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5076 (Pages : 2) Reg. No:........................

Name:............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

MT1C04– ODE & Calculus of variations

(2016 Admission onwards)

me: 3 hours Max. Weightage : 36

Part A(Short Answer Type Questions)

Answer all the questions

(Each Questions has weightage one)

Define the radius of convergence of a power series.

Find indicial equation and its roots of the differential equation

$4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0.$

Verify that $\log(1 + x) = xF(1,1,2,-x)$.

Show that $\frac{d}{dx}[x\, J_1(x)] = xJ_0(x)$.

Describe the phase portrait of $\begin{cases} \frac{dx}{dt} = -x \\ \frac{dy}{dt} = -y \end{cases}$

Determine the nature and stability of the critical point (0,0) for $\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$

Express $J_2(x), J_3(x), J_4(x)$ in terms of $J_0(x)$ *and* $J_1(x)$.

If $P(x)$ is a polynomial of degree n≥1 such that

$\int_{-1}^{1} x^k P(x)dx = 0$ *for* $k = 0,1, \ldots\ldots, n-1$, then show that $P(x) = cP_n(x)$ for some constant c.

Find critical points and show few of the paths of the non linear system

$\frac{dx}{dt} = -x\,, \frac{dy}{dt} = 2x^2y^2.$

Show that $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if

$a > 0$ *and* $b^2 - 4ac < 0.$

Show that $x^2y'' - 3xy' + 4\,(x + 1)y = 0$ has only one Frobenius series solution.

Show that $\Gamma\frac{1}{2} = \sqrt{\pi}$.

Distinguish between center and spiral, the critical points of an autonomous system.

Find a simple closed plane curve of length L enclosing the maximum area.

(1 4 x 1 = 14)

Part B(Paragraph Type Questions)
Answer Any Seven Questions
(Each Question has weightage two)

Find a power series solution of $y'' + \left(p + \frac{1}{2} - \frac{x^2}{4}\right)y = 0$ *where p is a constant.*

Show that a function of the form $ax^3 + bx^2y + cxy^2 + dy^3$ cannot be either positive definite or negative definite.

Solve $\frac{dx}{dt} = 5x + 4y$, $\frac{dy}{dt} = -x + y$.

Find the shortest curve joining two points (x_1, y_1) and (x_2, y_2) .

Find the general solution of Airy's equation using Bessel functions.

Find the general solution of the differential equation $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ near the singular point x=0.

If there exists a Liapunov's function $E(x, y)$ for the system $\frac{dx}{dt} = F(x,y)$ *and* $\frac{dy}{dt} = G(x,y)$ then show that the critical point (0,0) is stable.

If the solutions $x = x_1(t), y = y_1(t)$ *and* $x = x_2(t), y = y_2(t)$ of the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$ are linearly independent then show that $\begin{cases} x = c_1x_1 + c_2x_2 \\ y = c_1y_1 + c_2y_2 \end{cases}$ is the general solution.

3 Show that (0,0) is an asymptotically stable critical point of $\begin{cases} \frac{dx}{dt} = -y - x^3 \\ \frac{dy}{dt} = x - y^3 \end{cases}$.

4 Let $u(x)$ be any nontrivial solution of $u'' + q(x)u = 0$ *where* $q(x) > 0$ *for every* $x > 0$. If $\int_1^\infty q(x)dx = \infty$ *then show that* $u(x)$ has infinitely many zeros on the positive X-axis.

(7 x 2 = 14)

Part C(Essay Type Questions)
Answer Any Two Questions
(Each Question has weightage Four)

25 Solve the Legendre's equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$.

26 State and Prove orthogonal property of Bessel function.

27 Explain the nature of the critical point (0,0) of the autonomous system $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$ if the roots of corresponding auxiliary equation are real and distinct.

28 Explain and Solve the problem of brachistochrone.

(2 x 4 = 8)

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N16077 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

MT1C05– Discrete Mathematics

(2016 Admission onwards)

Time: 3 hours Max. Weightage : 36

Part A (Short Answer Questions)

Answer **all** questions.

Each question carries 1 weightage.

