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1M4M18198 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

MT4C15 – Functional Analysis II

(2016 Admission onwards)

Max. Time: 3 hours Max. weightage: 36

PART A

Answer all questions

Each question carries 1 weightage

1. State open mapping theorem.
2. Let X be a normed space over K and $A \in BL(X)$, then prove that $\sigma(A^{-1}) = \{k^{-1} : k \in \sigma(A)\}$.
3. Let X and Y be normed spaces and $F \in BL(X,Y)$. Define transpose of F.
4. Show that every reflexive normed space is a Banach space.
5. If X is a finite dimensional strictly convex normed space. Prove that X is uniformly convex.
6. Let $\{u_1, u_2, ...\}$ be an orthonormal set in an inner product space X and $f \in X'$. Prove that $\sum_n |f(u_n)|^2 \le \|f\|^2$.
7. Let H be a Hilbert space. $A \in BL(H)$ be irivertible, prove that $(A^*)^{-1} = (A^{-1})^*$.
8. Let H be a Hilbert space and $A, B \in BL(H)$ be self-adjoint prove that AB is self-adjoint if and only if A and B commute.
9. Let H be a Hilbert space and $A \in BL(H)$, then prove that $k \in \sigma(A)$ if and only if $\bar{k} \in \sigma(A^*)$
10. Let $A \in BL(H)$ be normal. Prove that eigen vectors corresponding to distinct eigen values are orthogonal.
11. Prove that numerical range and $A \in BL(H)$ need not be closed.
12. Let H = K^2 and the operator A on H defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Find $\sigma(A)$.
13. Let $A \in BL(CH)$ be a Hilbert-Schmidt operator, then prove that A^* is a Hilbert- Schmidt operator.
14. Define a compact linear map on a normed space

(14 x 1 = 14 weightage)

PART B

Answer any seven questions
Each question carries 2 weightage

15. Let X and Y be normed spaces and $F : x \to y$ be linear. Prove that F is an open ma there exists some t> 0 such that for every $y \in y$, there is some $x \in x$ with F(x)=y an $\|x\| \leq r\|y\|$.

16 State and prove Bounded Inverse theorem.

17 Let X be a Banach space. Prove that the set of all invertible operators is open in BL

18 Let X and Y be normed spaces and $F \in BL(X,Y)$. Prove that $F''J_x = J_y F$, where and J_y are the canonical embedding of X and Y into X'' *and* Y'' respectively.

19 For $1 < p < \infty$, prove that l^p is reflexive.

20. Let H be a Hilbert space and $f \in H'$. Prove that there is a unique $y \in H$ such that $f(x) = <x, y>, x \in H$

21. Let H be a Hilbert space and $A \in BL(H)$. Find a relation connecting adjoint A^* an transpose A' *of* A.

22 If $A \in BL(H)$ and A^* is bounded below, prove that R(A)=H.

23 IF $A \in BL(H)$ be normal, prove that $\|A^2\| = \|A^* A\| \|A\|^2$

24. Let $H \neq \{0\}$ *and* $A \in BL(H)$ be compact and self- adjoint, prove that $\|A\|$ OR $-\|A$ eigen value of A.

(7 x 2 = 14 weig

PART C

Answer any two questions
Each question carries 4 weightage

25 State and prove closed Graph theorem.

26 Let X be a normed space and $A \in BL(X)$ be of finite rank. Prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$

27. Let $A \in BL(H)$ be self- adjoint. Prove that A or –A is a positive operator if and or $|<Ax, y>|^2 \leq <Ax, x> <Ay, y>$ for all $x, y \in H$.

28 State and prove finite dimensional spectral theorem for self adjoint or normal oper

(2 x 4 = 8 wei

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

MT4C16 – Differential Geometry

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

Part A

Answer All questions.

Each question carries 1 weightage

1. Sketch the vector field on R^2: Where $X(p) = -p$
2. Show that the graph of any function $f: R^n \to R$ is a level set for some function $F: R^{n+1} \to R$.
3. Sketch the cylinder $f^{-1}(0)$ where $f(x_1, x_2) = x_1$.
4. Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 - x_2$
5. Let X and Y be smooth vector fields along parametrized curve $\alpha: I \to R^{n+1}$ and let $f: I \to R$ be smooth function along α. Verify that $(X \dot{+} Y) = \dot{X} + \dot{Y}$
6. Show that the unit n-sphere $x_1^2 + x_2^2 + \cdots x_{n+1}^2 = 1$ is connected for $n > 1$
7. Sketch the cylinder over the graph $f(x) = sinx$
8. Show that the two orientations on the unit n-sphere $x_1^2 + x_2^2 + \cdots x_{n+1}^2 = 1$ are given by $N_1(p) = (p, p)$ and $N_2(p) = (-p, p)$
9. Compute $\nabla_v f$ where $f: R^{n+1} \to R$ and $v \in R_p^{n+1}, p \in R^{n+1}$ are given by

