24

1M1N19078 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2019

MMT1C05– Number Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

(Answer all questions. Each question carries 1 weightage.)

1. Find all integers n such that $\varphi(n) = \frac{n}{2}$.
2. If f and g are arithmetical functions, prove that $(f * g)' = f' * g'$.
3. Prove that $\sum_{n\leq x} \Lambda(n)\left[\frac{x}{n}\right] = \log([x]!)$.
4. Prove that $\lim_{x\to\infty} \frac{\psi(x)}{x} = \lim_{x\to\infty} \frac{\vartheta(x)}{x}$.
5. Prove that for $x \geq 2$, $\vartheta(x) = \pi(x)\log x - \int_2^x \frac{\pi(t)}{t}\,dt$.
6. Prove that Legendre symbol is a completely multiplicative function.
7. Find the plain text of the cipher text 'HTWWXPPE' in the shift cryptosystem with enciphering key $b = 11$ (26-letter alphabet system)
8. Find the inverse of the matrix $\begin{bmatrix} 3 & 11 \\ 15 & 22 \end{bmatrix} (mod\ 29)$

(8 ×1= 8 weightage)

Part B

(Answer any two questions from each unit.Each question carries 2 weightage.)

UNIT 1

9. Prove that $\forall\, n \geq 1,\ \varphi(n) = \sum_{d/n} \mu(d)\frac{n}{d}$
10. $\forall\, x \geq 2$, prove that $\sum_{2\leq n\leq x} \frac{1}{n\log n} = \log(\log x) + B + O\left(\frac{1}{x\log x}\right)$, where B is a constant.
11. Prove that $\forall\, x \geq 2, \sum_{p\leq x} \left[\frac{x}{p}\right] \log p = x\log x + O(x)$

UNIT II

12. For $x > 0$, prove that $0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}$.

13. Prove that $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$ if and only if $\lim_{x\to\infty} \frac{\pi(x)\log \pi(x)}{x} = 1$.

14. Prove that $\lim_{x\to\infty} \left(\frac{M(x)}{x} - \frac{H(x)}{x\,log\,x}\right) = 0$.

UNIT III

15. Let p be an odd prime. Then prove that $\sum_{\substack{r=1 \\ (r|p)=1}}^{p-1} r = \frac{p(p-1)}{4}$ if $p \equiv 1(mod\ 4)$.

16. Determine the odd prime p for which 3 is a quadratic residue and those for which it is a non residue.

17. Explain (a) Frequency Analysis in cryptanalysis.

(b) Hash Function.

(6 × 2= 12 weightage)

Part C

(Answer any two questions. Each question carries 5 weightage.)

18. (a) Prove that for every $n \geq 1$, $\sum_{d/n} \lambda(d) = \begin{cases} 1 & if\ n\ is\ a\ square \\ 0 & otherwise \end{cases}$ and $\lambda(n) = |\mu(n)|$.

(b) State and prove Legendre's identity.

19. State and prove Abel's identity from this deduce Euler summation formula.

20. (a) State and prove quadratic reciprocity law for Legendre symbol.

(b) Determine whether 888 is a quadratic residue or non residue modulo 1999.

21. (a) Explain the advantages and disadvantages of public key crypto system as compared to classical crypto system.

(b)Describe RSA cryptosystem.

(2 × 5= 10 weightage)

25

1M1N19077 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2019

MMT1C04– Discrete Mathematics

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A (Short Answer Questions) (1-8)
Answer all questions.
Each question carries 1 weightage

1. Define a lattice. Give an example.
2. Define disjunctive normal form and give an example of it.
3. Draw the Hasse diagram of divisors of 45. Is it a chain? Justify your answer.
4. If G be a graph with n vertices and m edges and δ and Δ are, respectively, the minimum and maximum of the degree of a graph G, then show that $\delta \leq \frac{2m}{n} \leq \Delta$.
5. Prove that $e = xy$ of a graph G is a cut edge of a connected graph G if and only if, e does not belong to any cycle of G.
6. Prove that the Petersen graph is nonplanar.
7. Find a grammar that generates the language $L = \{a^n b^{2n} : n \geq 0\}$ on $\Sigma = \{a, b\}$.
8. Find a dfa that accepts all strings on {0, 1} except those containing the substring 001.

(8 x1 =8 weightage)

Part B

Answer any two questions from each unit. Each question has 2 weightage.