1. Draw the Hasse diagram of the set $(X, \mathcal{R})$ where X is the set of natural numbers less than 11 and $\mathcal{R}$ is defined by $x\mathcal{R}y$ when x divides y.
2. Let $(X,+,.,')$ be a be a Boolean algebra and $x, y \in X$ then Prove that $(x+y)' = x'.y'$ and $(x.y)' = x' + y'$.
3. Prove by help of an example that every partial order is not a total order.
4. Prove that every non-zero element of a Boolean algebra contains atleast one atom.
5. Prove that$x_1x_2 + x_3$ is symmetric with respect to x_1 and x_2.
6. Define the chromatic number of a graph find the chromatic number of k_5.
7. Prove that every tree with atleast two vertices has atleast two end nodes.
8. Find the dual of the dual k_4 .
9. For what value of n is the graph k_nEulerian ? Justify your answer.
10. If every vertex of G has degree atleast 2, then Prove that G contains a cycle.
11. Define connectivity of a graph. Prove that $\kappa(k_n) = n - 1$.
12. Find a grammar for $\Sigma = (a, b)$ that generates all strings with exactly one a.
13. Find a dfa that accepts all binary sequences that end with the digits 011.
14. What language does the grammar with these production generate?

$$S \rightarrow AA$$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

(14 x 1 = 14 weightage)

Part B

Answer any **seven** from the following ten questions.
Each question carries 2 weightage.

15. Let $(X, +, ., ')$ be aBoolean algebra. Prove that the corresponding lattice $(X, \leq)$ is complemented and distributive.
16. Let $(X, \leq)$ be a poset and A is a non empty finite subset of X. show that A has a maximum element if and only if it has a unique maximal element.
17. Show that every Boolean algebra gives rise to a lattice.
18. State and prove Eulers formula for connected planar graphs.
19. Prove that isomorphism relation is an equivalence relation on the set of simple graphs.
20. Prove that every closed odd walk contains an odd cycle .
21. Let $l(F_i)$ denotes the length of face F_i in a plane graph G. Prove that $2e(G) = \Sigma\, l(F_i)$.
22. Check whether $L = \{awa: w \in \{a, b\}^*\}$ a regular language. Prove that L^2 is regular.
23. Find a grammar that generate the language$\{a^{n+2}b: n \geq 1\}$.
24. Let $\Sigma = \{a, b, c\}$. Construct a dfa that accepts the language $a\Sigma^* b$.

(7 x 2 = 14 weightage)

Part C

Answer any **two** from the following four questions.
Each question carries 4 weightage.

25. (a)Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

    (b)If x and y are elements of Boolean algebra, Prove that $x = y \ iff \ \ xy' + x'y = 0$.

26. (a))If G is a simple graph , then prove that $\kappa(G) \leq \kappa'(G) \leq \delta(G)$.

    (b) Draw a graph with $\kappa(G) < \kappa'(G) < \delta(G)$.

27. (a) Prove that every graph with n-vertices and k edges has atleast $n - k$ components.

    (b)Prove that a graph is bipartite if and only if it has no odd cycle.

28. (a) Let L be the language accepted by the $nfa\ \ M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$.Prove that there exists a$dfa\ \ M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$such that $L = L(M_D)$.

    (b)Construct a dfa that accept all the strings with no more than $3a$'s.

(2 x 4 = 8 weightage)

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1M1N16074 (Pages : 3) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

MT1C02– Linear Algebra

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

(Answer all. 1 weightage each.)

1. Prove that the only subspaces of $\mathbb{R}^1$ *are* $\mathbb{R}^1$ and the zero subspace.
2. Let V be a vector space over the field F. Suppose there are a finite number of vectors in V which span V , then prove that V is finite dimensional.
3. Find the range and rank of zero transformation on a finite dimensional space V .
4. Find two linear operators T and U on $\mathbb{R}^2$ such that TU = 0 but UT $\neq$ 0.
5. Let W_1 and W_2 be subspaces of a finite dimensional vector pace V then prove that $(W_1 + W_2)^0 = W_1^0 + W_2^0$.
6. Let V be a finite-dimensional vector space. What is the minimal polynomial for the zero operator.
7. Let T be a linear operator on a finite dimensional vector space V . Suppose that $T_\alpha = c\alpha$ for all $\alpha \in V$. If f is any polynomial, then prove that $f(T)\alpha = f(c)\alpha$
8. Show that similar matrices have the same characteristic polynomial.
9. Let A be a 3×3 triangular matrix over the field F. Prove that the characteristic values of A are the diagonal entries of A.
10. Define minimal polynomial and give an example.
11. Show that inner product satisfies the parallelogram law.
12. Let V be a vector space over F. Prove or disprove that the sum of two inner products on V is an inner product on V .
13. Let V be a vector space and (|) an inner product on V then show that if $(\alpha|\beta) = 0$ for all β in V , then $\alpha = 0$.
14. If S is any subset of a vector space V , then show that its orthogonal complement is a subspace of V .

(14 x 1 = 14)

Part B

(Answer any seven. 2 weightage each.)

15. Prove that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.

16. Let V be the set of real numbers. Regard V as a vector space over the field of rational numbers, with the usual operations. Prove that this vector space is not finite dimensional

17. Let V be a vector space and T a linear transformation from V into V . Prove that the following statements are equivalent.

    a) the intersection of the range of T and the null space of T is the zero subspace of V .

    b) If $T(T\alpha) = 0$, then $T\alpha = 0$.

18. Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then prove that the space L(V,W) is finite dimensional and has dimension *mn*.

19. Let V be a finite-dimensional vector space over the field F. For each vector α in V define $L_\alpha(f) = f(\alpha), f \in V^*$. Then show that the mapping $\alpha \to L_\alpha$ is an isomorphism of V onto V**.

20. Let T be a linear operator on an n-dimensional vector space V . Then prove that the characteristic polynomial and minimal polynomial for T have the same roots, except for multiplicities.

21. Let W be an invariant subspace for T. Show that the characteristic polynomial for the restriction operator T_W divides the characteristic polynomial for T.

22. If E_1 and E_2 are projections onto independent subspaces, then $E_1 + E_2$ is a projection. True or false? Justify?

23. Let V be a finite dimensional vector space and let B = $\{\alpha_1, \ldots\ldots, \alpha_n\}$ be a basis for V . Let (|) be an inner product on V . If $c_1, \ldots\ldots, c_n$ are any *n* scalars, show that there is exactly c vector α in V such that $(\alpha|\alpha_j) = c_j, j = 1, \ldots\ldots, n$.

24. Define orthogonal sets and prove that an orthogonal set of non-zero vectors is linearly independent.

(7 x 2 =

Part C

(Answer any two . 4 weightage each.)

25. (a) If A is an m×n matrix with entries in the field F, then prove that row rank (A) = column rank (A).

(b) Describe explicitly the linear transformation T from $\mathbb{R}^2$ into $\mathbb{R}^2$ which has its range the subspace spanned by (1, 0,−1) and (1, 2, 2).

26. (a) Show that every n-dimensional vector apace over the field F is isomorphic to the space F^n.

(b) If W is a k-dimensional subspace of an n-dimensional vector space V , then prove that W is the intersection of (n − k) hyperspaces in V .

27. (a) If f is a non-zero linear functional on the vector space V , then prove that the null space of f is a hyperspace in V . Conversely, prove that every hyperspace in V is the null space of a non-zero linear functional on V .

(b) If W is an invariant subspace for T, then show that W is invariant under every polynomial in T. Also prove that the conductor S(α ;W) is an ideal in the polynomial algebraF[x] for each $\alpha \in V$.

28. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V . Then prove that

(a) T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

(b) T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1).....(x - c_k)$ *where* $c_1, ..., c_k$ are distinct elements of F.

(2 x 4 = 8)

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1M1N16075 (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

MT1C03– Real Analysis – I

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part- A

Answer *all* questions. Each question has *one* weightage.

1. Find a bounded subset of real numbers with a countable (infinite) number of limit points.
2. Prove that the open interval (a, b) is not compact by constructing an open cover that does not have a finite sub cover.
3. Give an example for an open subset of real numbers which is not connected.
4. Is every point of every open subset E of R^2 a limit point of E? Justify your answer.
5. Is there exist a function on the set of real numbers, which is discontinuous at all points? Justify your answer.
6. If f is a continuous mapping of a metric space X into a metric space Y, and if E is a dense subset of X, prove that $f(E)$ is a dense subset of Y.
7. Give an example to show that the mean value theorem for real valued functions is not valid for vector valued functions.
8. If $f(x) = |x|^3$, show that $f^{(3)}(0)$ does not exist.
9. Suppose that f is a bounded real valued function on $[a,b]$ and f^2 is Riemann integrable on $[a,b]$. Does it imply that f is Riemann integrable on $[a,b]$?
10. Prove that every continuous function on $[a,b]$ is Riemann integrable on $[a,b]$.
11. Show that the curve γ defined on $[0,2\pi]$ by $\gamma(t) = e^{it}$ is rectifiable.
12. Define uniform convergence of sequence of functions.
13. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
14. Give an example of sequences $\{f_n\}, \{g_n\}$ of uniformly converging functions such that $\{f_n g_n\}$ does not converge uniformly.

(14 x 1 = 14)

Part- B

Answer any *seven* from the following ten questions. Each question has *two* weightage

15. Let E be a subset of a metric space X. Prove that the complement of E° is the closure of the complement of E.

16. Show that the set of all sequences whose elements are the digits 0 and 1, is uncountable.

17. Prove that a finite point set has no limit points.

18. If E is a non compact subset of R, then prove that there exists a continuous function on E which is not bounded.

19. Show that the image of a connected set under a continuous function is connected.

20. If f is differentiable on $[a,b]$, then prove that the derivative f' cannot have any simple discontinuities on $[a,b]$.

21. Suppose f is defined and differentiable for every positive real x and $f'(x) \to 0$ as $x \to +\infty$. Let $g(x) = f(x+1) - f(x)$. Prove that $g(x) \to 0$ as $x \to +\infty$.

