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6M20287 (Pages : 2) Reg. No:...............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth SemesterB.Sc. Degree Examination, March/April 2020

BMAT6B09 – Real Analysis

(2017 Admission onwards)

me: 3 hours Max. Marks:120

Section A Answer all questions. Each question carries 1 mark.

1. Discuss the truthness of the statement :
"A continous real valued function on a bounded interval is bounded".
2. Give example of a continuous function with absolute minimum, but not with absolute maximum.
3. Give example of a continuous function that is not uniformly continuous on $(0, 1)$.
4. Find $\| \mathscr{P} \|$, the norm of the partition $\mathscr{P} = \{0, 0.1, 0.25, 0.38, 0.57, 0.73, 0.88, 1\}$ of $[0, 1]$.
5. Identify the type of the improper integral $\int_0^\infty xe^{-x}dx$.
6. Define the radius of convergence of a power series.
7. Write the relation between Beta and Gamma functions.
8. Write as a Beta Function the integral $\int_0^{\frac{\pi}{2}} \sin^5\theta\cos^3\theta\, d\theta$.
9. If $f_n(x) = \frac{x}{n}, x \in [0, 1]$, what is the uniform norm of f_n ?
10. Evaluate $\Gamma(5)$.
11. State the Weierstrass M-test for absolute convergence of a series of functions.
12. Write example of an improper integral of the third kind. $12 \times 1 = 12$ **marks.**

Section B Answer any 10 questions. Each question carries 4 marks.

13. If $f : [1, 3] \longrightarrow \mathbf{R}$ is given by $f(x) = x^2 + 5x + 2$, prove that $\exists \quad c \in [1, 3]$ such that $19 < f(c) < 21$.
14. Establish appropriately that the function $f(x) = x^3 - 3x + 1 = 0$ has a root between 0 and 1.
15. Prove that $\cos x$ is uniformly continuous on the set of real numbers.
16. State the non-uniform continuity criteria.
17. For the function $f(x) = \sin x$ on $[0, \frac{\pi}{2}]$, taking $\mathscr{P} = \{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\}$ as a partition and the right end point of eacch subinterval as the tags, write the Riemann sum.
18. State the Cauchy integrability condition. Give example of a function **not satisfying** the condition.
19. Explain the steps to prove that $\int_1^2 x^4\, dx = \frac{31}{5}$. State the theorem used.
20. State the Trapezoidal Rule.
21. Prove that the n^{th} term of a convergent series tends to 0. Is the converse true ? Give reason.
22. Find the radius of convergence of the power series $\sum_{n=1}^\infty \frac{x^n}{n^2+3n+4}$.

23. Find pointwise limit of the function $f(x) = x^n(1-x)$ on $[0, 1]$.

24. Give example of a sequence of functions converging pointwise, but not uniformly on the domain.

25. State and prove the symmetry property of Beta function.

26. Evaluate $\int_0^1 \frac{1}{\sqrt{x}}(1-x)^2\,dx$. $10 \times 4 = 40$ **marks.**

Part C

Answer any Six questions. Each question carries 7 marks.

27. State and prove the Uniform Continuity theorem.

28. Find the first four Bernstein polynomials of $f(x) = x^2$ on $[0, 1]$.

29. Define $f \in \mathscr{R}[a, b]$. Prove the uniqueness of the Riemann integral.

30. Prove that Riemann integrable functions are bounded.

31. State and prove the Cauchy Criterion for Riemann Integrability.

32. If $f : [a, b] \longrightarrow \mathbf{R}$ is continuous, then prove that $f \in \mathscr{R}[a, b]$.

33. Show that the sequence $\{f_n\}$ defined by $f_n(x) = \frac{x}{nx+1}$ is uniformly convergent on $[0, \infty)$.

34. State and prove the integral test for a series.

35. State and prove the relation between Beta and Gamma functions. $6 \times 7 = 42$ **marks.**

Part D

Answer any Two question. Each question carries 13 marks.

