1M3N18201 (Pages : 3) Reg. No: ....

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2018

MT3C14 – FUNCTIONAL ANALYSIS

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

PART A

Answer all questions (Each question has Weightage 1.)

1. Show that $d_p(x,y) = \left(\sum_{j=1}^{\infty} |x(j) - y(j)|^p\right)^{1/p}$ defines a metric on ℓ^p.
2. Prove that d is stronger than d′ if and only if any subset of X which is open with respect to d′ is also open with respect to d.
3. Give an example for an incomplete normed space.
4. If (x_n) and (y_n) are sequences in a metric space (X, d) converging to x and y respectively. Show that the sequence $(\, d(x_n, y_n)\,) \to d(x, y)$ in R with usual metric.
5. Justify: Completeness is not preserved under homeomorphisms.
6. Show that a normed space has no proper, open subspaces.
7. If X is a normed space over K and if $x_1, x_2 \in X$ with $x_1 \neq x_2$ Then show that there exists a $g \in X'$ such that $g(x_1) \neq g(x_2)$.
8. Give an example for a closed map which is not continuous.
9. If X is a normed space and if Z is a closed subspace of X, then show that the quotient map Q from X to X \ Z is continuous and open.
10. Prove that c_{00} is not a Banach space under any norm.
11. In an inner product space X, prove that for all x, y ∈ X,

    $$\|x+y\|^2 + \|x-y\|^2 = 2\,(\,\|x\|^2 + \|y\|^2\,).$$
12. Show that among all the p- norms, $\|.\|_p$ $1 \le p \le \infty$, on K^n $(n \ge 2)$, only the norm $\|.\|_2$ is induced by an inner product.
13. If $\{x_1, \ldots, x_n\}$ is an orthogonal set in an inner product space X, then show that

    $$\|x_1 + x_2 + \ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots + \|x_n\|^2.$$
14. Define an orthonormal basis for a Hilbert space.

(14 × 1 = 14 Weightage)

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MT3C14 – FUNCTIONAL ANALYSIS

(2017 Admission onwards)

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13. If $\{x_1, \dots, x_n\}$ is an orthogonal set in an inner product space X, then show that

$$\|x_1 + x_2 + \dots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \dots + \|x_n\|^2.$$

14. Define an orthonormal basis for a Hilbert space.

(14 × 1 = 14 Weightage)

PART B

Answer any seven questions (Each question has Weightage 2)

15. State and prove Minkowski's inequality for scalar sequences.

16. Show that for $1 \leq p < \infty$, the space $L^p(E)$ is complete ,where E is a measurable subset of R.

17. If x is a continuous K – valued function on $[-\pi, \pi]$ such that $x(\pi) = x(-\pi)$. Then prove that the sequence of arithmetic means of the partial sums of the Fourier series of x converges to x uniformly on $[-\pi, \pi]$.

18. State and prove Riesz Lemma.

19. Show that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

20. If X is a normed space and Y is a closed subspace of X. Then show that X is a Banach space if and only if Y and and X/Y are Banach spaces in the induced norm and the quotient norm, respectively.

21. If a normed space X has a Schauder basis, then prove that X is separable.

22. State open mapping theorem. Give examples to show that completeness condition on X and Y cannot be dropped in open mapping theorem.

23. In an inner product space X, prove that for all $x, y \in X$,

$$4\langle x, y\rangle = \langle x+y, x+y\rangle - \langle x-y, x-y\rangle + i\langle x+iy, x+iy\rangle - i\langle x-iy, x-iy\rangle$$

24. If X is a Hilbert space and if $\{u_1, u_2, ...\}$ is a countable orthonormal set in X ,and $k_1, k_2, ... \in K$ such that $\sum_n |k_n|^2 < \infty$, then prove that $\sum_n k_n u_n$ converges in X.

