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2M1N20103 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2020

MST1C01 – Analytical Tools for Statistics – I

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer any four(2 weightages each)

1. State inverse function theorem.
2. Define Lagrange's Multipliers.
3. Find the real and imaginary part of the complex function: $w = z^2+3z$; where $z=3+5i$.
4. State Cauchy's residue theorem.
5. What is an analytic function? Is $f(z) = e^z$ is analytic? Justify
6. Define residues and poles. Give one example.
7. Obtain the inverse Laplace transform of $\frac{8}{s(s^2+4)}$.

(2 x 4=8 weightages)

PART B

Answer anyfour(3 weightages each)

8. Describe the sufficient conditions for interchange of integration and differentiation of multivariable functions.
9. Obtain the Jacobian of transformation of $f(x,y) = yx^3 + xy^3 - x - y + 5$. Check whether f is invertible or not.
10. Find the Laurent series expansion of $1/(z^2+z-3z)$ in $1<|Z|<2$
11. State and prove Liouville's theorem.
12. Derive Cauchy's integral formula.
13. Find Laplace transformation $L(tCos^3 2t)$
14. Obtain the fourier series of $\frac{1}{(1-x)^2}$

(3x 4=12 weightages)

PART C

Answer any two(5 weightages each)

15. Suppose that D is a closed and bounded set in R^n. If $f: D \to R^m$ is continuous, then it is uniformly continuous in D.

16. (a) Derive necessary and sufficient condition for a function to be analytic

(b) Check whether the complex function $f(z) = az^2$ is analytic and Harmonic or not.

17. (a) Distinguish between zero and singularity of a complex function.

(b) Distinguish between the terms residue at a pole and residue at infinity.

18. Find Laplace transforms of following functions

(a) $t + Sin2t$

(b) e^{-at}

(c) $te^t + tsinht$

(5x2=10 weightages)

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Name: ……………………………

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2020

MST1C02 – Analytical Tools for Statistics – II

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. Define vector space and Subspaces.
2. Distinguish between basis and dimensions
3. Explain rank of product of matrix.
4. Define linear transformations.
5. Show that each eigen value of an idempotent matrix is either 0 or 1.
6. Define minimal polynomial and Characteristic polynomial of a matrix.
7. What are the rank and signature in a quadratic form.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Define linear dependence and independence. Prove that two vectors are linearly dependent if and only if one is a scalar multiple of the other.
9. Show that B= $\{1+x, x+x^2, 1+x^2\}$ is a basis for the set of polynomials of degree less than or equal to two.
10. Show that the null space of T is a subspace of V.
11. Let T be a linear operator on R^2 defined by $T(x, y) = (-y, x)$. Find the matrix of T in the ordered basis $\{(1, 2), (1, -1)\}$.
12. State and prove Cayley-Hamilton theorem
13. Define eigen values, eigen vectors and eigen spaces
14. Show that Moore-Penrose inverse is unique.

(4 x 3= 12 weightage)

Part C

Long Essay Type questions

(Answer any two questions. Weightage 5 for each question)

15. Let w_1 and w_2 are two subspaces of a vector space V over a field K Then Show that

   a) $w_1 + w_2$ is also a vector space over V.

   b) $w_1 \cap w_2$ is also a vector space over V.

   c) $w_1 \cup w_2$ is also a vector space over V, if and only if $w_1 \subseteq w_2$ or $w_1 \supseteq w_2$.

   d) $dim(w_1 + w_2) = dim(w_1) + dim(w_2) - dim(w_1 \cap w_2)$.

16. a) Distinguish between Symmetric and Skew symmetric matrices

   b) Let A be an $n \times m$ real matrix then show that, row rank (A) = column rank (A).

   c) Define Orthogonal and Unitary matrices.

   d) If A and B are two $n \times n$ square matrices, show that $(A+B) = t(A)+tr(B)$.

17. Find the characteristic polynomial and minimal polynomial for $A = \begin{bmatrix} -1 & 0 & 0 \\ 4 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

18. a) Write the necessary and sufficient condition for a non-homogeneous system of linear equation is said to be consistent.

   b) Define a quadratic forms and write the classifications of quadratic forms

   c) Find g-inverse for $A = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

(2 x 5= 10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2020

MST1C03 – Distribution Theory

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any fourquestions.Weightage 2 for each question)

1. Define probability generating function. How will you obtain mean and variance from it?
2. Obtain the distribution of the mean of a random sample taken from Gamma(α.p).
3. Define Pareto distribution. What are its important properties?
4. Obtain the harmonic mean of type-2 beta distribution.
5. Derive the joint distribution of all the order statistics of a random sample of size n from a uniform distribution over (-1,1).
6. Obtain the pdf of sample range based on a random sample of size n from a population with pdf $f(x)=e^{-x}$, $x>0$ and 0 elsewhere.
7. If (X,Y) have a joint pdf given by $f(x,y) = 4xy$, $0<x<1$, $0<y<1$ then find $P(X+Y<1)$.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Derive the pdf of a compound binomial distribution with a Poisson compounding density for the parameter n.
9. Define negative binomial distribution and give a practical situation where this distribution arises. Derive the expressions of its mean and variance.
10. Define multinomial distribution and obtain the correlation coefficient between its component random variables.