36. State and prove the Location of Roots theorem.

37. State and prove the additivity theorem for Riemann Integrals.

38. Define the Beta Function. Prove the convergence of the Beta Function. $2 \times 13 = 26$ **marks.**

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2020

BMAT6B10 – Complex Analysis

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A

Answeralltwelve questions

Each question carries 1mark

1. Show that $f(z) = z^n$, $n \geq 1$, n is an integer is differentiable for all z.
2. Define entire function.
3. Define C-R equation in polar form of the function $f(z) = u(r, \theta) + i\,v(r, \theta)$.
4. Evaluate $\int_0^{\pi/4} e^{it}\,dt$.
5. Define simply connected domain and give an example.
6. If v is a harmonic conjugate of u in a domain D, then prove that $-u$ is a harmonic conjugate of v in D.
7. Define absolute convergence of a series.
8. State Taylors Theorem.
9. Write Cauchy-Hadamard formula for radius of convergence.
10. Identify the singularity of $\frac{z}{\cos z}$.
11. Define the concept of zeros of order m of an analytic function f at $z = z_0$
12. Discuss the nature of singularity of the function $f(z) = \frac{1}{(z-4)^3}$.

(12 X 1= 12 marks)

Section B

Answer any ten out of fourteen questions

Each question carries 4mark

13. Show that the function $f(z) = \sqrt[3]{z}$ is differentiable everywhere. Also find $f'(z)$.
14. Suppose that a function f(z) and its conjugate are both analytic in a region D. then prove that f(z) is constant throughout D.
15. Find all values of z such that $e^z = 3 + 4i$.
16. Show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
17. State and prove Morera's Theorem.
18. If $|f(z)| \leq M$ everywhere on a contour C and L is the length of C then $\int_C |f(z)dz| \leq ML$.
19. Evaluate $\int_{|z|=3/2} e^z/(z-1)(z^2+4)$.
20. Integrate $\frac{z^2-1}{z^2+1}$ around $|z-i| = 1$ in the counter clockwise direction.
21. State Cauchy Goursat Theorem and evaluate $\int_{|z|=1} \tan z\,dz$.

22. Obtain the Taylor series representation for $\frac{1}{z}$ $about\ z = i$.

23. Show that Maclaurin series representation of $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} |z| < \infty$.

24. Check the nature of singularity of the function $\frac{\sin z}{z}$.

25. Find the Laurents series of $z^2 exp\left(\frac{1}{z}\right)$.

26. Find the residue of the function $\exp\left(\frac{1}{z^2}\right)$.

(10 X 4 = 40 marks)

Section C
Answer any six out of nine questions
Each question carries 7 mark

27. Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2+y^2}$ harmonic but u + iv is not analytic function of z

28. Let $u(x,y) = 4xy - x^3 + 3xy^2$, show that u is a harmonic function. Find its harmonic conjugate and hence the most general analytic function f(z) with u as real component.

29. For any complex valued function w(t), prove that $|\int_a^b w(t)dt| \leq \int_a^b |w(t)|dt$

30. Show that $\int_C (z - z_0)^m dz = \begin{cases} 2\pi i & if\ m = -1 \\ 0 & if\ m \neq -1 \end{cases}$ where m is an integer and C is the circle with center z_0 and radius r in the counter clockwise sense.

31. State and prove Cauchy's integral formula.

32. Prove that any n^{th} degree polynomial has at least one zero.

33. Find all Laurent series of $f(z) = -1/(z-1)(z-2)$ about $z = 0$.

34. Find the residue at the singular points of the function $\frac{1}{z(e^z-1)}$.

35. Evaluate $\int_0^{2\pi} \frac{d\theta}{13-5\sin\theta}$

(6 X 7 = 42 marks)

Section D
Answer any two out of three questions
Each question carries 13 mark

36. Suppose f(z) = u + iv and that $f'(z)$ exist at a point $z_0 = x_0 + iy_0$. Then the first order partial derivatives of u and v must exist at (x_0 , y_0) and they must satisfy the C.R. equations.