(7 × 2 = 14 Weightage)

PART C

Answer any two questions (Each question has Weightage 4)

25. (a). If X is a complete metric space, then show that the intersection of a countable number of dense open subsets of X is dense in X.

(b). Give an example to show that, in a non-complete metric space, the intersections of a denumerable number of dense open subsets need not be dense.

26. For any normed space X, prove that: For every subspace Y of X and every g in Y′ there is a unique Hahn-Banach extension of g to X if and only if X′ is ***strictly convex***, that is, for $f_1 \neq f_2$ in X′ with $\| f_1\| = 1 = \| f_2\|$, we have $\| f_1 + f_2 \| < 2$

27. (a) State and prove uniform boundedness principle.

(b) Give an example to show that we cannot drop the completeness condition on X in the uniform boundedness principle.

28. State and prove Bessel's inequality. Deduce Schwarz inequality from Bessel's inequality.

(2 × 4 = 8 Weightage)

PART B

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    (b) Give an example to show that we cannot drop the completeness condition on X in the uniform boundedness principle.

28. State and prove Bessel's inequality. Deduce Schwarz inequality from Bessel's inequality.

(2 × 4 = 8 Weightage)

Part B

Answer any seven questions. Each question carries 2 weights.

15. Let Ω be the set of all invertible linear operators on $\mathbf{R}^n$. Show that the mapping $A \to A^{-1}$ is continuous on Ω.
16. Let $f : E \to \mathbf{R}^m$, where E be an open subset of $\mathbf{R}^n$. Define the total derivative $f'(x)$ of f at $x \in E$ and show that it is unique.
17. Let $f : E \to \mathbf{R}$, E open subset of $\mathbf{R}^n$ and let $u \in \mathbf{R}^n$ be a unit vector. Show that $(D_u f)(x) = (\nabla f)(x) . u$.
18. If X is a complete metric space and if φ is a contraction of X into X, then show that φ has a unique fixed point.
19. Show that the total signed curvature of a closed plane curve is an integer multiple of 2π.
20. If $f : S_1 \to S_2$ is a diffeomorphism then show that $D_p f : T_pS_1 \to T_{f(p)} S_2$ is invertible.
21. Let $\sigma(u, v)$ be a surface patch with unit normal $N(u, v)$. Show that $N_u . \sigma_u = - L$ and $N_v . \sigma_v = - N$.
22. For a unit speed curve r show that $\ddot{r}$ is a linear combination of N and $N \times \dot{r}$.
23. Show that $\kappa^2 = \kappa_n^2 + \kappa_g^2$.
24. If κ_1 and κ_2 are the principal curvatures of a surface. Show that the Gaussian curvature $\kappa = \kappa_1 \kappa_2$.

(7 x 2 = 14 weightage)

Part C

Answer any two. Each question carries 4 weights

25. State Implicit Function Theorem and show that the system of equations

$$\begin{aligned} 3x + y - z + u^2 &= 0 \\ x - y + 2z + u &= 0 \\ 2x + 2y - 3z + 2u &= 0 \end{aligned}$$

can be solved for x, y, u in terms of z; for x, z, u in terms of y; for y, z, u in terms of x; but not for x, y, z in terms of u.

26. State Inverse Function Theorem and for the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(x, y) = (e^x \cos y, e^x \sin y)$ show that
    a) f is locally one to one on $\mathbf{R}^2$ but not one to one in $\mathbf{R}^2$
    b) for $a = (0, \pi/3)$, $b = f(a)$, let g be the continuous inverse of f, defined in a neighborhood of b such that $g(b) = a$.
27. Let U and $\tilde{U}$ be open subsets of $\mathbf{R}^2$ and $\sigma: U \to \mathbf{R}^2$ be a regular surface patch. Let $\varphi : \tilde{U} \to U$ be a bijective smooth map with smooth inverse. Show that $\tilde{\sigma} = \sigma_0 \varphi: \tilde{U} \to \mathbf{R}^3$ is a regular surface patch.
28. Let $\sigma: U \to \mathbf{R}^3$ be a patch of a surface S containing a point $p \in S$, and let (u, v) be coordinates in U. Show that the tangent space to S at p is spanned by σ_u, σ_v.

