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1M4M21557 (Pages : 3) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MST4C14 – Multivariate Analysis

(2019 Admission Onwards)

Time: 3 hours Max. Weightage : 30

Part- A

Answer any 4 questions.

Each question carries 2 weightage.

1. A bivariate normal density is given below. Find its mean vector and dispersion matrix.

$$f(x,y) = \frac{1}{2\pi}\exp[-\tfrac{1}{2}(2x^2 + y^2 + 2xy - 22x - 14y + 65)].$$

2. Define partial correlation and multiple correlation coefficients.

3. Let $\underset{\sim}{X}_\alpha \sim N_p(\mu, \Sigma)$, $\alpha = 1, 2, \ldots N$ and $\bar{\underset{\sim}{X}} = \frac{1}{N}\sum_{\alpha=1}^{N} X_\alpha$, derive the distribution of $\bar{\underset{\sim}{X}}$.

4. What is generalized variance? Write short note on the distribution of sample generalized variance based on multinormal distribution.

5. Describe Mahalnobis D^2 - statistic. What are its uses?

6. What is Bayes classification rule?

7. Define the terms: (i) Factor loadings and (ii) Factor rotation.

(4×2= 8 weightage)

Part - B

Answer any 4 questions.

Each question carries 3 weightage.

8. Let $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$ and $C=(c_{ij})_{p\times p}$ be a non-singular matrix with real elements. If $\underset{\sim}{Y} = C\underset{\sim}{X}$, derive the distribution of Y.

9. If $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix} \sim N_p(\mu, \Sigma)$, where $\underset{\sim}{X}^{(1)}$ contains the first q components of $\underset{\sim}{X}$ and $\underset{\sim}{X}^{(2)}$ the remaining, write down the conditional distribution of $\underset{\sim}{X}^{(1)}$ given $\underset{\sim}{X}^{(2)} = \underset{\sim}{x}^{(2)}$. Using this conditional distribution describe how do you find the regression coefficients of $\underset{\sim}{X}^{(1)}$ on $\underset{\sim}{X}^{(2)} = \underset{\sim}{x}^{(2)}$ and partial correlation coefficients of $\underset{\sim}{X}^{(1)}$ on $\underset{\sim}{X}^{(2)} = \underset{\sim}{x}^{(2)}$.

10. Write down the maximumum likelihood estimators of mean vector and dispersion matrix of multivariate normal distribution. Check whether the maximumum likelihood estimators are (i) unbiased (ii) consistent and (iii) sufficient for the parameters.

11. Let $A \sim W_p(n, \Sigma)$ and A be partitioned as $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ with usual notations. Derive the marginal distribution of A_{11}.

12. Let $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$, derive the likelihood ratio test for testing $H_0 : \Sigma = \Sigma_0$, where Σ_0 is a given positive definite matrix.

13. Describe how do you classify an observation $\underset{\sim}{X}$ into one of two multivariate normal populations when the parameters are unknown.

14. What are principal components? Describe how do you find the principal components of $\underset{\sim}{X}$, when the dispersion matrix of $\underset{\sim}{X}$ is unknown.

(4×3= 12 weightage)

Part - C

Answer any 2 questions.

Each question carries 5 weightage.

15. a) Let $\underset{\sim}{X} \sim N_3(\mu, \Sigma)$ where $\mu' = (1 \;\; 0 \;\; 1)$ and $\Sigma = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 5 & -3 \\ 3 & -3 & 5 \end{pmatrix}$. Let $\underset{\sim}{X}^{(1)} = \begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $X^{(2)} = X_2$. Find the conditional distribution of $\underset{\sim}{X}^{(1)}$ given $X_2 = x_2$.

b) If $\underset{\sim}{X}$ is a multivariate normal random vector, derive the condition for independence of a linear form $l'\underset{\sim}{X}$ and a quadratic form $\underset{\sim}{X}'A\underset{\sim}{X}$.

16. a) Let $\underset{\sim}{X}_\alpha \sim N_p(\mu, \Sigma)$, $\alpha = 1, 2, \ldots, N$ where μ is known, derive the maximumum likelihood estimator of Σ.

b) Derive the sampling distribution of partial correlation coefficient in the null case.

7. a) Define Hotelling's T^2- statistic. Prove that T^2 is invariant under non-singular transformation.

b) If $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$ and $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix}$ where $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are two sub vectors of $\underset{\sim}{X}$, derive the likelihood ratio test for testing the hypothesis that $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are independent.

8. a) Describe (i) Bayes and (ii) admissible procedures of classifications. With usual notation prove that if

$$P\{\frac{P_1(x)}{P_2(x)} = k|\pi_i\} = 0, i = 1, 2; \quad 0 \leq k \leq \infty,$$

then every admissible procedure is a Bayes procedure.

b) Describe Fisher's linear discriminant function. What is its distribution?

(2×5= 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MST4E01 – Operations Research – I

(2019 Admission onwards)

ime: 3 hours Max. Weightage : 30

PART A (Short Answer type)

(Answer any 4 questions. Weightage 2 for each question)

1. Define

   a) basic feasible solution b) non-degenerate basic feasible solution

   c) slack and artificial variables.

2. Write the dual of the LPP:

$$\text{Maximize } W = x + 2y + 2z$$

$$\text{Subject to } x + y + 2z <= 12$$

$$2x + y + 5z = 20$$

$$x + y - z >= 8$$

$$x >= 0;\ y >= 0,\ z \text{ unrestricted.}$$

3. Give the mathematical model of the assignment problem.
4. What is the importance of sensitivity analysis?
5. Give any two applications of zero-one programming.
6. What are pure and mixed strategies? Give one example each.
7. What is Dominance property in pay-off matrix?

(4 x 2 = 8 weightage)

PART B (Short Essay type questions)

(Answer any 4 questions. Weightage 3 for each question)

8. Prove that the set of feasible solutions of an LPP is a convex set and the vertices of the set are basic feasible solutions.
9. Prove that an LPP has an optimal solution if and only if its dual has an optimal solution, and in this case their optimal values are equal.
10. Explain the algorithm to find the optimal solution for a Transportation problem. You have to address the case of degeneracy also.
11. Discuss branch and bound algorithm to solve an Integer Programming problem.
12. Give an algorithm to solve a 3-machine n-job sequencing problem. You have to mention the assumptions you use, if any.

13. Show that a game problem can be formulated as an LPP and the problem for Player 1 is the dual of the problem of Player 2.

14. Solve the game graphically:

	Player B			
Player A	B1	B2	B3	B4
A1	2	2	3	-2
A2	4	3	2	6

(4 x 3 = 12 weightage)

PART C (Long Essay type questions)
(Answer any 2 questions. Weightage 5 for each question)

15. Solve using two phase simplex method:

MinimizeZ= 6x + 3 y

Subject to: x + y >= 1

2x – y >=1

3 y <= 2

x, y >=0

16. Solve: Maximize Z = 15 x + 45 y

Subject to x + 16 y <= 250; 5 x + 2 y <= 162. ; y <=50

and x, y >=0;

Determine the effect of discrete changes in the cost coefficients on the optimality of the optimum solution.

17. Solve using cutting plane method.

Maximize W = 3 x + y + 3 z

Subject to : - x + 2 y + z <= 4

4 y – 3 z <= 2

x – 3 y + 2 z <= 3.

x, y, z >=0 and integers.

18. Solve the two person game with pay-off matrix

$$P = \begin{pmatrix} 2 & -2 & -4 \\ -3 & 3 & -2 \\ -6 & -7 & 1 \end{pmatrix}$$

(2 x 5= 10 weightage)