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2M19085 (Pages : 2) Reg. No:..............................

Name: ..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Mathematics Degree Examination, March /April 2019**

**MT2C11- Operations Research**

(2018 Admission onwards)

e: 3 hours Max. Weightage : 36

## PART A

***Answer all 1 weightage each.***

Define convex function. Prove that $f(x) = x^2\ x \in \mathbb{R}$ is a convex function.

Prove that if f(X) is minimum at more than one of the vertices of $S_F$, then it is minimum at all those points which are the convex linear combination of these vertices.

Explain the role of simplex multipliers.

What is caterer problem?

Define integer vector. Give an example.

Give one disadvantage of the cutting plane method.

Define a path. Give an example.

What is maximum flow problem?

Define a loop in a transportation array.

0. State a necessary and sufficient condition for the existence of a saddle point.

1. State the fundamental theorem of rectangular games.

2. Find the saddle point of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 10 \end{bmatrix}$.

3. Explain dual simplex method.

4. What is the relation between the maximum flow and the capacity of cuts in a graph?

$(\mathbf{14 \times 1 = 14}\ \boldsymbol{Weights})$

## PART B

***Answer any seven. 2 weightage each.***

5. Show that the dual of the dual is the primal.

6. Write both the primal and dual linear programming problems corresponding to the rectangular game with the following payoff matrix. Hence solve the game. $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

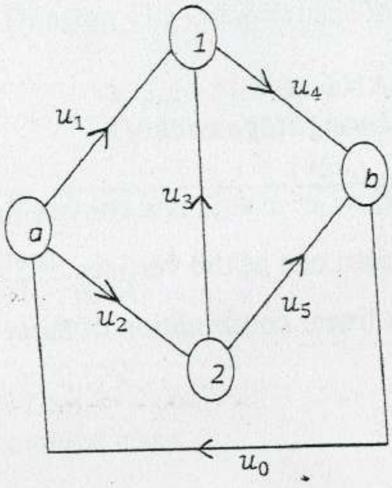
. Prove that if $a_i$ and $b_j$ are integral for all $i$ and $j$ the optimal solution of the transportation problem can be obtained in terms of integers. Also show that if $a_i$ and $b_j$ are rational, the problem can be reduced to the form in which they are integral.

18. Show graphically that at the optimal solution of the problem max $6x_1 - 4x_2$ subject to $x_1 - x_2 \leq 1,\ 3x_1 - 2x_2 \leq 6,\ x_1 \geq 0, x_2 \geq 0$. The value of $x_1$ and $x_2$ can be increased indefinitely while the value of the objective function remain unchanged.

19. Let $X \in E_n$ and $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semi definite, prove that $f(X)$ is a convex function.

20. Find the maximum flow in the following graph with constrains

$$2 \leq x_1 \leq 10, \quad 4 \leq x_2 \leq 12, \quad -2 \leq x_3 \leq 4, \quad 0 \leq x_4 \leq 5, \quad 0 \leq x_5 \leq 10$$




21. Describe how the changes in the values of $b_i$ affect the optimal solution of a linear programming problem.

22. Solve graphically the game whose payoff matrix is $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$.

23. Explain degeneracy in linear programming.

24. Describe the condition which terminates the branching in branch and bound method.

$(7 \times 2 = 14\ Weights)$

## PART C

***Answer any two. 4 weightage each.***

25. Solve by simplex method

Max $5x_1 + 3x_2 + x_3$ subject to $2x_1 + x_2 + x_3 \geq 3,\ -x_1 + 2x_3 \geq 4,$

$x_1, x_2, x_3 \geq 0$

26. Describe the algorithm for solving minimum path problem when

1) all arc lengths are non negative 2) arc lengths unrestricted in sign. Give example.

27. Describe the unbalanced transportation problem. Give example.

28. Solve by cutting plane method

Minimize $-2x_1 - 3x_2$, subject to $2x_1 + 2x_2 \leq 7,\ 0 \leq x_1 \leq 2, 0 \leq x_2 \leq 2,$

$x_1, x_2$ integers.