37. Suppose f(z) is a continuous complex valued function and assume that the integral of f(z) around a closed contour lying entirely in D have value zero, then f(z) has an antiderivative F(z) throughout D.

38. State and prove Cauchy's Residue theorem. Using this result evaluate $\int_{|z|=1} \frac{1}{z}\, dz$.

(2 x 13 =26 marks)

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1B6M20289 (Pages : 3) Reg. No:..............................
Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2020

BMAT6B11 – Numerical Methods

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A
Answer all *twelve* questions
(Each Question carries one mark)

1. Write Newton-Raphson formula for finding a root of $f(x) = 0$.
2. State Newton's forward difference interpolation formula.
3. What is mean by inverse interpolation.
4. Define the *averaging* operator, μ.
5. Show that $\Delta \equiv E - 1$, where Δ is the forward difference operator and E is the shift operator.
6. State Guass' backward formula.
7. Write the formula for computing the value of $\frac{dy}{dx}$ for non-tabular values of x using Newton's forward difference formula.
8. What is meant by complete pivoting.
9. Define spectrum of a square matrix.
10. Write fourth order Runge-Kutta formula.
11. State Newton's general interpolation formula with divided difference.
12. Define the characteristic equation of a square matrix.

(12 x 1 = 12 Marks)

Section B
Answer any *ten* out of fourteen questions.
(Each question carries 4 marks.)

13. Construct a forward difference table for (x_i, y_i); $i=0,1,2,......6$.
14. Show that $E = e^{hD}$, where E is the shift operator and $D=\frac{d}{dx}$.
15. Find the cubic polynomial which takes the following values: y(1)=24, y(3)=120, y(5)=336 and y(7)=720.

16. Find the Lagrange interpolating polynomialof degree two approximating the function *$y = \ln x$ defined by the following table of values:*

x	2	2.5	3.0
$y = \ln x$	0.69315	0.91629	1.09861

17. A solid of revolution is formed by rotating about the X-axis the area between the X-axis, the lines x=0 and x=1 and a curve through the points with the following coordinates:

x	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed.

18. Write Simpson's 3/8 formula.

19. Solve the system $0.0002x + 0.3003y = 0.1002$; $2.0000x + 3.0000y = 2.0000$ by partial pivoting.

20. Find three terms of y(x), given $y'' - xy' - y = 0$, $y(0) = 1, y'(0) = 0$ using Taylor's series method.

21. Explain Jacobi and Guass- Seidel method for solving linear systems by iterative methods.

22. Given $\frac{dy}{dx} = y - x$, $y(0) = 2$, find y(0.1) and y(0.2) by second order Runge-Kutta method.

23. Use Newton's divided difference formula find the value of $\log_{10} 301$, given

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.487

24. Find a real root of equation $f(x) = x^3 - 2x - 5 = 0$ using secant method.

25. Show that $\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$.

26. Write Lagrange's interpolation formula.

(10 x 4 = 40 Marks)

Section C

Answer any *six* out of nine questions.
(Each question carries 7 marks.)

27. Find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0,1] using iteration method correct to three decimal places.

28. Show that $e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots\right) = u_0 + u_1 x + u_2 \frac{x^2}{2!} + \cdots$

29. Using Lagrange's interpolation formula, find the form of the function *y(x)* from the following table:

x	*0*	*1*	*3*	*4*
y	*-12*	*0*	*12*	*24*

30. Solve the system $2x_1 + x_2 - x_3 = -1$, $x_1 - 2x_2 + 3x_3 = 9$, $3x_1 - x_2 + 5x_3 = 14$ by Gauss-Jordan Method.

31. Determine the largest eigenvalue and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

32. Solve the equation $y' = x + y^2$, subject to the condition $y = 1$ when $x = 0$ using Picard's method(Find two approximations).

