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1M2M20085 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2020

MMT2C10 – Operations Research

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer questions (1-8)

Answer all questions. Each question has 1 weightage.

1. Prove that the function $f(x) = x^2$ is a convex function.
2. Define basic feasible solution in linear programming problem.
3. Write the dual of the linear programming problem Maximise $x_1 + 6x_2 + 4x_3 + 6x_4$ subject to $2x_1 + 3x_2 + 17x_3 + 80x_4 \leq 48$, $8x_1 + 4x_2 + 4x_3 + 4x_4 = 2$, $x_1, x_2 \geq 0$ $and\ x_3\ and\ x_4\ are\ unstricted\ in\ sign.$
4. Prove that the dual of the dual of a linear programming problem is primal.
5. What is meant by loop s in a transportation array.
6. What is Caterer Problem in Operation Research.
7. Describe the notion of dominance in game theory.
8. What is Integer Linear Programming.

(8 x 1 = 8 Weightage)

Part B

Answer any two questions from each units (9-17)

Each question has weightage 2.

UNIT I

9. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. Prove that if f(x) is a positive semidefinte, then f(x) is aconvex function.
10. Prove that a basic feasible solution of a linear programming problem is a vertex of the convex set of feasible solutions.
11. Explain briefly the Simplex Algorithm for solving linear programming problem.

UNIT II

12. Prove that the Transpotation Array has a triangular basis.
13. Prove that the optimal value of the Primal if it exists is equal to the optimum value of the dual.
14. Prove that the value of the objective function for any feasible solution of the primal is not less than the value of the objective function for any feasible solution of the dual.

UNIT III

15. State and prove the fundamental. Theorem for rectangular games.

16. Define Spanning tree of a graph. Describe an algorithm for finding the spanning tree of minimum length of a graph.

17. Illustrate Branch and Bounded method through an example.

(6 x 2 = 12 Weightage)

PART C

Answer any two questions from the following four questions (18-21)
Each question has weightage 5.

18. a) What are simplex multiplers .

b) Maximize $f(x) = 5x_1 + 3x_2 + x_3$ subject to constrains $2x_1 + x_2 + x_3 = 3$; $-x_1 + 2x_3 = 4$; $x_1, x_2, x_3 \geq 0$.

19. a) Solve the transportation problem for minimum cost with cost coefficients demands and supplies as in the following table.

	D_1	D_2	D_3	
O_1	2	1	3	10
O_2	4	5	7	25
O_3	6	0	9	25
O_4	1	3	5	30
	20	20	15	

b) Describe degeneracy in transportation problem.

20. (a). Using cutting plane method solve, maximize $x_1 + x_2$ subject to $7x_1 - 6x_2 \leq 5$, $6x_1 + 3x_2 \geq 7$, $-3x_1 + 8x_2 \leq 6$, x_1 , x_2 are non negative integers.

(b). Describe an algorithm for solving the minimum path problem

21. (a). Solve graphically the game whose pay-off matrix is $\begin{bmatrix} 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix}$

(b). State and prove mini max theorem in theory of games.

(2 x 5 = 10 Weightage)

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1M2N20084 (Pages :3) Reg. No:……………………...

Name: ………………………..

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2020

MMT2C09 – ODE and Calculus of Variations

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all the questions. Each question has 1 weightage.

1. Locate and classify the singular points on the X axis of the differential equation $x^3(1-x)y''-2(1-x)y'+3xy=0$.
2. Show that $\tan x = x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$ by solving $y'=1+y^2$ $y(0)=0$ in two ways.
3. Verify$\cos x = \lim_{a\to\infty} F(a,a,\frac{1}{2},\frac{-x^2}{4a^2})$.
4. Determine the nature of the point $x=\infty$ for the differential equation
$$x^2y''+xy'+(x^2-p^2)y=0.$$
5. Describe the phase portrait of the following system.
$$\begin{cases}\frac{dx}{dt}=-x\\ \frac{dy}{dt}=-y.\end{cases}$$
6. Find the exact solution of the problem $y'=2x(1+y),\ y(0)=0$. Starting with $y_0(x)=0$, apply Picard's method to calculate $y_1(x), y_2(x)$ and$y_3(x)$.
7. Show that $f(x,y)=xy^2$ satisfies Lipschitz condition on any rectangle $a\le x\le b$ and $\infty \le y \le \infty$.
8. Find the stationary function of $\int_0^4 [xy'-(y')^2]dx$which is determined by the boundary condition $y(0)=0$and $y(4)=3$.