$(2 \times 4 = 8\ Weights)$

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2M2M19081 (Pages : 2) Reg. No:..............................

Name: ..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Mathematics Degree Examination, March /April 2019**

**MT2C07 - Algebra II**

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

**Part A**

**Answer all questions.**

**Each of question carries weightage 1**

1. Find an irreducible polynomial of degree 4 in $\mathbb{Z}_3[x]$.
2. Verify whether $\sqrt{2} - \sqrt{3}\,i$ is algebraic over $\mathbb{Q}$.
3. Give an example of a simple extension of $\mathbb{Q}$ which is not a finite extension. Justify your claim.
4. Prove that $\mathbb{Q}(\sqrt{2}, i) \neq \mathbb{Q}(\sqrt{2}\,i)$.
5. Prove that doubling the cube is impossible.
6. What is the relation between the primitive roots of unity in the field $\mathbb{Z}_p$ and the subgroups of the multiplicative group $\mathbb{Z}_p^*$?
7. Prove that the fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic.
8. Find the group of automorphisms of $\mathbb{Q}(i)$ leaving $\mathbb{Q}$ fixed.
9. Is $\mathbb{C}$ a splitting field extension of $\mathbb{R}$ ? Justify your answer.
10. Prove or disprove: Every finite extension of $\mathbb{Q}$ is a simple extension .
11. Give an example of a normal extension of $\mathbb{Q}$.
12. Find a proper extension F of $\mathbb{Q}$ such that $G(F/\mathbb{Q})$ is trivial.
13. Define cyclotomic polynomial. Find $\Phi_4(x)$ over $\mathbb{Q}$.
14. Is the regular 50-gon constructible ? Why ?

**(14 x 1 = 14 weightage)**

**Part B**

**Answer any seven questions.**

**Each of question carries weightage 2**

15. For which of the following polynomial f(x) in $\mathbb{Z}_3[x]$. $\mathbb{Z}_3[x]/\langle f(x)\rangle$ is a field ? Justify your claim.

    a) $f(x) = x^3+x+1$ b) $f(x) = x^2+x+2$

16. Prove that every finite extension of a field is an algebraic extension.

17. Prove that an algebraically closed field has no proper algebraic extension.

18. Argue that the field of constructible real numbers is an algebraic extension of $\mathbb{Q}$ but not a finite extension.

19. Prove that every finite field has a prime power order.

20. Find the fixed field F of the automorphism $\psi$ of $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ that maps $\sqrt{2}$ onto $-\sqrt{2}$, $\sqrt{3}$ onto $-\sqrt{3}$, and $\sqrt{5}$ onto $-\sqrt{5}$. What is[F : $\mathbb{Q}$] ?

21. Prove that if a finite extension E of F is a splitting field extension of F, then {E : F} = |G(E/F)|.

22. Prove that every field of characteristic 0 is perfect.

23. Find all the intermediate fields in between $\mathbb{Q}$ & $\mathbb{Q}(i, \sqrt{3})$. How can we argue that they are the only intermediate fields.

24. When a field E is said to be an extension of F by radicals ? Is the field $\mathbb{Z}_3[x]/\langle x^2+1\rangle$ an extension of $\mathbb{Z}_3$ by radicals ? Why ?

**(7 x 2 = 14 weightage)**

## Part C

**Answer any two questions.**
**Each of question carries weightage 4**

25. a) Prove that an ideal $\langle p(x)\rangle \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F.

b) Describe all the maximal ideals in $\mathbb{R}[x]$.

26. a) If E is a finite extension of a field F and K is a finite extension of E, then prove that K is a finite extension of F and [K : F]=[K : E][E : F].

b) Prove that if E is an extension of F and $\alpha, \beta$ belongs to E are algebraic over F, then $\alpha+\beta$ is algebraic over F.

27. a) If F is a finite field of characteristic p, prove that the map $\sigma_p: F \to F$ defined by $\sigma_p(a) = a^p$, for $a$ belongs to F, is an automorphism of F. Also prove that $F_{\{\sigma_p\}} \simeq \mathbb{Z}_p$.

b) Prove that any two algebraic closures of F are isomorphic.

