M1N16105 (Pages :2) Reg. No:....................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

ST1C04 – Regression & Linear Programming

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

PART A

Answer all questions each carry one weight.

1. Define estimable parametric function.
2. State Gauss Markov theorem?
3. Explain a simple linear regression model.
4. How to detect and mitigate the problem of multicollinearity?
5. Explain logistic regression Model.
6. Define Hat matrix and state its properties.
7. Define feasible, basic feasible and optimum basic feasible solution of a LPP.
8. Briefly describe the graphical method of solving a LPP.
9. Explain the concept of duality and its uses in LPP.
10. Explain transportation problem.
11. Distinguish between sensitivity analysis and parametric programming.
12. Define a saddle point. Is it necessary that a game should always posses a saddle point?

(12 X 1=12)

PART B

Answer any eight. Each carries two weights.

13. Obtain 95% confidence interval for the predicted value in the simple linear regression model.
14. Explain various methods for checking adequacy of a model.
15. Explain how you fit a polynomial regression model $Y=\beta_0+\beta_1X+\beta_2X^2+\epsilon$ to a given data.
16. Explain various probability plots to examine the normality assumption in regression analysis.
17. Explain the Poisson regression model and estimation of the parameters of this model.
18. Define the predicted and studentized residual sum of squares.
19. Prove that a basic feasible solution of the LPP is a vertex of the convex set of feasible solutions.

20. Use two-phase simplex method to solve the following LPP:
Max $Z = 3x_1 - x_2$ subject to $2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 2$, $x_2 \leq 4$, $x_1, x_2 \geq 0$.

21. Briefly describe revised simplex method and its advantages over simplex method.

22. Write short notes on the following
(i) Big- M method
(ii) Travelling salesman problem

23. Explain the Maxmin and Minimax principle used in game theory.

24. For the following pay off matrix of firm A, determine the optimal strategies for both the firms and the value of the game.

	Firm A		
Firm B	15	2	3
	6	5	7
	−7	4	0

(8 x 2=16)

PART C

Answer any two. Each carries four weights.

25. In the linear regression model $y = \alpha + \beta x + \epsilon$, obtain the best linear unbiased estimators of α and β. Also obtain their standard errors. Whether these estimates coincide with the MLE ?

26. What is Generalized Linear Model (GLM)? Explain the parameter estimation on GLM.

27. Define assignment problem and discuss various methods for solving it.

28. ABC limited has three production shops supplying a product to five ware houses. The cost of product varies from shop to shop and cost of transportation from one shop to a warehouse also varies. Each shop has a specific production capacity and each warehouse has certain amount of requirement. The cost of transportation is as given below.

		Warehouse					Capacity
		I	II	III	IV	V	
Shop	A	6	4	4	7	5	100
	B	5	6	7	4	8	125
	C	3	4	6	3	4	175
Required		60	80	85	105	70	

Find the optimum quantity to be supplied from each shop to different warehouses at minimum total cost.

(2 x 4 = 8)

1M1N16106 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

ST1C05 – Distribution Theory

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

PART-A

Answer all Questions

Each question carries a weightage of 1.

1. If X be a non negative random variable with distribution function F(.), then show that

$$E(X) = \int_0^\infty (1 - F(x))dx$$

2. A random variable X has probability distribution

$$p_k = P(X = k) = \frac{\alpha q^k}{k}, k = 1,2,\dots, 0 < q < 1, \qquad \alpha = \frac{-1}{\log(1-q)}$$

Find its probability generating function and hence the mean of X.

3. If X and Y are independently distributed random variables such that $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$ obtain the conditional distribution of X given X + Y

4. Let $X_1, X_2, \dots, X_n$ be a random sample of size 'n' taken from a population that follow Poisson distribution with mean λ. Derive the distribution of its sample mean.

5. Derive the lack of memory property possessed by geometric distribution.

6. If X has uniform distribution over (0, 1), derive the distribution of $Y = e^x$.

7. If X and Y are independently and identically distributed exponential random variables with mean $\frac{1}{\theta}$, identify the distribution of X – Y.

8. Derive the harmonic mean of X where X follows beta distribution of second type.

9. Let the bivariate random variable (X, Y) have joint probability density function $f(x,y) = kxy^2, 0 < x < y < 2$. Find the value of k ?

10. Show that in student's 't' distribution with 'n' degrees of freedom all odd and central moments are equal.

11. If F has snedecor's F distribution with degrees of freedom $n_1 = n_2 = n$. Then show that median of F occurs at F = 1

12. Define non central t distribution and when will this reduce to central t distribution

(12x1=12 weightage)

PART B

Answer *eight* Questions

Each question carries a weightage of 2.

