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FAROOK COLLEGE (AUTONOMOUS), DEPARTMENT OF STATISTICS

First Semester M.Sc Degree Examination, November 2018

MSTA1B01 – Measure Theory and Integration

Time : 3 hours Maximum Weightage : 36

Answer Scheme

Part A

(Answer **all** questions; each question carries 1 weightage)

1. Definition : 0.5 wt, Proof : 0.5 wt
2. $\sigma(C) = \{\phi, A_1, A_2, \Omega\}$: 1 wt
3. Definition : 0.5 wt, Example : 0.5 wt
4. Definition of f^+ and f^- : 0.75 wt, Proof : 0.25 wt
5. Definition : 0.5 wt, Example : 0.5 wt.
6. Statement : 1 wt
7. Definition : 0.75 wt, Example : 0.25 wt
8. Definition : 1 wt
9. Convergence of monotone sequence of sets : 0.75 wt, Convergence of arbitrary sequence of sets : 1 wt
10. Product sigma field : 0.5 wt, Product measure : 0.5 wt
11. Definition : 0.5 wt, Two properties : 0.5 wt
12. Definition : 1 wt

(12x1=12 weightage)

Part B

(Answer **any eight** questions; each question carries 2 weightage)

13. Complete proof : 2 wt
14. Defining an increasing sequence of simple functions : 1.5 wt, Complete proof : 2 wt
15. Four proofs : 0.5 wt each
16. Statement : 0.25 wt, Proof : 1.75 wt
17. Definition : 0.5 wt, Proof : 1.5 wt
18. Statement : 0.5 wt , Proof : 1.5 wt
19. Definition : 1 wt, Proof : 1 wt
20. Definition: 1 wt, Proof : 1 wt
21. Statement : 1 wt, Proof : 1 wt
22. Definition: 1 wt, Problem : 1 wt
23. Definitions of Lebesgue measure and Lebesgue Stieltjes measure : 1.5 wt, Proof : 0.5 wt
24. Statement : 1.5 wt, Importance : 0.5 wt

(8x2=16 weightage)

Part C

(Answer **any two** questions; each question carries 4 weightage)

25. i) Definition of integral of simple function : 1 wt, definition of integral of arbitrary measurable function : 1 wt
    ii) Proof : 2 wt
26. i) Statement : 0.5 wt, Proof : 1.5 wt
    ii) Definitions : 1 wt each
27. Definition : 1 wt, Statements : 1 wt, Proofs : 2 wt
28. Statement : 1 wt, Proof : 3 wt

(2x4=8 weightage)

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M1N18104 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MSTA1B02 – Analytical Tools for Statistics – I

(2017Admission onwards)

Max. Time: 3 hours Max. weightate : 36

Part A

(Answer ALL the questions. Weightage 1 for each question)

1. Define functions of bounded variation in an interval.
2. Briefly describe conditions for integrability.
3. State mean value theorem. Illustrate with an example.
4. Define point wise convergence of a sequence of functions.
5. State Weirstrass approximation theorem.
6. Find $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}+\sqrt{y}}$.
7. Find the values of any local maxima and minima of the function $f(x) = x^2 - 4$, for the domain $-2 \leq x \leq 2$.
8. Briefly explain the concept 'directional derivative'.
9. State inverse function theorem for a multivariable function.
10. Define Laplace transform and inverse Laplace transform.
11. Describe properties of the Fourier transform.
12. State Fourier integral theorem.

(12 x 1=12 weightage)

Part B

(Answer any EIGHT questions. Weightage 2 for each question)

13. Let $f_n(x) = \frac{x^2}{(1+x^2)^2}$, $x \in R$, $n \geq 1$ and $f(x) = \sum_n f_n(x)$. Check whether f_n is continuous.
14. State and prove the first fundamental theorem of integral calculus..
15. Show that the sequence of functions $\{f_n\}$, where $f_n(x) = \frac{x}{1+n\,x^2}$, $x \in R$ converges uniformly on any closed interval [a,b].
16. Prove: If $\{f_n\}$ be a sequence of functions such that $\lim_{n\to\infty} f_n(x) = f(x)$ for all $x \in [a,b]$ and let $M_n = \sup_{x\in[a,b]}|f_n(x) - f(x)|$, then $f_n \to f$ uniformly on [a,b] if and only if $M_n \to 0$ as $n \to \infty$.