28. Prove that

a) Every symmetric function of $y_1, y_2 \dots y_n$ over a field F is a rational function of the elementary symmetric functions $s_1, s_2 \dots s_n$.

b) $F(y_1, y_2 \dots y_n)$ is a finite normal extension of degree $n!$ of $F(s_1, s_2 \dots s_n)$.

c) The Galois group of $F(y_1, y_2 \dots y_n)$ over $F(s_1, s_2 \dots s_n)$ is isomorphic to $S_n$.

**(2 x 4 = 8 weightage)**

B2M19082 (Pages : 2) Reg. No:..............................

Name: ..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Mathematics Degree Examination, March /April 2019**

**MT2C08 - Real Analysis II**

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

**Part A**

**Answer All Questions**

**Each question carries 1 weightage**

1. Prove that for any set A there exist a measurable set E containing A and such that $m^*(A) = m(E)$.
2. Prove that continuous functions are measurable.
3. Define essential supremum and essential infimum of a measurable function.
4. Let f and g are integrable functions then prove that f+g is integrable and $\int (f+g)dx = \int f dx + \int g dx$.
5. Show that if f is an integrable function, then $|\int f dx| \le \int |f|\, dx$.
6. Prove that $D^+(-f) = -D_+(f)$ and $D^-(-f) = -D_-(f)$.
7. If $f \in L(a,b)$ then prove that $F(x) = \int_a^x f(t)dt$ in a continuous function on [a,b].
8. Define Lebesgue set of a function and prove that it contains any point at which the function is continuous.
9. Show that if μ is a σ-finite measure on R, then the extension $\bar{\mu}$ of μ to $S^*$ is also σ-finite.
10. Show that if $\varphi(E) = \int_E f d\mu$ then $\varphi$ is a signed measure.
11. Let [[x,s,μ]] be a measure space and let $\int f d\mu$ exist. Define ν by $\nu(E) = \int_E f d\mu$, for $E \in S$. Find a Hahn decomposition with respect to ν.
12. Give an example of a function which is continuous but not absolutely continuous.
13. State Radon-Nikodym theorem.
14. Show that if ν is a signed measure, $|\nu(E)| < \infty$ and $F \subseteq E$ then $|\nu(F)| < \infty$.

**(14 x 1= 14 weightage)**

## Part B

**Answer any seven from the following ten questions (15 - 24)**
**Each question has weightage 2**

15. Show that there exists uncountable set of zero measure.

16. Show that if $\{E_i\}$ is a sequence of measurable sets, $m(\bigcup_{i=1}^{\infty} E_i) < \infty$, and $\lim E_i$ exists, then $m(\lim E_i) = \lim m(E_i)$.

17. Prove that there exist a non-measurable set.

18. State and prove Fatou's Lemma.

19. Prove that $f \in BV[a.b]$ where a and b are finite if and only if the graph of f is rectifiable curve.

20. If $f \in L(a, b)$ then prove that (1) $F(x) = \int_a^x f(t)dt$ is continuous function on [a, b]. (2). $F \in BV[a, b]$.

21. Let $\mu^*$ be an outer measure on H(R) and let $S^*$ denote the class of $\mu^*$ measurable sets, then prove that $S^*$ is a $\sigma$ - ring.

22. Let $\nu$ be a signed measure and let $\mu$, $\lambda$ be measures on [[x,s]] such that $\lambda$, $\mu$, $\nu$ are $\sigma$-finite, $\nu << \mu$ and $\mu << \lambda$ then prove that $\frac{d\nu}{d\lambda} = \frac{d\nu}{d\mu}\frac{d\mu}{d\lambda}[\lambda]$.

23. Let g be a monotone increasing and absolutely continuous on *R*. Then prove that $M \subseteq \overline{S_g}$, and on M, $\overline{\mu_g} << m$.

24. Let $\{E_i\}$be a sequence of intervals and I an interval, if $I \subseteq \bigcup_{i=1}^{\infty} E_i$, then prove that $\mu(I) \leq \sum_{i=1}^{\infty} \mu(E_i)$.

