1N18077 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MT1C04– Number Theory

(2017 Admission onwards)

ax. Time: 3 hours Max. Weightage : 36

Part A

(Answer all questions.)

1. If $n \geq 1$, prove tat $\sum_{d/n} \mu(d) = \left[\frac{1}{n}\right]$.
2. Prove that for all ≥ 1, $\log n = \sum_{d/n} \Lambda(d)$.
3. Prove that the number of positive divisors of n is odd if and only if n is a square.
4. Prove that $\varphi^{-1}(n) = \prod_{p/n}(1-p)$.
5. Prove that $(f * g)' = f' * g + f * g'$.
6. Prove that $[x] + \left[x + \frac{1}{2}\right] = 2[x]$, for every real number.
7. Define Chebyshev'sψ-functionand ϑ-function and prove that $\psi(x) = \sum_{m \leq x} \vartheta\left(x^{\frac{1}{m}}\right)$.
8. Find the quadratic residues and non residuesmodulo 13.
9. Prove that Legendre's symbol $(n|p)$ is a completely multiplicative function of n.
10. Determine whether 5 is a quadratic residue or non residuemodulo 1111.
11. Using shift cryptosystem with 27-letter alphabet system (consisting of $A - Z$ and a blank, where A-Z have numerical equivalants $0 - 25$, blank=26) and enciphering key 9, find the cipher text of 'TODAY AT SIX PM'.
12. Describe about affine cryptosystem and find a formula for the number of different affine enciphering transformations with an N-letter alphabet.
13. Write a note on authentication in cryptography.
14. Find the inverse of the matrix $\begin{bmatrix} 15 & 17 \\ 4 & 9 \end{bmatrix} (mod\ 26)$.

(14×1= 14 weightage)

Part B

(Answer any 7 questions.

15. If $n \geq 1$, prove that $\varphi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$.

16. State and prove Mobius Inversion formula.

17. For every $n \geq 1$, prove that $\sum_{d/n} \lambda(d) = \begin{cases} 1, & \text{ifn is a square} \\ 0, & \text{otherwise} \end{cases}$.

18. Prove that for all $x \geq 2$, $log([x]!) = x\, logx - x + O(logx)$.

19. Prove that if $\lim_{x\to\infty} \frac{\pi(x) logx}{x} = 1$, then $\lim_{x\to\infty} \frac{\vartheta(x)}{x} = 1$.

20. Prove that there is a constant A such that $\sum_{p \leq x} \frac{1}{p} = \log\log x + A + O\left(\frac{1}{\log x}\right), \forall\, x \geq 2$.

21. State and prove Euler's criterion for Legendre's symbol.

22. Prove that the Diophantine equation $y^2 = x^3 + k$ has no solutions if k has the form $k = (4n-1)^3 - 4m^2$ where m and n are integers such that no prime $p \equiv -1(mod\ 4)$divides m.

23. Solve the system of simultaneous congruences

    $2x + 3y \equiv 1(mod\ 26)$

    $7x + 8y \equiv 2(mod\ 26)$

24. Describe about RSA cryptosystem.

(7×2= 14 weightag

Part C

(Answer **any** 2 questions.

25. Prove that the set of all arithmetical functions$f$ with $f(1) \neq 0$ is an abelian group und Dirichlet multiplication.

26. State Euler's summation formula. Hence deduce the following results

    a. $\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + O(x^{-s})$, if $s > 0, s \neq 1$

    b. $\sum_{n > x} \frac{1}{n^s} = O(x^{1-s})$ if $s > 1$

27. State Abel's Identity and deduce Euler's Summation formula from Abel's identity.

28. State and prove quadratic reciprocity law for Legendre's symbol.

(2× 4= 8 weightag

1N18078 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MT1C05– Discrete Mathematics

(2017 Admission onwards)

ax. Time: 3 hours Max. Weightage : 36

PART A

Answer all. 1 weightage each.

1. Prove that intersection of two chains is a chain.
2. Give an example to show that when a maximal element is unique, it need not be maximum element.
3. If x, y are elements of a Boolean algebra, prove that $x = y$ if, and only if, $xy' + x'y = 0$.
4. Define conjunctive normal form and give an example of it.
5. Show that in any group of two or people, there are always two with exactly same number of friends inside the group.
6. Show that if G is a self-complementary graph of order n, then $n \equiv 0$ or $1 \ (mod\ 4)$.
7. Define normal product of two simple graphs.
8. Show that a tree with at least two vertices contains at least two pendant vertices.
9. If $e = xy$ is not a cut edge of the graph G, prove that e belongs to a cycle of G.
10. Let G be a plane graph and f be a face of G. Then show that there exists a plane embedding of G in which f is the exterior face.
11. Let $\Sigma = \{a, b, c\}$ and $L = \{a, b\}$. Find L^+ and L^2.
12. Find a grammar that generates the language $\{a^n b^m : n \geq 0, m > n\}$.
13. Define non-deterministic accepter and give an example of it.
14. Find a language corresponding to the regular expression $(a + b)^*(a + bb)$.