13. Define probability generating function of a random variable. Let X_i, i = 1,2,…, n be a sequence of independently and identically distributed random variables such that $P(X_i = k) = q^k p$ for k = 0,1,2,3,… and i =1,2,…,n. Derive the pgf of $Y = X_1 + X_2 + \dots + X_n$ and identify the distribution of y.

14. Define hyper geometric distribution. Under conditions to be stated show that hyper geometric distribution becomes binomial distribution.

15. Suppose that (X_1, X_2, X_3) is multinomial with constants (n, p_1, p_2, p_3). Show that the conditional distribution of X_1 given $X_1 + X_2$ is binomial

16. Define power series distribution. Derive its cumulant generating function and hence its mean.

17. Let $X_i, i = 1,2,3, \dots, n$ be a set of identically and independently distributed random variables having Weibul distribution with probability density function $f(x) = cx^{c-1}e^{-x^c}$, $x > 0, c > 0$. Derive the probability density function of Y where $Y = X_{(1)}$.

18. Define Pareto distribution. Obtain the expression for its rth raw moment and hence its variance.

19. If X and Y are two independently distributed standard normal random variables deriv the distribution of U where $U = \frac{X}{Y}$.

20. Let X have a standard Cauchy distribution. Find the probability density function of where $Y = X^2$ and identify the distribution.

21. The bivariate random variable (X, Y) has joint probability density function f(x, y) = 6(1 – x - y $0 < x,\ y < 1$ and $x + y < 1$. Find Cov (X, Y) ?

22. Let $X_1, X_2, \dots, X_n$ be a random sample drawn from a population with uniform densi function over (0,1). Find the distribution of rth order statistic $X_{(r)}$ and hence its mean a variance.

23. Let X and Y be independently distributed according to Chi-square with parameters and n_2. Derive the distribution of $U = \frac{X}{X+Y}$.

24. Derive the mean and mode of F distribution and show that its mean is always greater th mode

25. Derive the moment generating function of X where X follows non central chi squ distribution with non centrality parameter λ.

(8x2=16 Weighta

PART-C

Answer *two* Questions

Each question carries a weightage of ***4***.

Mean $= e^{\mu+\frac{1}{2}\sigma^2}$

Median $= e^{\mu}$

Mode $= e^{\mu-\sigma^2}$

$\frac{1}{X} \to$ lognormal $(-\mu, \sigma^2)$

a)Define lognormal distribution with constants μ and σ. Show that it is positively skewed. If X has lognormal distribution with constants μ and σ, then obtain the distribution of 1/X

b) Let X have the conditional distribution given by $f(x/\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, x > 0$ and θ follows gamma distribution with parameters 'm' and 'n'. Obtain the unconditional distribution of X.

a)If 't' has student's 't' distribution with 'n' degrees of freedom show that t^2 follows F distribution with one and 'n' degrees of freedom

b) Derive the probability density function of non central student's 't' distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from $N(\mu, \sigma^2)$. Show that sample mean $\overline{X}$ and sample variance s^2 are independently distributed. Hence or otherwise obtain their individual distributions.

a)Define bivariate normal distribution. Let X and Y be independently distributed according to $N(\mu, \sigma^2)$. Define U = X+Y and V = X −Y . Examine whether U and V are independent.

b)Derive the probability density function of Pearson type I distribution and hence illustrate how does it generalize the beta distribution of the first type.

(2 x 4 = 8 Weightage)

M1N16104 (Pages : 2) Reg. No:…………………

Name: ………………………

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

ST1C03 – Analytical Tools for Statistics – II

(2016 Admission onwards)

ax. Time: 3 hours Max. Weightage : 36

Part A

(Answer **all** questions. Weightage 1 for each question)

Define linear independence of a set of vectors.

Check whether $S = \{(x_1, x_2) \in R^2 : x_1, x_2 \geq 0\}$ is a subspace of R^2.

Define basis of a vector space.

Define inner product of two vectors.

Define linear and orthogonal transformations.

Define triangular matrix. State any property of it.

What is singular value decomposition of a matrix?

Show that the modulus of each characteristic root of a unitary matrix is one.

Define indefinite quadratic forms.

Show that a real symmetric matrix has only real eigen values.

State spectral decomposition theorem for a real symmetric matrix.

If A is symmetric matrix then show that Moore-Penrose inverse of A is orthogonal.

(12 x 1= 12)

Part B

(Answer any **eight** questions. Weightage 2 for each question)

Let S be a subspace of a finite dimensional vector space. Prove that every generating set C of S contains a basis of S.

Prove that the intersection of any two subspaces S and T of a vector space V is a subspace of V.

Define rank of a matrix. If r(A) is the rank of A, then show that
(i) $r(AB) \leq \min[r(A), r(B)]$. (ii) $r(AB) = r(A)$ if B is a square and nonsingular matrix.