UNIT - I

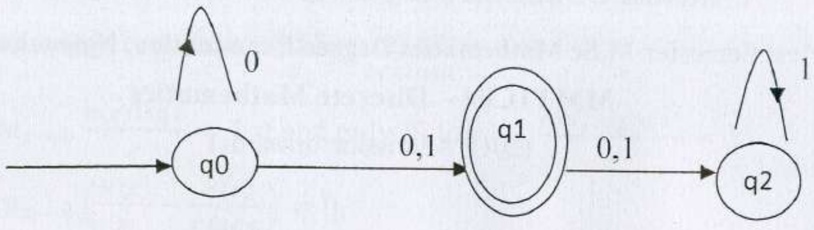
9. Let X = RU{*} where * is some point not on the real line. Define ≤ on X as $\{(x, y) \in R \times R : x \leq y \text{ in the usual order }\} \cup \{(*,*)\}$. Prove that ≤ is a partial order on X.
10. Write the following Boolean function in the disjunctive normal form
$F(x_1, x_2, x_3) = (x_1+x_2+x_3)(x_1'+x_2+x_3')(x_1+x_2'+x_3')(x_1'+x_2'+x_3')(x_1+x_2+x_3')$.
11. Prove that an edge is a cut edge if and only if it belongs to no cycle.

UNIT - II

12. Prove that every connected graph contains a spanning tree.
13. State and prove Whitneys theorem on 2-connected graphs.
14. Derive the Euler's formula for a connected plane graph.

UNIT - III

15. Find a dfa equivalent to the following nfa



16. Are the grammars $G_1 = (\{S\}, \{a,b\}, S, \{ S \to SS, S \to aSb, S \to \lambda, S \to bSa\})$ and $G_2 = (\{S\}, \{a, b\}, S, \{ S \to SS, S \to SSS, S \to aSb, S \to bSa, S \to \lambda \})$ are equivalent.

17. Let $\sum = \{a, b, c\}$. Construct a deterministic finite accepter that accepts the language $a\sum^*b$.

(6 x 2 = 12 weightage)

Part C

Answer any two from the following four questions (18-21)
Each question carries 5 weightage.

18. (a) Let (X, +, . ,') be a finite Boolean algebra , Prove that every element of X can be uniquely expressed as sum of atoms.

(b) Prove that $(X, \leq)$ is a lattice where (X,+, . ,') is the Boolean algebra and $\leq$ is defined in X by $x \leq y$ if and only if $x.y' = 0$. Find the maximum and minimum elements of this lattice.

19. Prove that the following statements are equivalent for a connected graph G

(a) G is Eulerian.

(b) The degree of each vertex of G is an even positive integer.

(c) G is an edge disjoint union cycles.

20. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

(b) Prove that for any loopless connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$

21. (a) Show that the language L={awa: $w \in \{a, b\}^*$} is regular.

(b) Let L be the language accepted by a non deterministic finite accepter $M_N = (Q_N, \sum, \delta_N, q_0, F_N)$. Then prove that there exist a dfa $M_D = (Q_D, \sum, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$.

(2 x 5 = 10 weightage)

26

1M1N19076 (Pages : 2) Reg. No:.............................

Name:...............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2019

MMT1C03– Real Analysis - I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A: Answer all questions (Each question carries 1 weightage)

1. Construct a bounded set of real numbers with exactly three limit points.
2. Let E^0 denote the set of all interior points of a set E. Prove that the complement of E^0 is the closure of the complement of E.
3. Give an example of an open cover of the segment $(0,1)$ which has no finite subcover.
4. Let f and g be continuous mapping of a metric space X into a metric space Y, and let E be a dense subset of X. Prove that $f(E)$ is dense in $f(X)$.
5. Suppose α increases on $[a,b]$, $a \leq x_0 \leq b$, α is continuous at x_0, $f(x_0) = 1$, and $f(x) = 0$ if $x \neq x_0$. Prove that $f \in \mathcal{R}(\alpha)$ and that $\int f d\alpha = 0$.
6. Define equicontinuous functions and give an example.
7. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
8. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, prove that $\{f_n + g_n\}$ converges uniformly on E.

(8 ×1= 8)

Part B- Answer any two from each unit (Each question carries 2 weightage)

Unit I

9. Show that a set E is open if and only if its complement is closed.
10. Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
11. Suppose f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then prove that $f(E)$ is connected.