11. Obtain the density of the geometric mean of a random sample of size n taken from a lognormal distribution with parameters (μ, σ^2).

12. Obtain the two population regression functions of bivariate normal distribution.

13. Derive the conditional distribution of $X_{(s)}$ given $X_{(r)}$, s>r, where $X_{(r)}$ and $X_{(s)}$ are the r-th and s-th order statistics of a random sample of size n taken from an exponential distribution with mean θ.

14. What is Pearson system of distributions? Obtain the distribution when the roots of the quadratic equation are real and of opposite signs.

(4 x 3= 12 weightage)

Part C
Long Essay Type questions
(Answer any two questions. Weightage 5 for each question)

15. Obtain the probability generating function of Poisson random variable. Deduce the first four factorial moments using the p.g.f. and hence obtain the measures of skewness and kurtosis.

16. (a) If $P(X_n=x|N=n)=1/(n+1)$; n=0,1,…,n and N is a random variable following Poisson with parameter β. Obtain the unconditional distribution of X_n .

(b) If $f(x,y)=(x+y)/4$; $|x|<1$, $|y|<1$ and 0 elsewhere then show that X and Y are not independent but X^2 and Y^2 are independent.

17. (a) Obtain the pdf of sample midrange if a random sample of size n taken from a continuous population with pdf f(x) and CDF F(x).

(b) Show that min $(X_1, X_2, \ldots, X_n)$ follows geometric distribution if and only if X_i 's follow geometric distribution.

18. Define noncentralt-distribution and derive its density.

19. Derive the joint distribution of the sample mean and the variance of a random sample taken from a normal population with mean μ and variance σ^2.

(2 x 5= 10 weightage)

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Name:.........................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2020

MST1C04 – Probability Theory

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. If $A_n \to A$, then show that $A_n^c \to A^c$.
2. Show that every function X on Ω is measurable with respect to the power set of Ω.
3. If $A_n \to A$, then show that $P(A_n) \to P(A)$.
4. If X has the pdf $f(x) = e^{-x}, x > 0$ then find the pdf of log (X)
5. If $X \geq 0$ and is integrable, then show that X can be infinite at most on a set of probability measure zero.
6. Give an example to show that convergence in distribution doesn't imply convergence of moments.
7. State Lindeberg-Feller form of Central Limit Theorem.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Examine the convergence of the sequence of the following sets and if convergent derive the limit.

a) $A_{2n} = \left(0, \frac{1}{2n}\right)$, $A_{2n+1} = \left(-1, \frac{1}{2n+1}\right)$;

b) $A_n = \{the\ set\ of\ rationals\ in\ \left(1 - \frac{1}{n+1},\ 1 + \frac{1}{n}\right)$.

9. Let $(\Omega, \mathcal{F}, P)$ be a probability space with $\Omega = (\omega_1, \omega_2, \omega_3, \omega_4)$. $\mathcal{F}$ is the class of all subsets of Ω and $P(\{\omega_i\}) = \frac{1}{10}, i = 1,2,3,4$. The random variable X is defined as $X(\omega_i) = \begin{cases} 1 \text{ if } i = 1,2 \\ 0 \text{ if } i = 3,4 \end{cases}$. Determine the probability space induced by X.

10. Show that $\varphi(t) = \frac{1}{8}(1 + 7e^{it}),\ t \in R$ is a characteristic function, but $|\varphi(t)|$ is not.

11. State and prove Helly-Bray Lemma.

12. State and prove monotone convergence theorem.

13. State and prove Kolmogorov's three series theorem

14. Let $\{X_n\}$ be independent with $P(X_n = n\alpha) = \frac{1}{4} = P(X_n = -n\alpha),\ P(X_n = 0) = \frac{1}{2}$, where α is a constant. Prove that Lindberg condition holds for $\{X_n\}$.

(4 x 3= 12 weightage)

Part C

Long Essay Type questions
(Answer any two questions. Weightage 5 for each question)

15. (a)Prove that sigma field is a monotone field. Verify whether the converse is true.

    (b) Prove that $\overline{lim}(A_n \cup B_n) = \overline{lim}(A_n) \cup \overline{lim}(B_n)$.

16. (a) State and prove continuity theorem for characteristic functions.

    (b) Find the distribution function of X if the characteristic function is given by

    (i). $\varphi(u) = \frac{1}{4}(1 + e^{iu})^2$ and (ii). $\varphi(u) = (2 - e^{iu})^{-1}$.

17. (a) With the help of an example show that the convergence in distribution need not imply the convergence in probability.

    (b) Prove that a sequence of random variables converges almost surely to a random variable if and only if the sequence converges mutually almost surely.

18. State and prove Kolmogorov inequality.

(2 x 5= 10 weightage)