(8 × 1 = 8 Weightage)

Part B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Find the general solution of the equation $(x^2-1)y''+(5x+4)y'+4y=0$ near its singular point $x=-1$.

10. Determine the nature of the point $x=\infty$ for the differential equation

$$x^2y''+xy'+(x^2-p^2)=0.$$

11. Find first three terms of the Legendre series of $f(x)=\begin{cases}0 & if\ -1\le x<0\\ x & if\ 0\le x\le 1\end{cases}$

Unit II

12. Show that for a positive integer $m\ge 0$, the Bessel functions $J_m(x)$ and $J_{-m}(x)$ are linearly dependent.

13. Solve the following system

$$\begin{cases}\dfrac{dx}{dt}=x+y\\ \dfrac{dy}{dt}=4x-2y.\end{cases}$$

14. Determine the nature and stability property of the critical point $(0,0)$ of the following system

$$\begin{cases}\dfrac{dx}{dt}=3x-4y\\ \dfrac{dy}{dt}=x-y\end{cases}$$

Unit III

15. Let $u(x)$ be a nontrivial solution of $u''+q(x)u'=0$ where $q(x)<0$. Show that $u(x)$ has at most one zero.

16. Discuss isoperimetric problems.

17. Find the normal form of Bessel's equation $x^2y''+xy'+(x^2-p^2)=0$ and hence show that every non trivial solution has infinitely many positive zeros.

$(6\times 2=12\ \boldsymbol{weightage})$

Part C

Answer any two questions. Each question carries 5weightage

18. (a) Find the general solution of the Gauss hyper geometric equation

$x(1-x)y'' + (c-(a+b+1)x)y' - aby = 0$near its singular point $x = 0$.

(b) Show that the equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius series solution and find it.

19. For the non-linear system $\begin{cases} \frac{dx}{dt} = y(x^2+1) \\ \frac{dy}{dt} = 2xy^2 \end{cases}$

a) Find the critical points.

b) Find the differential equation of paths.

c) Solve this equation to find paths.

d) Sketch a few of the paths and show the directions of increasing t.

20. a) Prove that $\int_0^1 xJ_p(\lambda_m(x))J_p(\lambda_n(x)) = \begin{cases} 0 & if\ m \neq n \\ \frac{1}{2}J_{p+1}(\lambda_n)^2 & if\ m = n \end{cases}$ where $\lambda_n'$are the positive zeros of some fixed Bessel function $J_p(x)$ with $p \geq 0$.

b) Show that between any two zeros of $J_0(x)$ there is a zero of $J_1(x)$ and between any two zeros of $J_1(x)$ there is a zero of $J_0(x)$.

21. State and prove Picard's theorem

$(2 \times 5 = 10\ \boldsymbol{weightage})$

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1M2M20083 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2020

MMT2C08 – Topology

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part-A

Answer all questions. Each question carries 1 weightage.

1. Distinguish between base and sub-base of a topological space.
2. Is it true that E and $\bar{E}$ have the same interior, where $E \subset \mathbb{R}$ and $\mathbb{R}$ is given with Usual topology?
3. Find all dense subsets of R with usual topology.
4. With necessary notations define product topology.
5. Give an example of a topological space which is first countable but not second countable.
6. Verify whether R within discrete topology is regular.
7. Is every connected space locally connected? Justify your answer.
8. Show that compact subset of a Hausdorff space is closed.

(8× 1 = 8 weightage)

Part-B

Answer any two questions from each Unit. Each question carries 2weightages.

Unit - I

9. Prove that metrisability is a hereditary property.
10. State and prove the necessary and sufficient condition for a subfamily of a topology to become a base for that topology.
11. For any subset A of a space ,prove that $\bar{A} = A \cup A'$.

Unit - II

12. Prove that every closed surjective map is a quotient map.
13. Prove that every continuous real valued function on a compact space is bounded and attains its extrema.
14. If a space $X$is locally connected, then prove that components of open subsets of X are open in .

Unit - III

15. Prove that all metric spaces areT_4.

16. Prove that a compact subset in a hausdorff space is closed.

17. Let X be a completely regular space. Suppose F is a compact subset of X,C is a closed subset of X and $F \cap C = \emptyset$.Prove that there exists a continuous function from X in to the unit interval [0,1]which takes the value 0 at all points of F and the value 1 at all points of C.

