and hence prove that the Weingarten map is self adjoint.

($2 \times 5 = 10$ *weightage*)

2M3N21313 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2021

MMT3C12 – Complex Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions

Each question carries 1 weightage

1. Define radius of convergence R and find R for the series $\sum_{n=0}^{\infty} z^{n!}$.
2. Find the point at which the function $\tan z$ is not analytic.
3. Find the cross ratio of (1,-1,i , -i).
4. If $z = x + iy$, prove that $|e^z| = e^x$.
5. When will you say that two curves homotopic to each other,
6. Determine the nature of singularity of the function $e^{1/z}$ at $z=0$. Justify your answer.
7. Find the residues of the function $f(z) = \frac{z^2-2}{z(z-2)}$ at $z = 2$ *and* $z = 0$.
8. Define: meromorphic function. Give an example.

(8 x 1 = 8 Weightage)

Part B

Answer any two questions from each unit

Each question carries 2 weightage

UNIT I

9. For a given power series $\sum_{n=0}^{\infty} a_n\ (z-a)^n$ define the number R by $\frac{1}{R} = \limsup|a_n|^{(1/n)}$, then prove that, if $|z-a| < R$ then the series converges absolutely otherwise it diverges
10. If G is open and connected and $f: G \to C$ is differentiable with $f'(z) = 0\ \forall z \in G$, then prove that f is constant
11. State and prove symmetric principle with respect to Mobius transformation.

UNIT II

12. Show that if $f: C \to C$ is continuous function such, f is analytic on [-1, 1]. Then prove that f is an entire function.
13. State and prove open mapping theorem.
14. If G is simply connected and $f: G \to C$ is analytic in G then prove that f has a primitive in G.

UNIT III

15. If f has an isolated singularity at $z = a$ then prove that the point $z=a$ is a removable singularity *iff* $\lim_{z\to a}(z-a)f(z) = 0$.
16. State and prove Argument principal.
17. Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$

(6 x 2 = 12 Weightage)

Part C

Answer any two questions
Each question carries 5 weightage

18. Let $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ have radius or convergence $R>0$ then prove that For each $k \geq 1, \sum_{n=k}^{\infty} n(n-1) \dots (n-k+1)a_n(z-a)^{n-k}$ the series has radius of convergence R and f is infinitely differentiable on $B(a;R)$.
19. If γ is a closed rectifiable curve in G such that $\gamma \sim 0$, then prove that $n(\gamma, w) = 0 \ \forall \gamma \in C - G$ where C is the entire complex plane.
20. State and prove Laurent series development of an analytic function
21. (a)state and prove Hadamad's three cycle theorem.

    (b) Show that $\int_0^{\infty} \frac{sinx}{x} dx = \frac{\pi}{2}$

(2 x 5 = 10 Weightage)

2M3N21314 (Pages : 3) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2021

MMT3C13 – Functional Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer all questions. Each question carries a weightage 1.

1. Show by an example that the completeness of a metric space may not be shared by an equivalent metric.
2. Let$1 \leq p < \infty$. Show that the closed unit ball in l^p is not compact.
3. Let f be a nonzero linear functional on a normed space X. Prove that if $Z(f)$ is dense in X then f is discontinuous.
4. If X and Y are two nonzero normed spaces then prove that $BL(X, Y)$ is nonzero.
5. Let X be a normed space over $\mathbb{K}$ and $0 \neq a \in X$. Prove that $\|a\| = \{|f(a)|: f \in \in X', \|f\| \leq 1\}$.
6. If Y is a proper dense subset of a Banach space X then prove that Y is not a Banach space in the induced norm.
7. Let X be a metric space. If Y and Z are metric spaces, $F: X \to Y$ is continuous and $G: Y \to Z$ is closed then prove that $G \circ F: X \to Z$ is closed.
8. Prove that every orthogonal subset of a nonzero elements in an inner product space is linearly independent.

(8×1=8 Weightage)

PART B

Answer any two questions from each unit
Each question carries a weightage 2.

UNIT I

9. If $X = \mathbb{R}$ and d denotes the usual metric on X then prove that every open subset in X is a disjoint union of a countable number of open intervals in X.
10. State and prove Riesz lemma for normed spaces.
11. Let X *and* Y be normed spaces and $F: X \to Y$ be a linear map. Prove that F is continuous if and only if F is bounded on $\overline{U}(0, r)$ for some $r > 0$.

