1B3N17085 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester BSc Degree Examination, November 2017

MAT3B03 - Calculus and Analytic Geometry

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART-A

Answer *all* Questions . Each question carries *one* mark

1. The derivative of $\ln(3x)$ is ....
2. Define the hyperbolic cosine function of x.
3. Find the derivative of $y = 6\sinh\left(\frac{x}{3}\right)$
4. Write the n^{th} term of the sequence 0,1,1,2,2,....
5. Evaluate the limit of the sequence $\left(\sqrt{\frac{n+1}{n}}\right)$
6. Evaluate $\lim \left(n^{\frac{2}{n}}\right)$
7. For what values of p, does the series $\sum_{n=1}^{p} \frac{1}{n^p}$ converge?
8. Find the focus of the parabola $y^2 = 10x$
9. Using discriminant test identify the conic section $x^2 + 2xy + y^2 + 2x - y + 1 = 0$
10. Write a parametric equation of the circle $x^2 + y^2 = 9$
11. Graph the polar region $-3 \le r \le 2, \theta = \frac{\pi}{4}$
12. Replace the polar equation $\frac{4}{2\cos\theta - \sin\theta}$ by the corresponding Cartesian equation.

(12x1=12 marks)

PART-B

Answer any *nine* questions. Each question carries two marks

13. Solve for x: $e^{2x-6} = 4$
14. Prove: $\cosh^2 x - \sinh^2 x = 1$
15. Evaluate $\int_0^{1/3} \frac{6\,dx}{\sqrt{1+9x^2}}$
16. Use sandwich theorem to find the limit of the sequence $\left(\frac{\cos n}{n}\right)$
17. Prove that $\lim_{n\to\infty} k = k$
18. Evaluate $\lim_{x\to 0} \frac{3x - \sin x}{x}$
19. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{2n}{n^2-n+1}$
20. Express the repeating decimal $\overline{0.234} = 0.234\ 234\ 234$ ...as the ratio of two integers.
21. Find the foci and vertices of the parabola $\frac{x^2}{9} + \frac{y^2}{16} = 1$

22. The position $P(x, y)$ of a moving particle in the xy-plane is given by the equations and parametric interval,

$$x = \sqrt{t}, y = t, t \geq 0.$$

Identify the path traced by the particle and describe the motion.

23. Find all polar coordinates of the point $P(2, \frac{\pi}{6})$
24. Find the length of the cardioid $r = 1 + \cos\theta$

(9x2=18 marks)

PART-C

Answer any *six* questions. Each question carry *five* marks

25. Prove that $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}, |x| < 1$
26. Check the convergence of the series $\frac{3}{4} + \frac{5}{9} + \frac{7}{16} + \frac{9}{25} + \cdots$
27. Check the convergence of $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$
28. Find the interval of convergence of the power series $\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n (x-2)^n$. What is the sum?
29. Find the Taylor series and Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.
30. Find the standard equation of a conic section hence find the center, foci, vertices, asymptotes as appropriate $y^2 - 4y - 8x - 12 = 0$
31. Find the tangent to the right hand hyperbola branch $x = \sec t, y = \tan t, -\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$, where $t = \frac{\pi}{4}$
32. Find the length of the parametric curve, $x = \cos t, y = t + \sin t, 0 \leq t \leq \pi$
33. Find the area inside the smaller loop of the limacon, $r = 2\cos\theta + 1$

(6x5=30 marks)

PART-D

Answer any *two* questions. Each question carries *ten* marks

34.

a) Graph the polar curve $r = 1 + \cos\frac{\theta}{2}$

b) Find the slope of the curve $r = -1 + \cos\theta$ at $\theta = \frac{\pi}{2}$

35.

a) Show that the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ is absolutely convergent.

b) Show that the Alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is conditionally convergent.

36.

a) Remove the cross product term in the equation and identify the curve: $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$. Sketch the graph.

b) Evaluate $\lim_{x\to\infty} x^{\frac{1}{x}}$

(2 x 10 = 20 Marks)

1B3N17086 (Pages : 2) Reg. No: ....

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester BSc Degree Examination, November 2017

MAT3C03 - Mathematics

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART A

Answer *all* questions. Each question carries *one* mark.