Show that a matrix is a projection matrix if and only if it is symmetric and idempotent.

Define Hermitian and skew Hermitian matrices. Show that eigen values of Hermitian matrix are real.

18. Define minimal polynomial. Prove that similar matrices have the same minimal polynomial.

19. Find the eigen values and eigen vectors of the matrix A= $\begin{bmatrix} 8 & -4 & 6 \\ 10 & -6 & 6 \\ 8 & -8 & 10 \end{bmatrix}$.

20. Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

21. Define trace of a matrix. Show that for any $m \times n$ matrix **A**, $n \times p$ matrix **B**, and $p \times q$ matrix **C**, tr(**ABC**) = tr(**B'A'C'**) = tr(**A'C'B'**).

22. Classify the following quadratic form as positive definite, positive semi-definite or indefinite.

$$x_1^{\,2} + 2x_2^{\,2} - x_3^{\,2} - 4x_1x_2 - 6x_1x_3 + 8x_2x_3.$$

23. Define the rank, signature and index of a real quadratic form. State the inter-relationship between them, if any.

24. Show that original matrix is the generalized inverse of its generalized inverse. Explain a method of finding generalized inverse of a given square matrix.

(8 x 2= 16)

Part C

(Answer any **two** questions. Weightage 4 for each question)

25. If A and B are two m x m idempotent matrix, then show that
    (a) $I_m - A$ is idempotent
    (b) Each eigen values of A is 0 or 1.
    (c) A+B is idempotent if and only if AB=BA= O
    (d) AB is idempotent if and only if AB=BA

26. Let $A_1, A_2, \ldots, A_n$ are symmetric matrices. If $\sum_{i=1}^{n} A_i$ is idempotent and $A_iA_j = O$ for i $\neq$ j. Then show that A_i *is idempotent* and rank $(\sum_{i=1}^{n} A_i) = \sum_{i=1}^{n} rank(A_i)$.

27. Let **A** represent an $m \times n$ nonnull matrix, let **B** represent a matrix of full column rank and **T** a matrix of full row rank such that **A** = **BT**, and let **L** represent a left inverse of **B** and **R** a right inverse of **T**.
    (a) Show that the matrix $\mathbf{R}(\mathbf{B'B})^{-1}\,\mathbf{R'}$ is a generalized inverse of the matrix **A'A** and that the matrix $\mathbf{L'}(\mathbf{TT'})^{-1}\mathbf{L}$ is a generalized inverse of the matrix **AA'**.
    (b) Show that if **A** is symmetric, then the matrix $\mathbf{R}(\mathbf{TB})^{-1}\,\mathbf{L}$ is a generalized inverse of the matrix $\mathbf{A}^2$.

28. (a) Define Moore-Penrose inverse of matrix and describe its properties.
    (b) Show that Moore-Penrose inverse of matrix is unique.

(2 x 4= 8)

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N16103 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

ST1C02 – Analytical Tools for Statistics – I

(2016 Admission onwards)

Time: 3 hours Max. Weightage : 36

Part A

(Answer **ALL** questions. Weightage 1 for each question)

1. Examine whether the limit of the function $f(x,y) = \frac{x^2y^2}{x^3+y^3}$ exists at the point (0,0).
2. State implicit function theorem.
3. Define directional derivative.
4. Define the limit of a multivariate function.
5. Define analytic function. Give an example.
6. State Cauchy's theorem for analytic function.
7. Examine whether $e^x \cos y$ is a harmonic function.
8. Define Pole and singularities of a function.
9. Define Laplace transform of a function.
10. State Poisson integral formula.
11. Obtain the Laplace transform of the function e^{at}.
12. If $L\{F(t)\} = f(s)$ then what is $L\{e^{at} F(t)\}$.

(12 x 1 = 12)

Part B

(Answer any **EIGHT** questions. Weightage 2 for each question)

13. Examine whether the limit of the function $f(x,y) = \frac{x^3y^3}{x^2+y^2}$ exist at (0,0).
14. Show that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & ; if\ (x,y) \neq (0,0) \\ 0 & ; if\ (x,y) = (0,0) \end{cases}$

    is continuous at the origin.
15. Examine the function $21x - 12x^2 - 2y^2 + x^3 + xy^2$ for maximum and minimum.
16. Find an analytic function whose real part is $e^x \cos y$.
17. State and prove Cauchy's integral formula.
18. Show that every analytic function satisfies Cauchy-Riemann equations.
19. State and prove Jordan's lemma.

20. Classify the nature and singularity of the function f(z) = $\frac{z^2}{(z-2)^2}$ and find the residue a

the singular point.