(2×2 = 4 Weightage)

UNIT II

12. Let X be a normed space over $\mathbb{K}$, and f be a nonzero linear functional on X. If E is an open subset of X then prove that $f(E)$ is an open subset of $\mathbb{K}$.
13. If every absolutely summable series is summable in a normed space X then prove that X is a Banach space.
14. Define the canonical embedding of a normed space X into its second dual X'' and prove that it is a linear isometry.

(2×2 = 4 Weightage)

UNIT III

15. Let X and Y be normed spaces. If Z is a closed subspace of X then prove that the quotient map $Q: X \to X/_Z$ is a continuous open map.
16. State and prove the bounded inverse theorem.
17. Let H *be a Hilbert space*, $\{u_1, u_2, \dots\}$ be a countable orthonormal set in H and $k_1, k_2 \dots$ belong to $\mathbb{K}$ be such that $\sum_n |k_n|^2 < \infty$. *Then prove that* $\sum_n k_n u_n$ converges in H.

(2×2 = 4 Weightage)

Part C

Answer any two question. Each question carries a weightage 5

18. (a)For $1 \leq p < \infty$, prove that the metric space l^p is separable.

    (b) Let X be normed space over $\mathbb{K}$ and $E_1,\ E_2$ be disjoint nonempty convex subsets of X, where E_1 is open in X. Prove that there exists an f in X' and $t \in \mathbb{R}$ such that $Ref(x_1) < t \leq Ref(x_2)$ for all $x_1 \in E_1$ and for all $x_2 \in E_2$.

19. (a) Let X and Y be normed spaces and $F: X \to Y$ be a linear map such that the range $R(F) of\ F$ is finite dimensional. Prove that F is continuous if and only if the zero space $Z(F)$ of F is closed in X.

    (b)Let $X$and Y be a Banach spaces and $F: X \to Y$ be a linear map which is closed and surjective. Prove that F is continuous and open.

20. (a) Let X and Y be normed spaces and $X \neq 0$. Prove that $BL(X, Y)$is a Banach space if and only if Y is a Banach space.

    (b).State and prove Schwarz inequality for inner product spaces.

21. (a) Prove that a Banach space cannot have a denumerable basis.

    (b) What is meant by a Schauder basis?. Illustrate.

    (c)Prove that if a normed space has a schauder basis then it is separable.

(2×5 = 10 Weightage)

2M3N21315 (Pages : 3) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2021

MMT3C14 – PDE & Integral Equations

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each question carries 1 weightage

1. Solve the equation $xu_x + (x+y)u_y = 1$ with the initial condition $u(1, y) = y$.
2. Find the domains where the following equation is hyperbolic, parabolic and elliptic.
$$u_{xx} + 2u_{xy} + [1 - q(y)]u_{yy} = 0, \text{ where } q(y) = \begin{cases} -1; & y < -1 \\ 0 ; & |y| \le 1 \\ 1 ; & y > 1 \end{cases}$$
3. Evaluate $u(1,4)$, if $u(x,t)$ is the solution of the Cauchy problem
$$u_{tt} - u_{xx} = 0 ; 0 < x < \infty, t > 0,$$
$$u(0,t) = t^2 ; t > 0,$$
$$u(x,0) = x^2 ; 0 \le x < \infty,$$
$$u_t(x,0) = 6x ; 0 \le x < \infty,$$
4. Let $u(x,y)$ be a harmonic function in a domain D, show that $u \in C^\infty(D)$.
5. Describe Separated and Periodic boundary conditions in heat conduction problems.
6. Consider the kernel $K(x, \xi) = x + \xi$ in the interval$(0, 1)$. Find the resolvent kernel associated with the kernel in the form of a power series in λ.
7. Define separable kernel. Is $e^{x\xi}$ separable? Justify your answer.
8. Show that the kernel $K(x, \xi) = \sin x . \cos \xi$ has no characteristic numbers associated with $(0, 2\pi)$.

(8 x 1 = 8 weightage)

Part B

Answer any two questions from each unit. Each question carries 2 weightage

Unit I

9. Find a coordinate system that transforms the equation $u_{xx} + 6u_{xy} - 16u_{yy} = 0$ into its canonical form. Hence find its general solution.

