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B2M19079 (Pages : 3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, March /April 2019

BMAT2B02 - Calculus

(2018 Admission onwards)

Time: 3 hours Max. Marks: 80

PART-A

(Answer *all* questions. Each question carries *one* mark)

1. Give an example of a function having no absolute minimum.
2. When is a function f said to be increasing on an interval I?
3. The graph of a differentiable function $y = f(x)$ is concave up on an interval where y' is........
4. A linear asymptote for the curve that is neither vertical nor horizontal is called......
5. Find dy if $y = x^2 + \sin x$.
6. Find the norm of the partition $P = [0, 1.2, 1.5, 2.3, 2.6, 3]$ of $[0, 3]$.
7. Suppose that f is continuous and that $\int_0^3 f(x)dx = 3$ and $\int_0^4 f(x)dx = 7$. Find $\int_3^4 f(x)dx$.
8. State the Mean Value Theorem for definite integrals.
9. Find $\frac{dy}{dx}$ if $y = \int_0^x t\sqrt{1 + t^2}dt$.
10. Suppose f is continuous on the symmetric interval $[-3, 3]$ and $\int_0^3 f(x)\, dx = 5$, then find $\int_{-3}^0 f(x)dx$ if $f(x)$ is even.
11. When is a function said to be smooth?
12. The volume of a solid of known integrable cross-section area $A(x)$ from $x = a$ to $x = b$ is given by.....

(12 x 1 = 12 Marks)

PART-B

(Answer any *seven* questions. Each question carries *two* marks)

13. State Rolle's theorem.

14. Find the critical points of $f(x) = x^{\frac{1}{3}}(x-4)$.

15. Find the asymptotes of the graph of $f(x) = \frac{x^2-3}{2x-4}$.

16. Find the linearization of $f(x) = \sqrt{x}$ at $x = 4$.

17. Find the Riemann sum $\sum_{k=1}^{4} f(c_k)\Delta x_k$ of $f(x) = x^2 - 1$ over $[0,2]$ by dividing interval into four subintervals of equal length and taking c_k as the left-hand endpoin each subinterval.

18. Show that if f is continuous on $[a, b]$, $a \neq b$, and if $\int_a^b f(x) = 0$, then $f(x) = 0$ at lea once in $[a, b]$.

19. Find the volume of the solid so generated by revolving the part of the curve $x^2(y - x^2) = 3$ between $x = 1$ and $x = 2$ about the x-axis.

20. Find the length of the arc of the curve $y = \frac{\sqrt{7}x}{3}$, $2 \leq x \leq 4$.

21. Show that the centre of mass of a straight, thin strip or rod of constant density lies h way between its two ends.

(7 x 2 = 14 Mar

PART-C

(Answer any *six* questions. Each question carries *five* marks)

22. Find the points of inflection on the curve $y = \frac{a^2x}{x^2+a^2}$ and show that they lie on a strai line.

23. Find two positive numbers whose sum is 20 and whose product is as large as possible.

24. Show that the function

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is artional} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable over $[0, 1]$.

25. Graph the function $f(x) = 2x - x^2$ over $[0, 3]$. Find the area of the region between graph and the x-axis.

26. Find the volume of the solid that lies between planes perpendicular to the x-axis $x = -1$ and $x = 1$, where the cross sections perpendicular to the x-axis on the inter $-1 \leq x \leq 1$ are circular disks whose diameters run from the parabola $y = x^2$ to parabola $y = 2 - x^2$.

Find the area of the region enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$.

Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $y = 0$ to $y = 2$.

Find the volume of the solid generated by the revolution about the x-axis of the loop of the curve $y^2 = x^2\left(\frac{3a-x}{a+x}\right)$.

(6 x 5 = 30 Marks)

PART-D

(Answer any *three* questions. Each question carries *eight* marks)

Using the algorithm for graphing, graph the function $y = 4x^3 - x^4$. Include the coordinates of any local extreme points and inflection points.

The cost function at a soft drink company is $c(x) = x^3 - 6x^2 + 15x$ (x in thousands of units). Is there a production level that minimizes average cost? If so, what is it?