$$f(x_1, x_2) = 2x_1^2 + 3x_2^2, v = (1,0,2,1)$$

10. Let S be an n-surface in R^{n+1}.let $\alpha: I \to S$ be a parametrized curve and let X and Y are vector fields tangent to S along α. Verify that $(fX)' = f'X + fX'$
11. Show that the two orientations on the unit n-sphere $x_1^2 + x_2^2 + \cdots x_{n+1}^2 = 1$ are given by $N_1(p) = (p, p)$ and $N_2(p) = (-p, p)$
12. Find the length of the parametrized curve $\alpha: I \to R^{n+1}$ given by

$$\alpha(t) = (t^2, t^3), I = [0,2], n = 1$$

13. Find the Gaussian curvature $K: S \to R$ where S is given by

$$\left(\frac{x_1^2}{a^2}\right) + \left(\frac{x_2^2}{b^2}\right) - \left(\frac{x_3^2}{c^2}\right) = 0,$$

14. Define oriented n-surface. Give an example. **(14 x 1 = 14 weightage)**

Part B

Answer any seven questions.

Each question carries 2 weightage.

15. Find the integral curve through p=(1,1) of the vector field $X(x_1, x_2) = (x_2, x_1)$

16. Let U be an open set in R^{n+1} and let $f: U \to R$ be smooth. Let $p \in U$ be a regular point of f, and let $c = f(p)$. Then show that the set of all vectors tangent to $f^{-1}($ p is equal to $[\nabla f(p)]^{\perp}$.

17. Show that the set S of all unit vectors at all points of R^2 forms a 3-surface in R^4.

18. Sketch the tangent space at a typical point of the level set $f^{-1}(1)$ where
$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$$

19. Prove that, in an n-phase, parallel transport is path independent.

20. Let S be a compact connected oriented n- surface in R^{n+1}. Then show that the G Kronecker curvature K(p) of S at p is non-zero for all $p \in S$ if and only if the sec fundamental form s_p of S at p is definite for all $p \in S$

21. Show that the unit n- sphere $x_1^2 + x_2^2 + ... + x_{n+1}^2 = 1$ is connected if n > 1.

22. Let S be an oriented n-surface in R^{n+1} which is convex at $p \in S$.Show that the s fundamental form of S at p is semi-definite.

23. Let S be the unit n-sphere $\sum_{i=1}^{n+1} x_i^2 = 1$oriented by the outward unit normal vect field.Prove that the Weingarten map of S is multiplication by -1.

24. State and prove the Inverse function theorem for n-surfaces. **(7 x 2 = 14 weigh**

Part C

Answer any two questions.

Each question carries 4 weightage

25. Let S be a compact connected oriented n-surface in R^{n+1} exhibited as a level set $f^{-1}(c)$ of a smooth function $f: R^{n+1} \to R$ with $\nabla f(p) \neq 0$ for all $p \in S$. Then s that the Gauss map maps S onto the unit sphere S^n.

26. Let S be an n-surface in R^{n+1}, let $p \in S$and $\in Sp$. Then show that there exist ar interval containing 0 and a geodesic $\alpha: I \to S$ such that

    (i) $\alpha(0) = p$ and $\dot{\alpha}(o) = v$

    (ii) If $\beta: \tilde{I} \to S$ is any other geodesic in S with $\beta(0) = p$ and $\dot{\beta}(0) = v$, th $\tilde{I} \subset I$ and $\beta(t) = \alpha(t)$ or all $t \in \tilde{I}$

27. (i) Let S be an n-surface in R^{n+1}, oriented by the unit vector field N. Let $p \in Sa$ $v \in Sp$. Then show that for every parametrized curve $\alpha: I \to S$, with $\dot{\alpha}(t_0) =$ some $t_0 \in I, \ddot{\alpha}(t_0). N(p) = L_p(v). v$

    (ii)Show that the Weingarten mapL_p is self adjoint.

28. Let S be an n –surface in R^{n+1} and let $p \in S$. Then there exists an open set V al in R^{n+1} and a parametrized n- surface $\varphi: U \to R^{n+1}$ such that φ is one to one m from U onto $V \cap S$.

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Name:..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

MT4E03 – Measure & Integration

(2016 Admission onwards)

Max. Time: 3 hours Max. weightage: 36

Part A

(Answer all questions (1-14). Each questions has one weightage .)