($6 \times 2 = 12$ weightage)

Part-C

Answer any two question. Each question carries 5 weightages.

18. Let X be a space and $A \subseteq X$. Prove that $int(A)$ is the union of all open sets containedin A.

(b)Prove that every closed and bounded interval is compact.

19. (a) State and prove Lebesgue covering lemma

(b) Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces and $f: X \to Y$ be a function. Then prove that f is continuos if and only if $f^{-1}(V)$ is open in X for every open subset V in .

20. (a) If $\{Ci\}$is a collection of connected subsets in a space $X$and if$\cap\ C\ i \neq \Phi$,then prove that $U\ Ci$ is connected.

(b)Prove that every quotient space of a locally connected space is locally connected.

21. State and prove Urysohn characterisation of normality.

($2 \times 5 = 10$ weightage)

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1M2M20082 (Pages : 3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2020

MMT2C07 – Real Analysis II

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part- A

Answer *all* questions. Each question has *one* weightage.

1. Show that any σ – algebra of subsets of R that contains all open intervals, also contains all closed intervals.
2. Show that every singleton set has outer measure zero.
3. Show that the characteristic function χ_A is measurable if and only if A is measurable.
4. Give an example to show that point-wise limit of a sequence of Riemann integrable functions need not be Riemann integrable.
5. Show that Monotone Convergence Theorem may not hold for decreasing sequence functions.
6. If f is integrable over a measurable set E, then show that for each $\varepsilon > 0$, there exists a set E_0 of finite measure for which $\int_{E-E_0} |f| < \varepsilon$.
7. If the function f is Lipschitz on a closed and bounded interval $[a,b]$, then show that f is absolutely continuous on $[a,b]$.
8. Show that the function f defined by $f(x) = x\cos(\pi/2x)$ if $0 < x \leq 1$ and $f(0) = 0$, is not of bounded variation on $[0,1]$.

(8 × 1 = 8 Weightage)

Part- B

Answer any *two* from each unit. Each question has *two* weightage

Unit - I

9. If E_1 and E_2 are measurable sets, prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

10. Show that every interval is measurable.

11. Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e. on E to the function f. Then prove that f is measurable.

Unit –II

12. State and prove Fatou's Lemma.

13. If f, g are integrable functions over E and if α, β are scalars then show that $\alpha f + \beta g$ is integrable over E and $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.

14. Define convergence in measure. If $\{f_n\}$ is a sequence of measurable functions on a set E of finite measure that converges point wise a.e. on E to f and f is finite a.e. on E, show that $\{f_n\} \to f$ in measure on E.

Unit – III

15. Show that a function f is of bounded variation on the closed, bounded interval $[a, b]$ if and only if f is the difference of two increasing functions on $[a, b]$.

16. If f is absolutely continuous on the closed, bounded interval $[a, b]$, then prove that it is of bounded variation on $[a, b]$.

17. If E is a measurable set and if $1 \le p \le \infty$, then show that $L^p(E)$ is a Banach space.

($6 \times 2 = 12$ Weightage)

Part- C

Answer any *two* from the following four questions.
Each question has *Five* weightage.

18. (a). Define Lebesgue outer measure. Prove that outer measure of an interval is its length.

(b) If E is a measurable set of real numbers, then prove that for any $\varepsilon > 0$, there exists an open set O containing E for which $m^*(O - E) < \varepsilon$.

19. (a) Define measurable functions. If f is an extended real- valued measurable function on E and if $f = g$ *a.e.* on E, then show that g is measurable on E.

(b) If f and g are measurable functions defined on a measurable set E that are finite *a.e.* on E, then show that the functions $\alpha f + \beta g$ and fg are measurable on E, where $\alpha, \beta \in R$.

20. (a) State and prove the Lebesgue Dominated Convergence Theorem.

(b) If f is a bounded function on a set E of finite measure, then show that f is Lebesgue integrable over E if and only if it is measurable.

21. (a) Show that a function f is absolutely continuous on the closed, bounded interval $[a, b]$ if and only if it is an indefinite integral over $[a, b]$.