(14 x 1 = 14 weightage)

PART B

Answer any seven. 2 Weightage each.

15. Let $X = R \cup \{*\}$ where $*$ is some point not on the real line. Define $\leq$ on X as $\{(x, y) \in R \times R : x \leq y$ in the usual order$\} \cup \{(*,*)\}$. Prove that $\leq$ is a partial order on X.
16. Prove that the characteristic numbers of a Boolean function completely determine it.
17. Write the Boolean function $f(a, b, c) = a + b + c'$, in their disjunctive normal form.
18. Show that a graph is bipartite if, and only if, it contains no odd cycle.

19. Show that the connectivity and edge connectivity of a simple cubic graph G are equal.

20. Show that a tree with n vertices has $n-1$ edges and conversely, a connected graph with n vertices and $n-1$ edges is a tree.

21. If G is a simple planar graph with at least three vertices, prove that $m(G) \leq 3n(G) - 6$. Use this result to show that the complement of a simple planar graph with 11 vertices is non-planar.

22. Construct a grammar for the language $L = \{a^n b^m : n \geq 0, m > n\}$.

23. Construct a non deterministic accepter that accepts the language $\{ab, abc\}^*$.

24. Find a regular expression for the language

$L = \{w \in \{0,1\}^* : w \text{ has no pair of consecutive zeros}\}$

(7 x 2 = 14 weightage)

PART C
Answer any Two. 4 Weightage each

25. Let $(X, +, ., ')$ be Boolean algebra. If $x, y \in X$ define $x \leq y$ if $x.y = 0$. Prove that $(X, \leq)$ is a lattice. Find the maximum and minimum elements of this lattice.

26. (a). Prove that a graph G with at least three vertices is 2-connected if, and only if, any two vertices of G are connected by at least two internally disjoint paths.

(b). Show that in a 2-connected graph G, any two longest cycles have at least two vertices in common.

27. For a connected graph G, show that the following statements are equivalent.

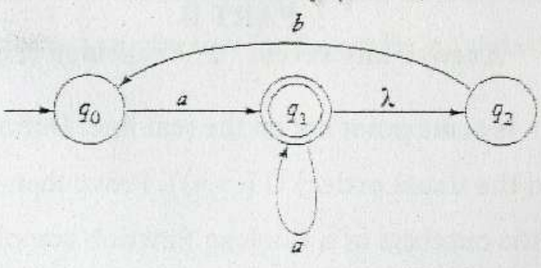
i. G is Eulerian

ii. The degree of each vertex of G is an even positive integer.

iii. G is an edge-disjoint union of cycles.

28. (a). Let L be the language accepted by a non-deterministic finite accepter. Then prove that there exists a deterministic finite accepter M_D such that $L = L(M_D)$.

(b). Find a dfa equivalent to the following nfa



(2 x 4 = 8 weightage)

34 A

M1N18074 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MT1C01– Algebra – I

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer all the questions (Each Questions has weightage one)

1 Show that every group of order 15 is cyclic.

2 Define decomposable and indecomposable groups.

3 Find the order of the factor group $z_4 \times z_2/\langle(2,1)\rangle$.

4 Find the center of the group S_3.

5 Show that the polynomial $25x^5-9x^4-3x^2-12$ is irreducible over $\mathbb{Q}$

6 Give an example of a normal series of z.

7 Show that the group S_3 is solvable.

8 Find the class equation of S_3.

9 Find all abelian groups of order 360(up to isomorphism).

10 If H is a subgroup of a group G, then show that G is an H-set.

11 Find the multiplicative inverse of $a_1+a_2i+a_3j+a_4k$, not all $a_i=0$, in quaternions $\mathbb{H}$ under quaternion multiplication.

12 Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$, and let C be the subring of F consisting of all the constant functions in F. Is C is an ideal in F ?.

13 Let $H=\langle 4\rangle$ and $N=\langle 6\rangle$ be subgroups of z_{24}. Find H+N and H∩N.

14 Find the reduced form and the inverse of the reduced form of the word, $a^2b^{-1}b^3a^3c^{-1}c^4b^{-2}$.

(14 x 1=14 Weightage)

Part B

Answer Any Seven Questions
(Each Question has Weightage two)

15 If G has a composition series, and if N is a proper normal subgroup of G, then show that there exists a composition series containg N.