21. Show that if f(z) is an entire function, which is bounded for all values of z, then it is constant.

22. If L(F(t)) = f(s) show that L{t^nF(t)} =$(-1)^n \frac{d^n}{ds^n} f(s)$ where L represents the Laplace transform.

23. Find the Fourier transform of F(x) = x if a≤x≤b.

24. Integrate $\frac{1}{z^4-1}$ around the circle |z+3| = 1.

(8 x 2 = 1

Part C

(Answer any **TWO** questions. Weightage 4 for each question)

25. i) State and prove Laurent's lemma.

ii) Derive the Laurent's series expansion of f(z) , where

$$f(z)=\frac{2}{(z-1)(z-3)}, 0<|z-1|<2.$$

26. State and prove the necessary and sufficient condition for a function to be analytic.

27. Evaluate

1. $\int_0^\infty \frac{x^2}{x^4+5x^2+6}\,dx$

2. $\int_{-\infty}^\infty \frac{1}{(x^2+1)(x^2+9)}\,dx$

28. i. Explain the Lagrange multiplier method.

ii. Maximize 36-x^2-y^2 subject to x+7y = 25.

(2 x 4 =

27

16102 (Pages : 2) Reg. No:...........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2016

ST1C01 – Measure theory & integration

(2016 Admission onwards)

Time: 3 hours Max. Weightage : 36

PART A

(Answer All questions. Weightage 1 for each question.)

Define Reimann - Stieltje's integral.

Describe uniform convergence of a sequence of functions.

Identify conditions under which the following equality holds for a sequence of functions $\{f_n\}$: $\lim_{n\to\infty}\lim_{x\to c} f_n(x) = \lim_{x\to c}\lim_{n\to\infty} f_n(x)$.

Prove or disprove: " Union of two fields is again a field".

If f and g are measurable functions, prove that max(f, g) is also a measurable function.

Distinguish between finite and σ-finite measures.

Let (X,F, μ) be a measurable space and $A \in F$. If f(x) = $I_A(x)$, for all x ∈ X, find $\int_X f d\mu$.

State Lebesgue dominated convergence theorem.

What you mean by L_p space?

Describe L_p convergence.

Define product measure.

State Fubini's theorem and describe its significance.

(12 x 1 = 12)

PART B

(Answer any EIGHT questions. Weightage 2 for each question.)

If $f \in R(\alpha)$ *and* $f \in R(\beta)$ *on* $[a,b]$, for any constants c_1, c_2, prove that

$$f \in R(c_1\alpha + c_2\beta) \text{ and } \int_a^b f d(c_1\alpha + c_2\beta) = c_1\int_a^b f d\alpha + c_2\int_a^b f d\beta.$$

Suppose α is increasing on [a, b] and $f \in R(\alpha)$, prove that $f^2 \in R(\alpha)$ *on* $[a,b]$.

State and prove fundamental theorem of calculus.

Define minimal σ-field over a given class of sets. Describe how do you construct Borel's σ-field in R, beginning with a class of subsets of R.

17. Beginning with the integral of simple functions, describe the systematic development of integral of arbitrary measurable functions.

18. State and prove monotone convergence theorem.

19. Let $\{f_n\}$ be almost uniformly Cauchy sequence, prove that there exists a measurable function f such that $\{f_n\}$ converges almost uniformly and almost every where to f.

20. If $\{f_n\}$ converges almost uniformly, prove that $\{f_n\}$ converges in measure. Is the converse true? Justify your answer.

21. State and prove Egoroff's theorem.

22. State and prove Jordan decomposition theorem

23. State Radon - Nikodym theorem. What is Radon - Nikodym derivative? What are the properties of Radon - Nikodym derivative.

24. Define Lebesgue - Stieltje's measure. Derive probability measure as its particular case.

(8 x 2 =

PART C

(Answer any TWO questions. Weightage 4 for each question.)

25. State and prove Weierstrass theorem.

26. (a) Let f be an arbitrary measurable function. Define the positive and negative part of f. Prove that positive and negative part of f are measurable functions.

(b) If f and g are measurable functions on (X, A, μ), prove that:

$$\int_X (f+g)d\mu = \int_X f d\mu + \int_X g d\mu$$

27. (a) Define convergence in measure.

If $f_n \to^m f \; and \; g_n \to^m g, prove\; that\; f_n + g_n \to^m f + g$

(b) State and prove Holders inequality.

28. (a) State Caratheodory extension theorem.

(b) Find the Lebesgue - Stieltje's measure generated by the following function g, wh

$$g(x) = 0, x < 0; g(x) = \frac{1}{4}, 0 \leq 1;\; g(x) = \frac{1}{2}, 1 \leq x < 2;\; and\; g(x) = 1, x \geq 2.$$

(2 x 4