33. From the following table, find the value of $e^{1.17}$ using Guass' forward formula:

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

34. Determine the value of y when $x = 0.1$, given that $y' = x^2 + y$, y(0)=1, using modified Euler's method, take h = 0.05.

35. Find the inverse of $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, using Guass elimination.

(6 x 7= 42 Marks)

Section D

Answer any *two* out of three questions.
(Each question carries 13 rnarks)

36. (a) Using Newton's forward difference formula, fing the sum $s_n = 1^3 + 2^3 + 3^3 + \cdots + n^3$.
(b) Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$, choose $x_0 = 2$.

37. Solve the equations $2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8$, by LU decomposition.

38. (a) The differential equation $y' = x^2 + y^2 - 2$ satisfies the following data:

x	*-0.1*	*0*	*0.1*	*0.2*
y	*1.0900*	*1.0000*	*0.8900*	*0.7605*

Use Milne's method to obtain the value of $y(0.3)$.

(b) Show that $\mu^2 = \frac{1}{4}(\delta^2 + 4)$.

(2 x 13 = 26 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2020

BMAT6B12– Number Theory & Linear Algebra

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Part A: Answer all 12 questions. Each question carries 1 Mark.

1. $3n^2 - 1$ is never a perfect square, where n is an integer. Is it true?
2. Find the greatest common divisor of (-8, -36)
3. If $a \equiv b(\text{mod}\, m)$, $a \equiv b(\text{mod}\, n)$, then $a \equiv b(\text{mod}\, mn)$. Is it true?
4. Define the least common multiple of two non zero integers a, b.
5. Find an integer solution, if there is any, of $2x + 8y = 13$
6. Find a prime number, which is of the form $n^2 - 4$
7. If n is a an odd pseudoprime, then $2^n - 1$ is ......
8. State Wilson's theorem.
9. Let $S = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}\}$ such that $c = 1$. Is S a vector space?
10. Is union of a set of subspace of a vector space V a subspace?
11. Find the subspace of R^3 spanned by the singleton set $\{(1, 0, 0)\}$
12. If a linear map $f : V \to W$ is injective, then what is $Ker\, f$

(12×1=12 marks)

Part B: Answer any *TEN* questions. Each question carries 4 Marks.

13. State and prove Euclid's Lemma
14. Prove that if a is any odd integer, $32|(a^2 + 3)(a^2 + 7)$
15. Express the number 15435 in the canonical form
16. Show that $n^7 - n$ is divisible by 42
17. Show that Euler totient function $\phi(n)$ is an even integer, if $n > 2$
18. Define repunit. Give an example of a repunit and write the standard form of a repunit.
19. Show that the expression of any odd integer $\frac{a(a^2+2)}{3}$ is an intger for all $a \geq 1$.

20. Find the remainder obtained uppon dividing the sum $1! + 2! + 3! + \cdots + 99! + 100!$ by 12.

21. If p and q are distinct primes with $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then show that $a^{pq} \equiv a \pmod{pq}$

22. Show that the set W of complex metrices $\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & -\alpha \end{bmatrix}$ is a real vector space of dimension 4.

23. Let $f : V \to W$ be linear and if X is a subspace of V, then show that $f^{\to}(X)$ is a subspace of W.

24. Prove that the mapping $P_{r_i} : R^n \to R$ is defined by $P_{r_i}(x_1, x_2, \ldots, x_i \ldots x_n) = x_i$ is linear.

25. Prove that the set of upper triangular $n \times n$ metrices is a subspace of the vector space $Mat_{n \times n}(F)$

26. Let V be a vector space of dimension $n \geq 1$ over a field F. Then show that V is isomorphic to vector space F^n

(10×4 = 40 Marks)

Part C: Answer any *SIX* questions. Each question carries 7 Marks.