Unit II

12. Let f be defined on $[a,b]$; if f has a local maximum at a point $x \in (a,b)$, and it $f'(x)$ exists, then show that $f'(x) = 0$.
13. Suppose f is continuous mapping of $[a,b]$ into $\mathbb{R}^k$ and f is differentiable in (a,b) . Then prove that there exists $x \in (a,b)$ such that $|f(b) - f(a)| \leq (b-a)|f'(x)|$.

14. Suppose f is bounded on $[a, b]$, f has only finitely many points of discontinuity on $[a, b]$, and α is continuous at every point at which f is discontinuous . Then prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$

Unit III

15. If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|dt$.

16. State and prove Cauchy criterion for uniform convergence.

17. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

(6 ×2= 12)

Part C - Answer any two (Each question carries 5 weightge)

18. (a) If a set E in $\mathbb{R}^k$ has one of the following three properties, then prove that it has the other two:
    a) E is closed and bounded.
    b) E is compact.
    c) Every infinite subset of E has a limit point in E.

    (b) Construct a compact set of real numbers whose limit points form a countable set.

19. (a) Show that $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$ is not differentiable at 0.

    (b) State and prove L'hospital's rule.

20. (a) If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$ then prove that $fg \in \mathcal{R}(\alpha)$, $|f| \in \mathcal{R}(\alpha)$ and $| \int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$.

    (b) State and prove the fundamental theorem of Calculus.

21. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. $If \{f'_n\}$ converges uniformly on $[a, b]$ then show that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f, and $f'(x) = \lim_{n\to\infty} f'_n(x)$ $(a \leq x \leq b)$.

    (b) Give an example of a sequence of pointwise bounded functions on a set E which is not uniformly bounded.

(2 ×5 = 10)

27

1M1N19075 (Pages : 2) Reg. No:………………………...

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2019

MMT1C02– Linear Algebra

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part- A

Answer *all* questions. Each question has *one* weightage.

1. Is the collection of all ordered pairs (x, y) of real numbers with the operations $(x, y)+(x_1, y_1)=(x+x_1, y+y_1)$ and $c(x, y)=(x, cy)$, a vector space over $\Re$?
2. Is there a linear transformation $T:\Re^3 \rightarrow \Re^2$ such that $T(1,-1,1)=(1,0)$, and $T(1,1,1)=(0,1)$? If yes, find one such linear transformation.
3. Let $T: R^3 \rightarrow R^3$ be the linear map defined by $T(x, y, z)=(x+y, 0, z)$. Find the rank of T.
4. Let T be the linear operator on R^3 defined by $T(x, y, z)=(-z, x, -y)$. Find the matrix of T in the standard ordered basis for R^3.
5. If A and B are $n \times n$ complex matrices, show that $AB-BA=I$ is impossible.
6. If V is an n-dimensional vector space over F, find the characteristic polynomials for identity operator and zero operator.
7. Prove that if E is the projection on R along N, then (I – E) is the projection on N along R.
8. Let V be an inner product space and let α, β be vectors in V. Show that $\alpha=\beta$ if and only if $(\alpha \mid \gamma)=(\beta \mid \gamma)$ for every γ in V.

(8 × 1 = 8 Weightage)

Part- B

Answer any *two* from each unit. Each question has *two* weightage

Unit - I

9. Show that a linear transformation maps a linearly dependent set in to a linearly dependent set. Is it true that any linearly independent set is mapped to another linearly independent set under a linear transformation?
10. Show that any vector space of dimension n over F is isomorphic to F^n.
11. Find a basis and the dimension of the linear subspace of $\Re^n$ given by $\{(x_1, x_2, \ldots, x_n) \in \Re^n / x_1=0,\ x_2+\ldots+x_n=0\}$.

Unit -II

12. If W is a subspace of a finite dimensional vector space V, then show that

$dim\ W + dim\ W^0 = dim\ V$

13. Let V and W be finite dimensional vector spaces over the field F, and let T be a linear transformation from V into W. Show that range of T' is the annihilator of the null space of T.

14. Let T be a linear operator on V. If every subspace of V is invariant under T, show that T is a scalar multiple of the identity operator.

Unit – III

15. Let E be a projection of V and let T be a linear operator on V. Prove that both the range and null space of E are invariant over T if and only if $ET = TE$.

16. Show that $(\alpha | \beta) = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$, where $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$, defines an inner product on $\Re^2$.