**(7 x 2 = 14 weightage)**

## Part C

**Answer any two from the following four questions (25 -28)**
**Each question has weightage 4**

25. Prove that there exists a non-measurable set $\mu$ and not every measurable set is a Borel set.

26. If $\mu$ is a $\sigma$-finite measure on a ring R, then prove that it has a unique extension to the $\sigma$-ring S( R) .

27. State and prove Lebesgue's Differentiation theorem.

28. If f and g are left continuous function on the finite interval [a, b] and f, $g \in BV[a, b]$ then prove that $\int_{[a,b]} f(x+)d\overline{\mu_g} + \int_{[a,b]} gd\overline{\mu_f} = f(b+)g(b+) - f(a)g(a)$.

**(2 x 4 = 8 weightage)**

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M19083 (Pages : 2) Reg. No:..............................

Name: ..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Mathematics Degree Examination, March /April 2019**

**MT2C09- Topology**

(2018 Admission onwards)

ne: 3 hours Max. Weightage : 36

**Part- A**

**Answer *all* questions.**

**Each question has *one* weightage.**

1. Prove or disprove: The sequence $\{(-1)^n\}_{n=1}^{\infty}$ is convergent when R is given the discrete topology.
2. What is the closure of $(\sqrt{2},\ 5)$ w.r.to scattering topology on $R$ (where open sets are of the form $A \cup B$, where $A$ is open in the usual topology on $R$ and $B \subset R \setminus Q$.) ?
3. Let $\tau_1$ be the co-finite topology and $\tau_2$ be the co-countable topology on R. What is the relation between these two topologies?
4. Let $E \subset R$ and $R$ is given with usual topology. Is it always true that if $\overline{E}$ is connected, then so is $E$ ?
5. Does there exist a continuous function $f : R \to R$, whose range is the set of rational numbers? Justify.
6. Let $\tau_1, \tau_2$ be topologies on a set X with $\tau_1 \subseteq \tau_2$ . Let $X_1 = (X, \tau_1)$ and $X_2 = (X, \tau_2)$. If $X_2$ is Hausdorff, then is it true that $X_1$ is also Hausdorff ? . Justify.
7. Give an example of a topological space which is not separable.
8. Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{\phi, X, \{2,3\}, \{1,2,3\}, \{2,3,4,5\}\}$ be a topology on X. Find the interior of $\{1, 3, 4\}$ in $(X, \tau)$.
9. Are the subspaces [0,1] and (0,1) of the real line homeomorphic? Justify.
10. Suppose X is an infinite set given with discrete topology. Is $X$ compact? Justify.
11. Define quotient topology.
12. What do you mean by a Tychnoff space?
13. Is it true that limit of a convergent sequence in a topological space, uniquely exists? Justify.
14. Give an example of a $T_1$-space which is not $T_2$.

**(14 x 1 = 14 weightage)**

## Part- B

**Answer any *seven* from the following ten questions.**
**Each question has *two* weightage**

15. Prove or disprove: The usual topology on R is finer than the co-finite topology on R.
16. Define basis for a topology. Give one example. Is a basis itself a topology on a set?
17. Define closure of a subset of a topological space. For a subset $A$ of a topological space $X$, show that $\overline{A} = A \cup A'$.
18. Prove that, for a subset $A$ of a topological space, $\text{int}(A) = X - \overline{(X - A)}$.
19. Show that continuous image of a compact space is compact.
20. Show that every second countable space is first countable. Is the converse true?
21. Prove that all metric spaces are $T_4$.
22. Show that every completely regular space is regular.
23. Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.
24. Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Show that X is normal.

**(7 x 2 = 14 weightage)**

## Part- C

**Answer any *two* from the following four questions.**
**Each question has *four* weightage.**

25. (a) Prove that metrizability is a hereditary property.
    (b) Suppose that X is a metrizable space and A is a subset of X. Prove that a point $y \in \overline{A}$ if and only if there is a sequence $x_n \in A$ such that $x_n \to y$ in $X$.
26. (a) Prove that continuous image of a connected space is connected.
    (b) Show that every path connected space is connected. Is the converse true? Justify.
27. (a) Show that every continuous real valued function on a compact space is bounded and attains its extrema.
    (b) Prove that every closed subspace of a compact space is compact.
28. Prove that every regular Lindeloff space is normal. Deduce that every compact Hausdorff space is $T_4$.