22. If f is a bounded real function on $[a,b]$ and if α is monotonically increasing on $[a,b]$, then prove that $f \in \Re(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$, there exists a partition P of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$

23. If K is a compact metric space, if f_n are continuous functions on K and if $\{f_n\}$ converges uniformly on K, then show that $\{f_n\}$ is equicontinuous on K.

24. If $\{f_n\}$ is a sequence of continuous functions on a compact set K, such that $f_n(x) \ge f_{n+1}(x)$ for all $x \in K, n = 1,2,3,...$, and if $\{f_n\}$ converges pointwise to a continuous function f on K, then prove that $f_n \to f$ uniformly on K.

(7 x 2 = 14)

Part- C

Answer any *two* from the following four questions. Each question has *four* weightage.

(a) Prove that every connected metric space with at least two points is uncountable.

(b) Prove that a subset E of R^k is compact if and only if E is closed and bounded.

(a) If f is a continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

(b). Give an example of a *bounded* function which is *continuous, but not uniformly continuous.*

7. (a) If $\{f_n\}$ is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E then prove that f is continuous on E.

(b) If $f_n \in \Re(\alpha)$ on $[a,b]$ and if $f_n \to f$ uniformly on $[a,b]$, then show that $f \in \Re(\alpha)$.

28. (a) If K is a compact metric space, if f_n are continuous functions on K and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then show that $\{f_n\}$ has a uniformly convergent subsequence.

(b) Prove that the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$ converges uniformly on every bounded interval in R.

(2 x 4 = 8)

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1M1N16073 (Pages : 2) Reg. No:……………………….

Name:……………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

MT1C01– Algebra – I

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer ALL the 14 questions. Each carries 1 weightage

1. Prove that the group $Z_3 \times Z_3$ is not cyclic.
2. Define: Action of a group G on a set X. Give one example.
3. Define: Solvable group. Prove that the group S_3 is solvable.
4. How many distinguishable ways can seven people be seated at a round table?
5. If H and N are subgroups of a group G, and N is normal in G, show that $H \cap N$ is normal in H
6. Define: Sylow p-subgroup of a group G, where p is a prime. Give one example.
7. Prove that no group of order 20 is simple.
8. State the evaluation homomorphism for field theory.
9. How many polynomials are there of degree ≤ 3 in $Z_2[x]$
10. Find all zeroes of $x^3 + 2x + 2$ in Z_7.
11. Find all units in $Z_7[x]$.
12. Show that $f(x) = x^3+3x+2$ in $Z_5[x]$ is irreducible in Z_5.
13. Show that the fields R and C are not isomorphic.
14. Give an example to show that a factor ring of an integral domain may be a field.

(14 x 1 = 14)

Part B

Answer any SEVEN questions. Each carry 2 weightage.

15. Define: Decomposable group.
Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.

16. Find the isomorphic refinements of the series $\{0\}<8Z<4Z<Z$ and $\{0\}<9Z<Z$

17. Let X be a G-Set. Explain an orbit in X under G. Show that $G_x = \{g \in G / gx = x\}$ is a subgroup of G for each $x \in X$.

18. State and prove third isomorphism theorem.

19. Show that no group of order 48 is simple.

20. Explain the class equation of a group G.
Write the class equation for S_3.

21. Prove that if D is an integral domain, then D[x] is also an integral domain.

22. Show that the equation $x^2 = 2$ has no solution in rational numbers.

23. State the division algorithm for the ring of polynomials F[x].
Show that the element $a \varepsilon F$ is a zero of $f(x) \in F[x]$ if and only if (x—a) is a factor of f(x) in F[x].

24. Show that if R is a ring with unity and N is an ideal of R, such that $N \neq R$, then R/N is a ring with unity.

(7 x 2 = 14)

Part C

Answer any two questions. Each questions carry 4 weightage

25. (a) Prove that the group $Z_m \times Z_n$ is isomorphic to Z_{mn} if and only if m and n are relatively prime.
(b) Find the order of (8, 4 ,10) in the group $Z_{12} \times Z_{60} \times Z_{24}$.

26. (a) State and prove first sylow theorem.
(b) Show that the 2-sylow subgroup of S_3 are conjugates.

27. (a) Prove that for any prime p, every group G of order p^2 is abelian.
(b) If p and q are distinct primes with $p < q$, then prove that every group G of order pq has a single subgroup of order q and this subgroup is normal in G.
(c)If G is not congruent to 1 modulo p, prove that G is abelian and cyclic.

28. (a) State and prove Eisenstien condition for irreducibility.
(b) Prove that the cyclotomic polynomial $\phi_p(x) = 1+x+x^2 \ldots x^{p-1}$ is irreducible over Q, for any prime p.

(2 x 4 = 8)