17. State and prove Bessel's inequality.

(6× 2 = 12 Weightage)

Part- C

Answer any *two* from the following four questions. Each question has *Five* weightage.

18. (a) If W_1 and W_2 finite dimensional subspaces of a vector space V, then show that

W_1+W_2 is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

(b) Suppose that $\{x_1, \ldots, x_m\}$ are linearly independent vectors in a vector space V, but $\{y\} \cup \{x_1, \ldots, x_m\}$ is linearly dependent. Then show that y can be written as a unique linear combination of $\{x_1, \ldots, x_m\}$.

19. (a) State and prove *rank – nullity* theorem.

(b) If T is a linear operator on an n – dimensional vector space V whose range and null space are identical, then show that n is even.

20. (a) Show that minimal polynomial and characteristic polynomial for a linear operator have the same roots except for multiplicities.

(b) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then, show that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \cdots (x - c_k)$ where $c_1, \ldots, c_k$ are distinct elements of F.

21. If V is a real or complex vector space with an inner product $(|)$, then for any α, β in V, prove that (a) $(\alpha | \beta) = 0, \forall \beta \in V \Rightarrow \alpha = 0$.

(b) $|(\alpha | \beta)| \le \|\alpha\| \, \|\beta\|$, and

(c) $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$

(2 × 5 = 10 Weightage)

28

1M1N19074 (Pages : 2) Reg. No:.............................

Name:.............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2019

MMT1C01– Algebra – I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions (Each question carries 1 weightage)

1. Give an example of an isometry of the plane which leaves the X – axis fixed..
2. Find all proper nontrivial subgroups of $Z_2 \ X \, Z_2$.
3. Prove that a direct product of abelian groups is abelian.
4. A Sylow 3-subgroup of a group of order 54 has order____________.
5. Find the reduced form and the inverse of the reduced form of $a^2b^{-1}b^3a^3c^{-1}c^4b^{-2}$.
6. Determine whether the polynomial $25x^5-9x^4-3x^2 - 12$ is irreducible over Q.
7. List all the elements of the group whose presentation is $(a, b : a^2 = 1, b^2 = 1, ab = ba)$
8. Let G = {e, a, b} be a cyclic group of order 3 with identity element e.

   Write the element (2e + 3a + 0b) (4e + 2a + 3b) in the group algebra $Z_5(G)$ in the form re + sa + tb for r, s, t $\in Z_5$.

(8 x 1 =8 weightage)

Part B

Answer any two from each unit (Each question carries 2 weightage)

UNIT I

9. Find the order of $5 + < 4 >$ in the group $Z_{12} /< 4 >$.
10. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.
11. State and Prove Burnside's Formula.

UNIT II

12. Let $\phi : Z_{12} \rightarrow Z_3$ be the homomorphism such that $\phi(1) = 2$

   a. Find the kernel K of ϕ.

   b. List the cosets in Z_{12} / K, showing the elements in each coset.

13. Give the isomorphic refinements of the two series:

   $\{0\} < 10Z < Z$ and $\{0\} < 25Z < Z$

14. Prove that no group of order p^r is simple for $r > 1$.

UNIT III

15. Consider the evaluation homomorphism $\phi_3 : Z_7[x] \rightarrow Z_7[x]$.

   Compute $\phi_3[(x^4 + 2x)(x^3 - 3x^2 + 4)]$.

16. Let G be a finite group of the multiplicative group $(F^*, .)$. Prove that G is cyclic.

17. Let R be a commutative ring and let $a \in R$. Show that $I_a = \{x \in R / ax = 0\}$ is a ideal of R.

(6 x 2 =12 weightage)

Part C

Answer any two (Each question carries 5 weightage)

18. Let X be a G – set and let $x \in X$.

   a. Prove that $G_x = \{g \in G / gx = x\}$ is a subgroup of G.

   b. Prove that $|Gx| = (G: G_x)$, where Gx is the orbit of x.

19. a. Prove that the group $Z_m \times Z_n$ is cyclic and is isomorphic to Z_{mn} if and only if m and n are relatively prime.

   b. State and Prove First Sylow Theorem.

20. a. Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Prove that G is isomorphic to $H \times K$.

   b. For a prime number p, Prove that every group G of order p^2 is abelian.

21. a. State and Prove Division Algorithm for F[x], where F is a field.

   b. State and Prove Factor Theorem.

(2 x 5 =10 weightage)