10. Find a compatibility condition for the Cauchy problem

$$u_x{}^2 + u_y{}^2 = 1, \qquad u(\cos s, \sin s) = 0, \qquad 0 \le s \le 2\pi$$

Also solve the problem.

11. Solve the Cauchy problem

$$u_{tt} - 9u_{xx} = e^{x} - e^{-x}\,;\; -\infty < x < \infty, t > 0$$

$$u(x,0) = x,\; u_t(x,0) = \sin x\,;\; -\infty < x < \infty.$$

Unit II

12. Solve the problem

$$u_{tt} - 4u_{xx} = 0\,;\; 0 < x < 1, t > 0,$$

$$u_x(0,t) = u_x(1,t) = 0\,;\; t \ge 0$$

$$u(x,0) = \cos^2 \pi x;\; 0 \le x \le 1$$

$$u_t(x,0) = \sin^2 \pi x \,.\cos \pi x\,;\; 0 \le x \le 1$$

13. Solve the Laplace equation $\nabla^2 u = 0$ in the square $0 < x, y < \pi$, subject to the boundary condition $u(x,0) = u(x,\pi) = 1$, $u(0,y) = u(\pi,y) = 0$.

14. Using the energy method prove uniqueness of the problem

$$u_t - ku_{xx} = F(x,t)\,;\; 0 < x < L, t > 0,$$

$$u(0,t) = a(t), \qquad u(L,t) = b(t)\,;\; t \ge 0$$

$$u(x,0) = f(x)\,;\; 0 \le x \le L$$

Unit III

15. For the homogeneous Fredholm integral equation $y(x) = \lambda \int_a^b K(x,\xi)\, y(\xi)\, d\xi$, with symmetric kernel $K(x,\xi)$,show that the characteristic functions corresponding to distinct characteristic numbers are orthogonal over (a,b).

16. Write a note on Neumann series.

17. Derive the formula $\underbrace{\int_a^x \cdots \int_a^x}_{n\ times} f(x)\, dx \ldots dx = \frac{1}{(n-1)!}\int_a^x (x-\xi)^{n-1} f(\xi) d\xi$

(6 x 2 = 12 weightage)

Part C

Answer any two questions. Each question carries 5 weightage

18. (a) Derive d'Alembert's formula for the Cauchy problem for the one-dimensional homogenous wave equation.

    (b) State and prove the existence and uniqueness theorem for the Cauchy problem of first order Quasilinear equations.

19. (a) Solve the Heat equation with homogeneous boundary conditions by method of separation of variables.

    (b) State and prove The weak maximum principle and The strong maximum principle.

20. (a) Using Green's function transform the boundary value problem

    $y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$to an Integral equation.

    (b) If y satisfies the Volterra equation$y(x) = \int_0^x \xi(\xi - x) y(\xi)\, d\xi + \frac{1}{2}x^2$,show that y also satisfies an initial value problem. Is the converse is true? Justify your answer.

21. (a) Using Lagrange method analyze the problem

    $$xuu_x + yuu_y = x^2 + y^2;\ x > 0,\ y > 0,\ u(x,x) = \sqrt{2}x.$$

    Determine whether there exists a unique solution, infinitely many solutions or no solution at all. If there is a unique solution, find it; if there are infinitely many solutions, find at least two of them.

    (b) Prove that the equation $y(x) = \frac{1}{\pi}\int_0^{2\pi} \sin(x+\xi)\ y(\xi)\, d\xi + F(x)$ possesses no solution when $F(x) = x$, but that it possesses infinitely many solutions when $F(x) = 1$. Determine all such solutions.

(2 x 5 = 10 weightage)

2M3N21316 (Pages : 3) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2021

MMT3E03 – Measure and Integration

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Section A

Answer ALL questions. Each question carries 1 weight.

1. Prove or disprove : "the set $\mathfrak{M}$ of all finite subsets of a countable set X is a σ-algebra.

2. Let μ be a positive measure on a σ-algebra $\mathfrak{M}$ and $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$ be sets in $\mathfrak{M}$.

   If $A = A_1 \cap A_2 \cap A_3 \cap \cdots$ and if $\mu(A_1) < \infty$, then prove that $\mu(A_n) \longrightarrow \mu(A)$ as $n \longrightarrow \infty$.