(b) If f is integrable over the closed, bounded interval $[a, b]$, then show that

$$\frac{d}{dx}\left[\int_a^x f\right] = f(x) \quad \text{for almost all } x \in (a, b)$$

(2 × 5 = 10 Weightage)

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1M2M20081 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2020

MMT2C06 – Algebra II

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each question carries 1 weightage.

1. Show that a field contains no proper nontrivial ideals.
2. Prove that algebraically closed field F has no proper algebraic extensions.
3. Prove that it is not always possible to construct with a straightedge and a compass, a square having area equal to the area of the given circle.
4. Verify whether $F = \{0, 1, \alpha, 1+\alpha\}$ is a subfield of $\overline{\mathbb{Z}_2}$ where $\overline{\mathbb{Z}_2}$ is an algebraic closure of $\mathbb{Z}_2$ containing a zero α of $x^2 + x + 1$.
5. Let $\{\sigma_i : i \in I\}$ be a collection of automorphisms of a field E then show that the set $E_{\{\sigma_i\}}$ of all $\alpha \in E$ left fixed by every σ_i for $i \in I$ form a subfield of E.
6. Describe all extensions of automorphism $\psi_{\sqrt{3},-\sqrt{3}}$ of $\mathbb{Q}(\sqrt{3})$ to an isomorphism mapping $\mathbb{Q}(i, \sqrt{3}, \sqrt[3]{2})$ on to a subfield of $\overline{\mathbb{Q}}$.
7. Define splitting field over a field F. Give one example.
8. Find the number of elements in $G\left(K/_{\mathbb{Q}}\right)$ where K is the splitting field of $x^3 - 1$ over $\mathbb{Q}$.

$(\mathbf{8 \times 1 = 8}\ \boldsymbol{weightage})$

Part B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Prove that $\mathbb{Q}(\sqrt{3}+\sqrt{7}) = \mathbb{Q}(\sqrt{3}, \sqrt{7})$.
10. Show that if F is a field every ideal in $F[x]$ is principal.
11. Prove that if α and β are constructible real numbers then so are $\alpha+\beta, \alpha-\beta, \alpha\beta,$ and α/β if $\beta \neq 0$.

Unit II

12. State and prove Frobenius Automorphism theorem.
13. Define separable extension and show that if K is a finite extension of E and E is a finite extension of F then K is separable over F if and only if K is separable over E and E is separable over F.
14. Show that if F and F' are finite field having the same number of elements then F and F' are isomorphic.

Unit III

15. The regular n-gon is constructible with a compass and a straightedge if and only if all thec primes dividing n are Fermat primes whose square donot divide n.

16. Show that the Galois group of the 5thcyclotomic polynomial over $\mathbb{Q}$ is a cyclic group of or

17. Let $y_1 \dots y_5$ be independent transcendental real numbers over $\mathbb{Q}$. Then show that the polyr $f(x) = \prod_{i=1}^{5}(x - y_i)$ is not solvable by radicals over $F = \mathbb{Q}(s_1 \dots s_5)$ where $s_i$is the i^{th} elementary symmetric function in $y_1 \dots y_5$.

$(6 \times 2 = 12 weigh$

Part C

Answer any two questions. Each question carries 5 weightage.

18. .

a. Let R be a commutative ring with unity, then show that M is a maximal ideal of R only if R/M is a field.

b. Find all $c \in \mathbb{Z}_5$ such that $\mathbb{Z}_5[x] / < x^2 + cx + 1 >$ is a Field.

19. .

a. If F is a finite field of characteristic p with algebraic closure $\bar{F}$, then show that x^{p^n} has p^n distinct zeroes in $\bar{F}$.

b. Show that a finite field GF(p^n) of p^n elements exist for every prime power p^n.

20. .

a. Define primitive nth root of unity in $\mathbb{C}$.

b. Let ξ be a primitive 5th root of unity in $\mathbb{C}$.

i. Show that $\mathbb{Q}(\xi)$ is the splitting field of $x^5 - 1$over $\mathbb{Q}$.

ii. Show that every automorphism of $K = \mathbb{Q}(\xi)$ maps ξonto some power ξ^r o

iii. Using part ii describe the elements of $G\left(K/_{\mathbb{Q}}\right)$.

iv. Give the group and field lattice diagram for $\mathbb{Q}(\xi)$over $\mathbb{Q}$.

21. .

a. Show that every field of characteristic 0 is perfect.

b. State and prove primitive element theorem.

$(2 \times 5 = 10 weig$