1. Solve $y' = -2xy$..
2. Show that the equation$(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$ is exact.
3. Verify that $x^4 + y^4 = 1$ is a solution of $x^3 + y^3y' = 0$.
4. Find the rank of the matrix $\begin{bmatrix} 5 & 1 \\ 2 & 3 \\ 7 & 4 \end{bmatrix}$.
5. Find the eigenvalues of the matrix $\begin{bmatrix} 2 & 9 & 2 \\ 0 & -7 & 1 \\ 0 & 0 & 6 \end{bmatrix}$.
6. Is the matrix $\begin{bmatrix} 14 & 4 \\ 7 & 2 \end{bmatrix}$ a singular matrix? Give reason.
7. If $\vec{a} = [2,5,8]$,then find$\vec{a}.\vec{a}$ and $\vec{a} + \vec{a}$.
8. Define a unit vector and give an example.
9. Find a unit vector in the direction of the vector from $P(0,1,-1)$ to $Q(-1,2,2)$.
10. Sketch the unit vector obtained by rotating the unit vector $\hat{j}$ anticlockwise $\frac{\pi}{2}$ rad about the origin.
11. If $\phi(x, y, z) = 3x^2 + 4y^3 + xyz$, then find $\nabla\phi$ at $(1,1,1)$.
12. The vector $\vec{r}(t) = 4\cos t\,\hat{\imath} + 4\sin t\,\hat{\jmath} + t\hat{k}$ gives the position of a moving body at time t. Find the velocity of the body when $t = 4$.

$(12 \times 1 = 12$ marks)

PART B

Answer *any nine* questions. Each question has *two* marks.

13. Define singular solution of a differential equation. Give an example, and explain.
14. Find the curve through the point $(1,1)$ in the xy −plane having at each points the slope $-\frac{y}{x}$.
15. Find an integrating factor of $(y - 2x^3)dx - x(1 - xy)dy = 0$ and solve it.
16. Find the rank of the matrix A by reducing in to row canonical form, where $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 1 & 0 \\ 1 & -3 & -6 \\ 5 & 5 & 6 \end{bmatrix}$.
17. Find the rank of the matrix A by reducing in to normal form, where $A = \begin{bmatrix} 12 & 24 & 36 & 72 \\ 14 & 22 & 36 & 2 \\ 2 & -2 & 0 & -70 \end{bmatrix}$
18. Using Cayley-Hamilton theorem find A^{-1}, where $A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$.
19. Find first and second partial derivatives with respect to x of vector function $[x^2y, y^2z,\ z^2x]$.
20. Find the unit tangent vector of the curve $\vec{r}(t) = \sqrt{2}\cos t\,\hat{\imath} + \sqrt{2}\sin t\,\hat{\jmath} + \sqrt{3}\hat{k}$ at the point $(\sqrt{2}, 0, \sqrt{3})$.

21. Show that $\vec{u}(t) = \sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \sqrt{7}\hat{k}$ is orthogonal to its derivative.

22. Find the directional derivative of the function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ at (1,1) in the direction of the vector $\vec{u} = \hat{\imath} - \hat{\jmath}$.

23. Find the length of one turn of the helix $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\hat{k}$.

24. Find the curl of $\overrightarrow{F} = x^2yz\hat{\imath} + xy^2z\,\hat{\jmath} + xyz^2\hat{k}$at $(-1, -1, -1)$.

($9 \times 2 = 18$ marks)

PART C

Answer *any six* questions. Each question has *five* marks.

25. Experiments show that a radioactive substance at a rate proportional to the amount present. Starting with 2 grams of substance at time $t = 0$, what can be said about the amount available at a later time.

26. Solve $xy' + y = xy^3$.

27. Find the orthogonal trajectories of the family of circles $x^2 + (y - c)^2 = c^2$.

28. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

29. For what values of α does the system of equations

$$x + 2y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = \alpha$$

has a solution? Find the general solution when αtakes this value.

30. If $\vec{v}$ is a differentiable vector function, then prove that $\text{div}(\text{curl } v) = 0$.

31. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\,\hat{\imath} - 5z\,\hat{\jmath} + 10x\,\hat{k}$along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 0$ to $t = 2$.

32. Calculate $\iint_R f(x, y)dx\,dy$ for $f(x, y) = 1 - 6x^2y$ and $R : 0 \le x \le 2\,;\ 0 \le y \le 1$.

33. If$\vec{A} = 2xy\,\hat{\imath} + yz^2\hat{\jmath} + xz\,\hat{k}$ and S is a rectangular parallelepiped bounded by $x = 0, y = 0, z = 0\,; x = 1, y = 2, z = 3$, then evaluate $\iint_S \vec{A}.n\ dA$.

($6 \times 5 = 30$ marks)

PART D

Answer *any two* questions. Each question has *ten* marks.

34. Verify that the eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and A^T are the same.

35. Verify Gauss's divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\,\hat{\jmath} + (z^2 - xy)\,\hat{k}$ over the rectangular parallelepiped $0 \le x \le 2\,;\ 0 \le y \le 21\,;\ 0 \le z \le 2$.

36. Verify Stoke's theorem for the function $\vec{F} = x^2\hat{\imath} + xy\,\hat{\jmath}$ integrated around the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = 2, y = 2$.

($2 \times 10 = 20$ marks)