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B2M17200 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, March 2017

MAT2B02 - Calculus

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART-A

(Answer *all* questions. Each question carries *one* mark)

1. When a function f is said to have a local maximum value at an interior point c of its domain?
2. Why Rolle's theorem is not applicable for $f(x) = |x|$ in the interval $[-1, 1]$?
3. Give an example of a curve which is concave up on every interval.
4. $\lim_{x\to\infty}\left(\frac{2}{x} - 3\right) = \cdots$
5. Find dy if $y = x^7 + 100x$.
6. Write a partition of the closed bounded interval $[0, 2]$ having norm 0.3.
7. The average value of $f(x) = x$ on $[0, 2]$ is....
8. State the Mean Value Theorem for definite integrals.
9. If $y = \int_0^x \sqrt{1+t^2}\,dt$, then $\frac{dy}{dx} = \cdots$
10. Suppose $\int_0^1 f(x)\,dx = 3$, then find $\int_{-1}^0 f(x)dx$ if $f(x)$ is even.
11. When a function is said to be smooth?
12. The work done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$ is given by.....

(12 x 1 = 12 Marks)

PART-B

(Answer any *seven* questions. Each question carries *two* marks)

13. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.
14. Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.
15. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.
16. When a function f is said to be Riemann integrable over $[a, b]$?
17. Show that the value of $\int_0^1 \sqrt{1+\cos x}\,dx$ cannot possibly be 2.
18. Evaluate $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx$
19. The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.
20. Find the length of the arc of the curve $y = \frac{x}{2}$, $2 \le x \le 6$.
21. Find the work W done by the force of $F(x) = \frac{1}{x^2}N$ directed along the x-axis from $x = 1m$ to $x = 10m$.

(7 x 2 = 14 Marks)

PART-C

(Answer any *six* questions. Each question carries *five* marks)

22. If f has a local maximum or minimum value at an interior point c of its domain, and if f is defined at c, then prove that $f'(c) = 0$.

23. Prove that the curve $y = \frac{x}{1+x^2}$ has three points of inflection and they are collinear.

24. Determine the smallest perimeter possible for a rectangle whose area is 16 square units.

25. Using limits of Riemann sums, prove that $\int_0^b x dx = \frac{b^2}{2}$

26. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$. Does f actually take on this value a some point in the given domain?

27. Sketch region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$. Also find th area of the region enclosed by the curves.

28. Find the volume of the solid generated by the revolution about the x-axis of the loop c the curve $y^2 = x^2 \left(\frac{3a-x}{a+x}\right)$.

29. Find the centre of mass of a wire of constant density δ shaped like a semicircle of radiu a.

(6 x 5 = 30 Marks

PART-D

(Answer any *three* questions. Each question carries ***eight*** marks)

30. Using the algorithm for graphing, graph the function $y = x^4 - 2x^2$. Include th coordinates of any local extreme points and inflection points.

31. The cost function at a soft drink company is $c(x) = x^3 - 6x^2 + 15x$ (x in thousands c units). Is there a production level that minimizes average cost? If so, what is it?

32. a) Find $\frac{dy}{dx}$ if $y = \int_0^{\sqrt{x}} \cos t \, dt$

    b) Find the area of the region between the x-axis and the graph of

$$f(x) = x^3 - x^2 - 2x, -1 \leq x \leq 2.$$

33. Prove that the length s of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ measured from $(0, a)$ to the point $(x, y$ is given by $s = \frac{3}{2}(ax^2)^{\frac{1}{3}}$. Also find the entire length.

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq \frac{1}{2}$ about th x-axis.

(3 x 8 = 24 Mark

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MAT2C02 - Mathematics

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PART-A

Answer all the **TWELVE** questions

Each question carries **ONE** mark

1. Write the formula cosh (2x) in terms of sinh(x) and cosh(x).
2. Express $sinh^{-1}$(x) in terms of natural logarithms.
3. State the limit comparison test for the convergence of an improper integral.
4. Write the n^{th} term test for the convergence of a series.
5. What is the value of $\lim_{n\to\infty} n^{1/n}$.
6. What is the condition on r so that the geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ = a+ar+ar^2+.... to Converges?
7. Define absolute convergence of a series.
8. Roughly graph the set of points whose polar coordinates satisfy $1 \leq r \leq 2$ *and* $0 \leq \theta \leq \frac{\pi}{2}$.
9. Give the polar equation of the circle through the origin, cantered on the x-axis at (a,0).
10. Write the cylindrical coordinate of the point (0, 1, 0) in rectangular system.
11. Define the continuity of the function f(x,y) at the point (x_0, y_0).
12. Find $\frac{\partial f}{\partial x}$ if f(x, y) = e^{xy}.

(12 x 1 = 12 Marks)

PART-B

Answer any **SEVEN** questions

Each question carries **TWO** marks.

13. Evaluate the integral $\int_0^1 sinh^2$(x).
14. Is the area under the curve y = $\frac{1}{\sqrt{x}}$ from x = 0 to x = 1 finite. If so what is it?
15. Using Sandwich theorem show that the sequence $\left\{\frac{1}{2^n}\right\}$ converges to zero.
16. Show that the series $\sum_{n=1}^{\infty} \frac{n}{2n+5}$ diverges.
17. Check the convergence of the alternating series 1 - $\frac{1}{2}$ + $\frac{1}{3}$ - $\frac{1}{4}$ +......
18. Find the Cartesian equation corresponding to the polar equation r^2 = 4r cosθ.
19. Find an equation for the hyperbola with eccentricity $\frac{3}{2}$ and diretrix x = 2.
20. Find the linearization of f(x,y,z) = x^2+y^2+z^2 at the point (1,1,1,).
21. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if f(x,y) = x^y

(7 x 2 = 14 Marks)

PART-C

Answer any **SIX** questions

Each question carries **FIVE** marks

22. Find the volume of the solid generated by revolving the region between the parab
$x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

23. Find the length of the curve $y = \frac{(x^2+2)^{\frac{3}{2}}}{3}$ from $x = 0$ to $x = 3$.

24. Test the convergence of the series $\sum_{n=0}^{\infty} \frac{n^3+1}{2^n+1}$.

25. Test for the convergence of the series $\sum a_n$ if $a_n = (\sqrt[n]{n} - 1)^n$.

26. Find the area of the region shared by the circle $r = 2$ and the cardioids $r = 2(1\text{-cos}$

27. Find the area of the surface generated by revolving the right-hand loop of the lemniscates $r^2 = \cos(2\theta)$, from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$, about the x-axis.

28. Show that the function $f(x,y) = \frac{x^4}{x^4+y^2}$ has no limit as $(x,y) \to (0,0)$.

29. Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

(6 x 5 = 30 M

PART-D

Answer any **THREE** questions

Each question carries **EIGHT** marks

30. a). Show that the improper integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is convergent and find its value.

b). Test for the convergence of the improper integral $\int_1^{\infty} \frac{\log x}{x+2}\, dx$.

31. a). Test for the convergence and absolute convergence of the series $1 - \frac{1}{4} + \frac{1}{7} - \frac{1}{10}$

b). Find the Taylor series of $f(x) = \sin x$ at $x = 0$.

32. a). Identify the function $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots$

b). Find the values of x for which the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$ Converges.

33. Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \le t \le 2\pi$.

34. a). Find f_x and f_y if $f(x,y) = \tan^{-1}(\frac{y}{x})$.

b). Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x+2y+z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln(s)$

(3 x 8 = 24