1. Define a continuous function and give an example.
2. Give an example of a Borel function.
3. Does there exist an infinite σ - algebra which has only countably many members? Prove your claim.
4. Show that the supremum of any collection of lower semicontinuous function is lower semicontinuous .
5. Show that the range of any $f \in C_c(X)$ is a compact subset of the complex plane.
6. Define regular measure and give an example.
7. Define σ -finite measure and give an example.
8. Is every Lebesgue measurable set a Borel set ? Prove your claim.
9. Is it true that every compact subset of $\mathbb{R}^1$ is the support of a continuous function? Prove your claim.
10. Let μ *and* λ be complex measures on the same σ - algebra *M*. Prove or disprove that $\mu + \lambda$ is a complex measure.
11. What do you mean by Jordan decomposition of a real measure.
12. What do you mean by polar representation of a complex measure.
13. Explain the meaning of " $L^q(\mu)$ is isometrically isomorphic to the dual space of $L^p(\mu)$ "
14. Define measurable rectangle and give an example.

(14 x 1 = 14 weightage)

Part B

(Answer any seven from the following ten questions (15-24).
Each questions has weightage 2.)

15. Define positive measure and give an example. Also prove that positive measure is monotonic and finitely additive.

16. State and prove Fatou's Lemma.

17. If $f \in L^1(\mu)$ then prove that $\left|\int_X f d\mu\right| \le \int |f| d\mu$.

18. Suppose $f : X \to [0, \infty]$ is measurable and $E \in M,\ and\ \int_E \int d\mu = 0$. Then prove t
$f = 0$.

19. Prove that if f is a real function on a measurable space X such that $\{x : f(x) \ge r\}$ is measurable for every rational r, then prove that f is measurable.

20. In a topological space show that a closed subset of a compact space is compact.

21. Let X be a locally compact Hausdorff space in which every open set is σ - compac λ be any positive measure on X such that $\lambda(K) < \infty$ for every compact set K. Then prove that λ is regular.

22. State and prove Lusin's theorem.

23. Suppose μ is a positive measure on M, $g \in L^1(\mu),\ and\ \lambda(E) = \int_E g d\mu,\ (E \in M)$ the prove that $|\lambda|(E) = \int_E |g| d\mu,\ (E \in M)$.

24 If $f \in L^1(\mathbb{R}^k)$, then , then prove that almost every $x \in \mathbb{R}^k$ is a Lebesgue point of f.

(7 x 2 = 14 weig

Part C

(Answer any two from the following questions (25-28) .
Each questions has weightage 4.)

25. (a) Give an example of a σ - compact space and prove your claim.
(b) State and prove the Vitali-Caratheodory theorem.

26. State and prove the Lebesgue-Radon-Nikodym Theorem.

27. Let I = [a, b], let $f : I \to \mathbb{R}^1$ be continuous and nondecreasing. Then prove that eac the following three statements about f implies the other two:
a) f is AC on I.
b) f maps sets of measure 0 to sets of measure 0.
c) f is differentiable a.e. on I, $f \in L^1,\ and\ f(x) - f(a) = \int_a^x f'(t)dt (a \le x \le b)$.

28. (a) Prove or disprove that the product measure is complete.
(b) Define convolution product of two $L^1(\mathbb{R}^1)$ functions and prove that it is again i $L^1(\mathbb{R}^1)$.

(2 x 4 = 8 weig

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

MT4E07 – Computer Oriented Numerical Analysis

(2016 Admission onwards)

Max. Time: 1 ½ hours Max. weightage: 18

Part A (Short Answer Questions)

(Answer all questions. Each question has weightage 1)

1. Draw a flow chart to find the H.C.F of two numbers.
2. Write a C++ program to evaluate Euler totient function.
3. What is meant by white space in C++ program.
4. Write a C++ program that uses for loop.
5. Write a short note on user defined functions in C++.
6. Write an algorithm to find the biggest from among n numbers.

(6x1=6 weightage)

Part B

(*Answer any four from the following six questions. Each question has weightage 2*)

7. Write a C++ program that uses arrays.
8. Write an algorithm to find the $\int_a^b f(x)dx$ using trapezoidal rule.
9. Explain Lagrange's interpolation algorithm.
10. Write an algorithm to solve the initial value problem using Runge Kutta method.
11. Write an algorithm for finding the integral of tabulated values.
12. Write an algorithm to find the dominant eigenvalue of a square matrix.

(4x2=8 weightage)

Part C

Answer any one from the following two questions. Each question has weightage 4

13. Write an algorithm and C++ programme to solve a system of equation having n equation and n unknowns.
14. Write an algorithm and C++ programme to find the integral using tabulated values by the method of Simpson rule.

(1x4=4 weightage)