(2 x 4 = 8 weightage)

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17. Let $f : E \to \mathbf{R}$, E open subset of $\mathbf{R}^n$ and let $u \in \mathbf{R}^n$ be a unit vector. Show that $(D_u f)(x) = (\nabla f)(x) . u$.
18. If X is a complete metric space and if φ is a contraction of X into X, then show that φ has a unique fixed point.
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20. If $f : S_1 \to S_2$ is a diffeomorphism then show that $D_p f : T_pS_1 \to T_{f(p)} S_2$ is invertible.
21. Let $\sigma(u, v)$ be a surface patch with unit normal $N(u, v)$. Show that $N_u . \sigma_u = - L$ and $N_v . \sigma_v = - N$.
22. For a unit speed curve r show that $\dddot{r}$ is a linear combination of N and $N \times \dot{r}$.
23. Show that $\kappa^2 = \kappa_n^2 + \kappa_g^2$.
24. If κ_1 and κ_2 are the principal curvatures of a surface. Show that the Gaussian curvature $\kappa = \kappa_1 \kappa_2$.

(7 x 2 = 14 weightage)

Part C

Answer any two. Each question carries 4 weights

25. State Implicit Function Theorem and show that the system of equations

$$\begin{aligned} 3x + y - z + u^2 &= 0 \\ x - y + 2z + u &= 0 \\ 2x + 2y - 3z + 2u &= 0 \end{aligned}$$

can be solved for x, y, u in terms of z; for x, z, u in terms of y; for y, z, u in terms of x; but not for x, y, z in terms of u.

26. State Inverse Function Theorem and for the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(x, y) = (e^x \cos y, e^x \sin y)$ show that
    a) f is locally one to one on $\mathbf{R}^2$ but not one to one in $\mathbf{R}^2$
    b) for $a = (0, \pi/3)$, $b = f(a)$, let g be the continuous inverse of f, defined in a neighborhood of b such that $g(b) = a$.
27. Let U and $\tilde{U}$ be open subsets of $\mathbf{R}^2$ and $\sigma: U \to \mathbf{R}^2$ be a regular surface patch. Let $\varphi : \tilde{U} \to U$ be a bijective smooth map with smooth inverse. Show that $\tilde{\sigma} = \sigma \circ \varphi: \tilde{U} \to \mathbf{R}^3$ is a regular surface patch.
28. Let $\sigma: U \to \mathbf{R}^3$ be a patch of a surface S containing a point $p \in S$, and let (u, v) be coordinates in U. Show that the tangent space to S at p is spanned by σ_u, σ_v.

(2 x 4 = 8 weightage)

1M3N18203 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2018

MT3C12 - Multivariable Calculus and Geometry

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer all questions. Each question carries 1 weight.

1. Let $A \in L(R^2), B \in L(R^2)$. Then AB need not be same as BA. True or False. Justify.
2. Let Ω be the set of all invertible linear operators on $\mathbf{R}^n$. If $A \in \Omega$, $B \in L(\mathbf{R}^n)$ and $\|B - A\| . \|A^{-1}\| < 1$ then show that $B \in \Omega$.
3. Let $U = \{(x, y) \in R^2 \ / \ y > 0\}$. Give an example of a differentiable function $f : U \to \mathbf{R}^3$ and find $f'(1,2)$.
4. Let $f : R^2 \to R$ be defined by $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} ; \text{ for } (x, y) \neq (0,0) \\ 0 \quad \text{at} \quad (0,0) \end{cases}$