17. Show that the series for which $S_n(x) = \frac{[illegible]}{1+n\,x}$ can be integrated term by term on [0,1], though they are not uniformly convergent.

18. Discuss the Taylor's theorem for a multivariable function.

19. Show that $f(x,y,z) = x^2 + y^2 + z^2$ is continuous at the origin.

20. Find all the local maxima, minima and saddle points of the function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.

21. Suppose f is monotonic and α is continuous and monotonically increasing. Then f ∈ R(α).

22. Find the Fourier integral representation of $f(x) = \begin{cases} 1, if\ |x| < 1 \\ 0, if\ |x| > 1. \end{cases}$

23. Determine the Laplace transform of (a) $t \cos h\, at$ (b) $e^{at} \sin \omega t$.

24. Determine inverse Laplace transform of (a) $\frac{1}{s^2+4s}$ (b) $\frac{9}{s^2}\frac{(s+1)}{(s^2+9)}$.

(8 x 2 =16 weightage)

Part C

(Answer any TWO questions. Weightage 4 for each question)

25. (a) Describe Fundamental Properties of the Riemann-Stieltjes Integral

(b) Let f be a bounded function on [a, b] with finitely many discontinuities. Suppose α is continuous at every point where f is discontinuous. Then f ∈ R(α).

26. Prove: Let $\{f_n\}$ be a sequence of differentiable functions on [a,b] such that it converges at least one point $x_0 \in [a,b]$. If the sequence of differentials $\{f'_n\}$ converges uniformly to G on [a,b], then the given sequence $\{f_n\}$ converges uniformly on [a,b] to f and f'(x)=G(x).

27. a) Give interpretation of Lagrangian multipliers.

(b) Find the extreme values of $f(x,y) = x + y^2 + 2z$ subject to $4x^2 + 9y^2 - 36z^2 = 36$.

28. (a) Describe the properties of Fourier series.

(b) Find the Fourier series expansion of $f(x) = \frac{x^2}{2}, -\pi < x < \pi$.

Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$.

(2 x 4=8 weightage)

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M1N18106 (Pages : 2) Reg. No:...............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MSTA1B04 – Sampling Theory

(2017 Admission onwards)

Max. Time: 3 hours Max. weightage : 36

PART A

(Answer ALL questions. Weightage 1 for each question)

1. What is sampling error? How do you measure it?
2. Define non probability sampling. Give *two* examples.
3. Illustrate the selection of a circular systematic sample by an example.
4. What are the advantages of stratified sampling over simple random sampling?
5. Explain optimum allocation in stratified sampling.
6. What do you mean by auxiliary variable techniques in estimation?
7. Define ratio estimate of population mean.
8. Define Des Raj ordered estimate of population total.
9. What do you mean by Murthy's unordered estimator?
10. Discuss the efficiency of cluster sampling over simple random sampling.
11. Define two phase sampling.
12. Write a note on non sampling errors .

(12 x 1 = 12 weightage)

PART B

(Answer any EIGHT questions. Weightages 2 for each question)

13. What are the principal steps in a large scale sample survey? Explain.
14. With usual notation, show that $E(s^2) = \sigma^2$ in simple random sampling with replacement from a finite population.
15. How do you estimate population proportion using simple random sample with replacement? Also derive the sampling variance of your estimate.
16. Explain the cost function in stratified random sampling. Also find the optimum allocation of sample size into various strata when the total cost of survey is given.