16 For a prime number p,show that every group G of order p^2 is abelian.

17 Show that $(x,y : y^2x=y , yx^2y=x)$ is a presentation of the trivial group of one element.

18 Show that an element $a \in F$ is a zero of $f(x) \in F[x]$ if and only if x-a is a factor of f(x) in F[x].

19 Show that the set End(A) of all endomorphisms of an abelian group A forms a ring under homomorphism addition and homomorphism multiplication.

20 Show that factor group of a cyclic group is cyclic.

21 If H and K are finite subgroups of a group G, then show that $|HK|=\frac{(|H|)(|K|)}{|H\cap K|}$

22 Show that the converse of the theorem of Lagrange is false.

23 If m is a square free integer then show that every abelian group of order m is cyclic.

24 Show that no group of order 48 is simple.

(7 x 2=14 Weightage)

Part C

Answer Any Two Questions
(Each Question has Weightage Four)

25 If G is a finite group and X is a finite G-set, then show that (number of orbits in X under G) $=\frac{1}{|G|}\sum_{g\in G}|X_g|$

26 a) Let P_1 and P_2 be Sylow p-subgroups of a finite group G. Then show that P_1 and P_2 are conjugate subgroups of G.

b) If G is a finite group and p divides |G|, then show that the number of Sylow p-subgroups is congruent to 1 modulo p and divides |G|.

27 State and prove division algorithm for F[x].

28 a) Show that M is maximal normal subgroup of G if and only if G/M is simple.

b) Compute the factor group $z_4 \times z_6/<(0,2)>$.

(2 x 4=8 Weightage)

34 B

M1N18075 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MT1C02– Linear Algebra

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part- A

Answer *all* questions.

Each question has *one* Weightage.

1. Is the set of all polynomials of degree exactly n a vector space over $\Re$?
2. Is the set $\{(1,0,-1),(2,1,2),(3,-2,0)\}$ forms a basis for $\Re^3$?
3. Consider the functions $f(x)=|x|e^{\alpha x}$ and $g(x)=xe^{\alpha x}$ for $x\in\Re$. Then prove that the pair $\{f,g\}$ is linearly independent on $\Re$.
4. Find the coordinate matrix of $(4,-1,-3)$ relative to the ordered basis $\{(1,0,-1),(1,1,1),(1,0,0)\}$ of $\Re^3$.
5. What is the rank and nullity of identity map on $\Re^3$?
6. Give an example of a non zero linear operator T such that $T^2 = 0$.
7. What do you mean by the dual space of a vector space?
8. What are the hyper spaces in $\Re^3$?
9. Define the transpose of a linear transformation
10. Show that similar matrices have the same characteristic polynomial.
11. If I denotes the identity linear transformation on an n-dimensional vector space, find the minimal polynomial for I.
12. State Cayley – Hamilton theorem.
13. In an inner product space X, prove that $(0\,|\,\beta)=0$ for all $\beta\in X$.
14. State Bessel's inequality.

(14 x 1 = 14 Weightage)

Part- B

Answer any *seven* from the following ten questions.

Each question has *two* Weightage

15. Prove that any set of vectors that includes the zero vector is linearly dependent.
16. Show that $\Re$ over Q is not finite dimensional.
17. Show that any set of linearly independent vectors in a vector space V is either a basis for V, or it can be extended to a basis for V.

18. Describe explicitly a linear transformation on $\Re^3$ whose range is spanned by $(1,0,-1)$ and $(1,2,2)$.

19. If $T:V \to W$ is a linear transformation, show that T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

20. Show that minimal polynomial and characteristic polynomial for a linear operator have the same roots except for multiplicities.

21. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then, show that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

22. Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T if and only if $ETE = TE$.

23. Show that an orthogonal set of non-zero vectors in an inner product space is linearly independent.

24. For any $\alpha \in \Re^2$, with standard inner product, show that $\alpha = (\alpha \mid e_1)e_1 + (\alpha \mid e_2)e_2$, where $\{e_1, e_2\}$ is the standard basis for $\Re^2$.

(7 x 2 = 14 Weightage)

Part- C

Answer any *two* from the following four questions.
Each question has *four* Weightage.

25. (a) If V is any n-dimensional vector space, then show that no subset of V that contains less than n vectors can span V.
(b) If V is a finite dimensional vector space, then show that any two basis of V have the same number of elements.

26. Define a linear transformation. If V is an $n-$dimensional vector space and W be an m dimensional vector space over the field F, show that $L(V, W)$ is finite dimensional and has dimension mn.