3. Define Lebesgue integral of a measurable function over a set in a σ-algebra.

   If $0 \leq f \leq g$, then prove that $\int_E f\,d\mu \leq \int_E g\,d\mu$.

4. Suppose $f : X \longrightarrow [0,\infty]$ is measurable, $E \in \mathfrak{M}$ and $\int_E f\,d\mu = 0$.

   Then prove that $f = 0$ a.e. on E.

5. Let μ be a complex measure $|\mu|$ its total variation measure.

   Prove that $\|\mu\| = |\mu|(X)$ defines a norm on the vector space of all complex measures.

6. What is meant by mutually singular measures ?

   Let λ_1 and λ_2 be measures and μ be a positive measure.

   If $\lambda_1 \perp \mu$ and $\lambda_2 \perp \mu$, then prove that $(\lambda_1 + \lambda_2) \perp \mu$.

7. Explain the Hahn Decomposition with all necessary details.

8. State Fubini's theorem and explain all the terms involved. **$8 \times 1 = 8$ Weights.**

Section B

Answer any TWO questions from each unit. Each question carries 2 weights.

UNIT I

9. Define Borel sets, F-σ sets and G-δ sets. Give example of an F-σ set that is not closed. Also give example of a G-δ set that is not open.

10. State and prove Fatou's Lemma.

11. Explain the concept of "a property almost everywhere" with respect to a measure. In the collection of measurable functions, prove that, equality almost everywhere is an equivalence relation.

UNIT II

12. Let X be a locally compact, σ-compact Hausdorff space. Let $\mathfrak{M}$ be a σ-algebra containing all Borel subsets of X. Let μ be a regular Borel measure on $\mathfrak{M}$. If $E \in \mathfrak{M}$ and $\varepsilon > 0$, prove that there is a closed set F and an open set V such that $F \subset E \subset V$ and $\mu(V - F) < \varepsilon$.

13. Prove that every set of positive measure has non-measurable subsets.

14. Suppose μ and λ are measures on a σ-algebra $\mathfrak{M}$, μ is positive and λ is complex. Prove that the following two statements are equivalent.

    a) $\lambda \ll \mu$

    b) To every $\varepsilon > 0$ corresponds a $\delta > 0$ such that $|\lambda(E) < \varepsilon$for all $E \in \mathfrak{M}$ with $\mu(E) < \delta$.

UNIT III

15. Let $(X, \mathscr{S})$ and $(Y, \mathscr{T})$ be measurable spaces. Prove that $\mathscr{S} \times \mathscr{T}$ is the smallest monotone class that contains all elementary sets.

16. Let $(X, \mathscr{S})$ and $(Y, \mathscr{T})$ be measurable spaces. If $E \in \mathscr{S} \times \mathscr{T}$, then prove that $E_x \in \mathscr{T}$ and $E^y \in \mathscr{S}$.

17. Define Lebesgue Point of an $f \in L^1(\mathbf{R}^k)$. Prove that for an $f \in L^1(\mathbf{R}^k)$, almost every $\overline{x} \in \mathbf{R}^k$ is a Lebesgue Point of the f.

$6 \times 2 = 12$ Weights.

Section C

Answer any TWO questions. Each question carries 5 weights.

18. a) Define a measurable function.

    b) Prove that every measurable non-negative extended real valued function is the limit of an increasing sequence of simple measurable functions.

19. State and prove Lusin's theorem.

20. a) Prove that the total variation measure of a complex measure is a positive measure.

b) If μ is a complex measure on a non-empty set X, prove that $|\mu|(X) < \infty$.

21. Let $(X, \mathscr{S}, \mu)$ and $(Y, \mathscr{T}, \lambda)$ be σ-finite measure spaces, f an $\mathscr{S} \times \mathscr{T}$-measurable function on $X \times Y$.

a) Prove that for each $x \in X$ the function f_x defined as $f_x(y) = f(x, y)$ is a $\mathscr{T}$-measurable on Y.

b) Let $Q \in \mathscr{S} \times \mathscr{T}$. If $\phi(x) = \lambda(Q_x)$ and $\psi(y) = \mu(Q^y)$ for every $x \in X$ and $y \in Y$ then prove that ϕ is $\mathscr{S}$-measurable and ψ is $\mathscr{T}$-measurable. Also prove that

$$\int_X \phi \, d\mu = \int_Y \psi \, d\lambda.$$

2×5 = 10 Weights.