17. Obtain confidence interval for population total using stratified random sampling.
18. Define Hartley and Ross unbiased ratio type estimator. Show that it is unbiased.
19. Explain the method of estimating population mean using regression estimator. Obtain the relative bias of your estimator.
20. Show that Lahiri's method ensures PPS sample.
21. Define Horvitz-Thompson estimator of population total and show that it is unbiased.
22. How would you estimate population mean $\overline{Y}$ using a simple random sample of n clusters from N clusters each of M elements. Derive the sampling variance of your estimate in terms of intraclass correlation coefficient ρ.
23. Explain various estimators of population mean in cluster sampling with unequal clusters
24. Explain two-stage sampling. Give advantages of it over one stage cluster sampling.

(8 x 2 = 16 weightage)

PART C

(Answer any TWO questions. Weightages 4 for each question)

25. (a) If the loss function due to an error in $\bar{y}$ is $\lambda(\bar{y}-\overline{Y})^2$ where λ is constant and if the cost function is $C = c_0 + c_1 n$, show that the most economical value of n is

$$n = \sqrt{\frac{\lambda S^2}{c_1}}.$$

(b) With usual notation, show that systematic sampling is efficient than simple random sampling if and only if $S^2_{wsy} \geq S^2$.

26. Explain the method of estimating gain due to stratification over simple random sampling using stratified random sample.
27. (a)Explain combined ratio estimator and separate ratio estimator of population total.
(b) When do you prefer regression method over ratio method? Why?
28. (a)Construct an unbiased estimator of population mean using two-stage sampling.
(b) Derive Yates-Grundy form of estimated variance of Horvitz-Thompson estimator of population mean under PPSWOR.

(2 x 4 = 8 weightage)

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M1N18107 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2018

MSTA1B05 – Distribution Theory

(2017 Admission onwards)

Max. Time: 3 hours Max. weightage : 36

PART-A

Answer all Questions

Each question carries a weightage of 1.

1. Define moment generating function. If the r^{th} raw moment of a random variable X is $\mu_r' = (r+1)!2^r, r = 0,1,2,...$ find the moment generating function of X.
2. If X is a Poisson variate with mean λ show that $E(X^2) = \lambda\, E(X+1)$. Also show that if X and Y are independent Poisson variates, the conditional distribution of X given X+Y is binomial.
3. Show that Poisson distribution is a special case of power series distribution.
4. If X is distributed according to U(0,1), show that -2log X is distributed according to chi square.
5. Obtain the distribution of X If $Y = \frac{(X-\mu)^c}{\lambda}$ has exponential distribution with p.d.f $f(y) = e^{-y}$
6. X is a random variable following Pareto distribution of first kind. Show the relationship $P(X > uv / X > u) = P(X > v)$ holds for all u,v > 0. Is the converse true?
7. Distinguish between Beta distribution of the first kind and Beta distribution of the second kind. If X follow the latter, show that $Y = \frac{1}{1+X}$ follow the former.
8. If X and Y are independent standard Cauchy random variables then find the distribution of $X+Y$.
9. Define the log normal distribution and derive its p.d.f. Examine whether the distribution is symmetric.
10. The joint distribution of a bivariate discrete vector (X_1,X_2) is specified by
$$P(X_1 \geq x_1, X_2 \geq x_2) = p_1^{x_1} p_2^{x_2} \theta^{x_1 x_2},$$
$x_1,x_2 = 0,1,2,...;\ 0 \leq \theta \leq 1,\ 0 < p_1, p_2 < 1$. Obtain the marginal pmf of X_1 and X_2. When will X_1 and X_2 be independent?
11. Define non- central t distribution. When will this reduce to the central t?
12. Define F statistic and write its probability density function.

(12 x1 = 12 Weightage)

PART B

Answer eight Questions

Each question carries a weightage of 2.

13. If X is a non-negative integer valued random variable then find the p.g..f of
(i) $P(X \leq n)$ (ii) $P(X = 2n)$
14. Let X and Y be independent random variables following the negative binomial distributions, $NB(r_1,p)$ and $NB(r_2,p)$ respectively. Show that the conditional probability mass function of X given $X + Y = t$ is hypergeometric.