27. (a) If V is a finite dimensional vector space over the field F and if W is a subspace of V, prove that $\dim W + \dim W^0 = \dim V$.
(b) If S is any subset of a finite dimensional vector space V, then show that $(S^0)^0$ is the subspace spanned by S.

28. (a) In an inner product space V, prove that $|(\alpha \mid \beta)| \leq \| \alpha \| \| \beta \|$ for all $\alpha, \beta \in V$.
(b) Apply Gram – Schmidt process to the vectors $(1,0,1), (1,0,-1)$ and $(0,3,4)$ to obtain an orthonormal basis for $\Re^3$ with the standard inner product.

(2 x 4 = 8 Weightage)

34 C

M1N18076 (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MT1C03– Real Analysis – I

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part- A

Answer *all* questions. Each question has *one* weightage.

1. Show that the closure of a set E in a metric space X is the smallest closed set in X that contains E.
2. Give an example for a closed subset of real numbers which is not connected.
3. Define perfect sets. Give an example for a perfect set which is not compact.
4. Is $d(x, y) = |x^2 - y^2|$ defines a metric on R ? Justify your answer.
5. If $f:[a,b] \to [a,b]$ is a continuous function, then prove that $f(x) = x$ for at least one $x \in [a,b]$.
6. Give examples for functions with discontinuity of first and second kind.
7. Show that a uniformly continuous function of a uniformly continuous function is uniformly continuous.
8. State Taylor's theorem.
9. If f is defined on $[a,b]$ and if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then show that $f'(x) = 0$.
10. Suppose that f is a continuous, non negative real valued function on $[a,b]$ and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all $x \in [a,b]$.
11. Define rectifiable curves. Give one example.
12. If $\{f_n\}$ and $\{g_n\}$ converges uniformly on a set E, then prove that $\{f_n + g_n\}$ converges uniformly on E.
13. Is it true that every uniformly bounded sequence of functions has a uniformly convergent subsequence? Justify.
14. What do you mean by an equicontinuous family of functions?

(14 x 1 = 14)

Part- B

Answer any *seven* from the following ten questions.
Each question has *two* weightage

15. Show that a set is closed if and only if its complement is open.
16. If E is an infinite subset of a compact set K, then show that E has a limit point in K.
17. Prove that every closed subset of a compact set is compact.
18. Let f be a continuous real valued mapping on a metric space X and let $Z(f)$ be the set of all $p \in X$ such that $f(p) = 0$. Show that $Z(f)$ is closed.
19. Show that monotone functions have no discontinuities of second kind.
20. If f is a continuous mapping of $[a,b]$ into R^k, and if f is differentiable in (a,b), then show that there exists $x \in (a,b)$ such that $|f(b) - f(a)| \le (b-a)|f'(x)|$.
21. Show that $f \in \Re(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$, there exists a partition P of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$.
22. If $f \in \Re(\alpha)$ on $[a,b]$, then prove that $|f| \in \Re(\alpha)$ and $\left|\int_a^b f d\alpha\right| \le \int_a^b |f| d\alpha$.
23. State and prove the Cauchy criterion for uniform convergence of a sequence of functions.
24. Show that there exists a real continuous function on the real line which is nowhere differentiable.

(7 x 2 = 14)

Part- C

Answer any *two* from the following four questions.
Each question has *four* weightage.

25. (a) Prove that a subset E of the real line is connected if and only if E has the property: If $x, y \in E$ and $x < z < y$, then $z \in E$.

    (b) Show that every non empty perfect set in R^k is uncountable.

26. (a) If f is a real differentiable function on $[a,b]$ and if $f'(a) < \lambda < f'(b)$, then show that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

    (b) Suppose that $\{f_n\}$ is a sequence of functions defined on E, and that $|f_n(x)| \le M_n$ for all $x \in E$ and for $n = 1,2,3,...$, then show that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

27. (a) Suppose that $f \in \Re(\alpha)$ on $[a,b]$, $m \leq f \leq M$, ϕ is continuous on $[m,M]$ and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in \Re(\alpha)$ on $[a,b]$.

(b) Give an example to show that limit of the integral need not be equal to the integral of the limit.

28. (a) If $\{f_n\}$ is an equicontinuous sequence of functions on a compact set K and if $\{f_n\}$ converges pointwise on K, then prove that $\{f_n\}$ converges uniformly on K.

(b) If K is a compact metric space, if f_n are continuous functions on K and if $\{f_n\}$ converges uniformly on K, then show that $\{f_n\}$ is equicontinuous on K.

(2 x 4 = 8)