   Find partial derivatives of f at (0,0).
5. Check whether $f : [-1,1] \to \mathbf{R}$ by $f(x) = x^2$ is a contraction or not.
6. $A : \mathbf{R}^4 \to \mathbf{R}^3$ defined by $A(x_1, y_1, y_2, y_3) = (x_1 + y_3, y_1 + y_3, y_2 + y_3)$. Write down the linear map A_y and check whether A_y is invertible or not.
7. Show that any reparametrization of a regular curve is regular.
8. Let r be a regular curve and let $\tilde{r}$ be a unit speed reparametrization of r, $\tilde{r}(u(t)) = r(t)$ for all t, where u is a smooth function of t. Show that $u = \pm s + c$ where s is the arc length of r starting at any point and c is a constant.
9. Define a surface in $\mathbf{R}^3$ and give an example.
10. Let $f : S_1 \to S_2$ be smooth between the surfaces. Define the derivative D_pf of f at $p \in S_1$.
11. Find the first fundamental form of $\sigma(u, v) = (\cosh u, \sinh u, v)$.
12. For a unit speed curve r on an oriented surface S show that the normal curvature is $\langle \ddot{r}, r \rangle$.
13. Show that a point p is an umbilic point of a surface S if and only if the Weingarten map is a scalar multiple of the identity map.
14. Compute the second fundamental form of the coordinate patch $\sigma(u, v) = \mathbf{a} + up + vq$ where $\mathbf{a}$ is a point on the plane and p, q are two unit vectors that are parallel to the plane and perpendicular to each other.

(14 x 1 = 14 weightage)

Part B

Answer any seven questions. Each question carries 2 weights.

15. Let Ω be the set of all invertible linear operators on $\mathbf{R}^n$. Show that the mapping $A \to A^{-1}$ is continuous on Ω.
16. Let $f : E \to \mathbf{R}^m$, where E be an open subset of $\mathbf{R}^n$. Define the total derivative $f'(x)$ of f at $x \in E$ and show that it is unique.
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18. If X is a complete metric space and if φ is a contraction of X into X, then show that φ has a unique fixed point.
19. Show that the total signed curvature of a closed plane curve is an integer multiple of 2π.
20. If $f : S_1 \to S_2$ is a diffeomorphism then show that $D_p f \; T_pS_1 \to T_{f(p)} S_2$ is invertible.
21. Let $\sigma(u, v)$ be a surface patch with unit normal $N(u, v)$. Show that $N_u . \sigma_u = - L$ and $N_v . \sigma_v = - N$.
22. For a unit speed curve r show that $\dddot{r}$ is a linear combination of N and $N \times \dot{r}$.
23. Show that $\kappa^2 = \kappa_n^2 + \kappa_g^2$.
24. If κ_1 and κ_2 are the principal curvatures of a surface. Show that the Gaussian curvature $\kappa = \kappa_1 \kappa_2$.

(7 x 2 = 14 weightage)

Part C

Answer any two. Each question carries 4 weights

25. State Implicit Function Theorem and show that the system of equations

$$\begin{aligned} 3x + y - z + u^2 &= 0 \\ x - y + 2z + u &= 0 \\ 2x + 2y - 3z + 2u &= 0 \end{aligned}$$

can be solved for x, y, u in terms of z; for x, z, u in terms of y; for y, z, u in terms of x; but not for x, y, z in terms of u.

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28. Let $\sigma: U \to \mathbf{R}^3$ be a patch of a surface S containing a point $p \in S$, and let (u, v) be coordinates in U. Show that the tangent space to S at p is spanned by σ_u, σ_v.

(2 x 4 = 8 weightage)

1M3N18202 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2018

MT3C15- PDE and Integral Equations

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Section A

Answer *all* questions. Each question carries 1 weightage.