Using limits of Riemann sums, establish the equation $\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}$.

Prove that the length of the arc of the parabola $y^2 = 4ax$ cut off by the latus rectum is $2a[\sqrt{2} + \log(1 + \sqrt{2})]$.

Show that the centre of mass of a thin plate of constant density covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis.

(3 x 8 = 24 Marks)

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BMAT2C02 - Mathematics

(2018 Admission onwards)

...e: 3 hours Max. Marks: 80

Answer all questions. Each question has ONE mark

1. Define a partition of $[a, b]$.
2. Give an example for a non-integrable function.
3. $\frac{d}{dx}\left(\int_0^x \frac{1}{1+t^2}\,dt\right) = \ldots\ldots\ldots\ldots\ldots$
4. Set up an integral for the area of the surface generated by revolving the curve $y = \tan x, 0 \leq x \leq \frac{\pi}{4}$ about the x- axis.
5. $\cosh 2x = \ldots\ldots\ldots\ldots$
6. Show that $\operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x}$
7. Find a formula for the n^{th} term of the sequence 0,1,1,2,2,3,3,4, ...
8. Find the sum of the series $\sum_{n=1}^{\infty}(-1)^n \frac{7}{4^n}$
9. State the absolute convergence test.
10. Graph the set of points whose polar coordinates satisfy the conditions $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$.
11. Identify the conic $r = \frac{6}{2+3\cos\theta}$.
12. Find the slope of the cardiod $r = -1 + \sin\theta$ at $\theta = 0$.

(12 × 1 = 12 Marks)

II. Answer any SEVEN questions. Each question has TWO marks

13. Without evaluating show that the value of $\int_0^1 \sqrt{1+\cos x}\,dx$ is less than $^3/_2$.
14. State mean value theorem for definite integrals.
15. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta \cot\theta\, d\theta$.
16. Find the volume of the torus (doughnut) generated by revolving a circular disk of radius a about an axis in its plane at a distance $b \geq a$ from its center.
17. Show that $\cosh^2 x - \sinh^2 x = 1$.

18. Show that the alternating harmonic series converges conditionally.

19. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $x = 2$.

20. Find the equivalent Cartesian point corresponding to a point whose polar coor

given by $\left(5, \tan^{-1}\left(\frac{4}{3}\right)\right)$.

21. Find the area of the region in the plane enclosed by the cardiod $r = a(1 + \cos\theta$

$(7 \times 2 = 14$

III. Answer any SIX questions. Each question has FIVE marks

22. Find the area of the region between the x-axis and the graph of

$f(x) = x^3 - x^2 - 2x, -1 \leq x \leq 2$.

23. A pyramid 3 m high has a square base that is 3 m on a side. The cross-sectio pyramid perpendicular to the altitude x m down from the vertex is a square x m o Find the volume of the pyramid.

24. Find the length of the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from $y = 1$ to $y = 2$.

25. Evaluate $\int_{-\ln 2}^{0} \cosh^2\left(\frac{x}{2}\right) dx$.

26. Discuss the convergence of $\{a_n\}$ with $a_n = \frac{\ln n}{n^{1/n}}$.

27. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$.

28. Find the points of intersection of the curves $r^2 = 4\cos\theta$ and $r = 1 - \cos\theta$.

29. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.

$(6\times 5 = 30$

IV. Answer any THREE questions. Each question has EIGHT marks

30. (i). Find the mean value of $f(x) = \sqrt{4 - x^2}$ on $[-2,2]$.

(ii). Find the area of the region enclosed by the parabola $y = 2 - x^2$ and

$x + y = 0$.

31. (i). Evaluate $\frac{d}{dx}(\cosh^{-1}(\sec x)), 0 < x < \frac{\pi}{2}$

(ii). Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, 1 about the x-axis.

32. Discuss the convergence of the alternating series $\sum \frac{(-1)^n}{1+\sqrt{n}}$.

33. Find the radius of convergence and interval of convergence of the powe

$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$.

34. Find the area of the surface generated by revolving the right hand loop of $r^2 =$ about the y-axis.

$(3 \times 8 = 24$