**(4 x 2 = 8 weightage)**

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2B2M19084 (Pages : 3) Reg. No:..............................

Name: ..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Mathematics Degree Examination, March /April 2019**

**MT2C10- ODE and Calculus of variations**

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

**Part A(Short Answer Type Questions)**

***Answer all the questions***

***Each question has weightage 1.***

1. Express $\tan x$ in the form of a power series $\sum a_n x^n$ by solving $y' = (1+y^2)$, $y(0) = 0$ in two ways.
2. Locate and classify the singular points on the $X$ axis of the differential equation$x^3(1-x)y'' - 2(1-x)y' + 3xy = 0$.
3. Define the hypergeometric equation and hypergeometric series.
4. Find the first three terms of the Legendre series of f(x) $= e^x$.
5. State the orthogonality property of Legendre's polynomial.
6. Show that $\frac{d}{dx}(J_0(x)) = -J_1(x)$.
7. Define node and saddle point of an autonomous system.
8. Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are solutions of the homogeneous system

   $$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases}$$
9. Determine whether the function $-2x^2 + 3xy - y^2$ is positive definite, negative definite or neither.
10. Describe the phase portrait of the following system.

    $$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = 0 \end{cases}$$
11. State Sturm comparison theorem.
12. Find the shortest curve joining two points $(x_1, y_1)$and $(x_2, y_2)$.
13. Discuss the isoperimetric problem.
14. Show that $f(x,y) = xy^2$ satisfies Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$.

**$(14 \times 1 = 14$ weightage)**

Type equation here.

## Part B(Paragraph Type Questions)

*Answer any seven questions*

***Each question has weightage 2.***

15. Show that the confluent hypergeometric equation has $x = \infty$ as an irregular singular point.

16. Show that the equation $x^2y'' + xy' + (x^2 - 1)y = 0$ has only one Frobenius series solution.

17. Prove that $(1 + x)^p = F(-p, b, b, -x)$.

18. Show that between any two positive zeros of $J_1(x)$, there is a zero of $J_0(x)$.

19. Let $u(x)$ be a nontrivial solution of $u'' + q(x)u' = 0$ where $q(x) < 0$. Show that $u(x)$ has at most one zero.

20. Show that $(0, 0)$ is an asymptotically stable critical point of the system

$$\begin{cases} \dfrac{dx}{dt} = -2x + 3y + xy \\ \dfrac{dy}{dt} = -x + y - 2xy^2. \end{cases}$$

21. Explain the following.

    a) Euler's Equation

    b) Stationary function

    c) Stationary value

    d) Extremals

22. How we can transform a second order linear equation into a system of linear equations?

23. Solve the following system

$$\begin{cases} \dfrac{dx}{dt} = x + y \\ \dfrac{dy}{dt} = 4x - 2y. \end{cases}$$

24. Find the exact solution of the problem $y' = (x + y)$, $y(0) = 1$. Starting with $y(x) = 1$, apply Picard's method to calculate $y_1(x), y_2(x)$ and $y_3(x)$.

**($7 \times 2 = 14$ weightage )**

## Part C(Essay Type Questions)

*Answer any two questions*

*Each question has weightage 4.*

25. Derive Rodrigue's formula for Legengre's polynomials and show that $P_n(x)$ given by Rodrigue's formula satisfies the Legendre's equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$.
26. State and prove the orthogonality properties of Bessel's function.
27. State and prove Picard's theorem; (Local existence).
28. Determine the nature and stability properties of the critical point $(0,0)$ of the system

$$\begin{cases} \dfrac{dx}{dt} = -3x + 4y \\ \dfrac{dy}{dt} = -2x + 3y. \end{cases}$$

**(2 × 4 = 8 weightage )**