1. Define a Curve. Give an example of a Curve and its parametric representation.
2. Show that the partial differential equation corresponding to the surface $F(x-z, y-z) = 0$ is $p + q = 1$
3. Find the complete integral of a partial differential equation $z - px - qy - p^3 + q^3 = 0$
4. Find the general solution of the p.d.e $px + qy = z$
5. Define a Paffian differential equation. Give an example
6. Give a brief note on Monge Cone.
7. Find the characteristic strip of a non linear partial differential equation $xp + yq - pq = 0$, through the initial curve $z = \frac{x}{2}$, $y = 0$.
8. What type of the the second order semi linear partial differential equation is $u_{xx} + 2u_{xy} + 17u_{yy} = 0$
9. What is meant by " Range of influence" in the case of one dimentional wave equations.
10. State any one of the boundary value problem.
11. State Harnack's Theorem.
12. Classify all the integral equations.
13. Transform $\frac{d^2y}{dx^2} + \lambda y = 0$ with the conditions $y(0) = 1, y'(0) = 0$ as an integral equation.
14. State any two properties of Greens function corresponding to $L_y = \frac{d}{dx}[p(x)\frac{d}{dx} + q(x)]y$ in the interval (a, b).

(14×1=14 Weightage)

Section B

Answer any *SEVEN* questions. Each question carries 2 Weightage.

15. Show that the singular integral is also a solution of the partial differential equation $f(x, y, z, p, q) = 0$

16. Find the complete integral of the partial differential equation $f = z^2 - pqxy = 0$ by Charpit's method

17. Find the integral corresponding to a partial differential equation $yzdx + 2xzdy - 3xydz = 0$.

18. Show that $xp - yq - x = 0$ and $x^2p + q - xz = 0$ are compatible and find a one parameter family of common solutions

19. Reduce the equation $x^2u_{xx} - y^2u_{yy} = 0$ into the canonical form.

20. Suppose that $u(x, y)$ is harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$. Then show that u attains its maximum on the boundary B of D.

21. Show that the Riemann function for the equation $L[u] = u_{xy} + \frac{1}{4}u = 0$ is $v(x, y, \alpha, \beta) = J_0(\sqrt{(x-\alpha)(y-\beta)})$.

22. Show that the characteristic number of a Fredholm equation with a real symmetric kernal are real.

23. Find d' Alembert's solution which describes the vibrations of finite string.

24. Find the resolvant kernal, where the kernal of the integral equation is $K(x, \xi) = 1 - 3x\xi$ in the interval (0,1)

(7×2 = 14 Weightage)

Section C

Answer any *TWO* questions. Each question carries 4 Weightage.

25. a, Find the general solution of $x^2p + y^2q = (x + y)z$.
    b, Find a complete integral of $p^2 + q^2 = x + y$

26. Find integral surface of the quasi linear equation $zz_x + z_y = 0$ containing the initial data curve $x_0 = s, y_0 = 0, z_0 = f(s)$ where $f(s) = 1$, if $s \leq 0$, $1 - s$ if $0 \leq s \leq 1$ and is 0 if $s \geq 1$.

27. a, The solution of the following problem if it exist, then show that it is unique. $u_{tt} - c^2u_{xx} = F(x, t),\ 0 < x < l,\ t > 0$; $u(0, t) = u(l, t) = 0$, if $t \geq 0$; $u(x, 0) = f(x),\ 0 \leq x \leq l$; $u_t(x, 0) = g(x),\ 0 \leq x \leq l$.
    b, What is the solution of the above equation if $F(x, t) = 0$

28. a, Show that $\frac{d^nI_n}{dx^n} = (n-1)!f(x)$, where $I_n(x) = \int_a^x((x-\xi)^{n-1}f(\xi)d\xi$
    b, Show that $\int_a^x \int_a^{x_n} \int_a^{x_{n-1}} \cdots \int_a^{x_3} \int_a^{x_2} f(x_1)dx_1dx_2 \cdots dx_n = \frac{1}{(n-1)!} \int_a^x((x-\xi)^{n-1}f(\xi)d\xi$

(2×4=8 Weightage)