27. Show that 561 is an absolute pseudoprime

28. Show that there is no x for which both the linear congruence $x \equiv 29 \pmod{52}$ and $x \equiv 19 \pmod{72}$

29. Find the highest power of 7 which devides 900!

30. Find the last two digits in the decimal representation of 3^{256}

31. Solve the linear congruence $17x \equiv 9 \pmod{276}$

32. If V is a vector space with dim V=10 and X, Y are subepaces of V, dim X=9 and dim Y=8 then find the smallest possible value of dim $X \cap Y$

33. Define basis of a vector space. Show that in R^2, the subset $\{(1,1), (1,-1)\}$ is a basis

34. Prove that any line L that passes through the orign is a subspace of the vector space R^2.

35. If V has a finite basis B, then show that evey basis of V is finite and has the same number of elements as B.

(6×7=42 Marks)

Part D: Answer any *TWO* questions. Each question carries 13 Marks.

36. Using Euclidean algorithm calculate the the $gcd(12378, 3054)$. Also represent the gcd as a linear combination of 12378 and 3054

37. State and prove Fermat's theorem

38. If S is a subset of V, then show that the following statement are equivalent.
(1) S is a basis
(2)S is a maximal independent subset
(3) S is a minimal spanning set.

(2×13 = 26 Marks)

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1B6M20291 (Pages : 4) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2020

BMAT6B13(E01)– Linear Programming

(2017 Admission onwards)

Time: 3 hours Max. Marks: 80

SECTION A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Show that $S = \{(x_1, x_2) \mid 2x_1 + 3x_2 = 7\} \subset \mathbb{R}^2$ is a convex set.
2. Define convex hull of a set.
3. Define optimum solution of an LPP.
4. Define slack variable.
5. Write the unsymmetric primal-dual problem.
6. The second constraint of a primal is an equation, then —— dual variable is ——.
7. Define a non-degenerate BFS of the system of linear equations $AX = B$.
8. There exists a balanced transportation problem without an optimal solution. Justify your answer.
9. Does the set of cells $\{(1,2), (2,4), (4,6), (6,3), (3,5), (5,1)\}$ form a loop in a 5×6 transportation problem.
10. What do you mean by degeneracy in a transportation problem.
11. What is an assignment problem.
12. State König theorem.

(12 x 1= 12 marks)

SECTION B

Answer any **nine** *out of twelve questions.*
Each question carries 2 mark.

13. State the Fundamental theorem of linear programming.

14. Plot the feasible region in the plane for the LPP constraints: $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$ and $x_1, x_2 \geq 0$.

15. Write the standard form of an LPP.

16. Show that every vertex of the convex set of feasible solutions to an LPP is a basic feasible solution.

17. Show that dual of the dual of an LPP is the primal.

18. Write the dual of the LPP, Minimize $z = 3x_1 - 2x_2 + 4x_3$

$$\begin{aligned} \text{subject to: } & 3x_1 + 5x_2 + 4x_3 \geq 7 \\ & 6x_1 + x_2 + 3x_3 \geq 4 \\ & 3x_1 + 5x_2 + 4x_3 \geq 7 \\ & 7x_1 - 2x_2 - x_3 \leq 10 \\ & x_1 - 2x_2 + 5x_3 \geq 3 \\ & 4x_1 + 7x_2 - 2x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

19. Suppose optimality criteria of an LPP satisfied. What is indicated by one or more artificial vectors are in the basis at positive level.

20. What is the role of a pivot element in a simplex table.

21. Write the matrix notation of a transportation problem.

22. Find an IBFS by NWCR:

	I	II	II	IV	Available
A	6	4	1	5	14
B	8	9	2	7	16
C	4	3	6	2	5
Requirement	6	10	15	4	35

23. A balanced transportation problem possesses a finite feasible solution and an optimal solution. Justify your answer.

24. Write a note on restrictive Assignment problem.

(9 x 2= 18 marks)

SECTION C

Answer any **six** *out of nine questions.*
Each question carries 5 mark.

25. Prove that the closed half spaces of $\mathbb{R}^n$ are convex sets.

26. Solve graphically, Minimize $z = x_1 + \frac{3}{2}x_2$

$$\begin{aligned} \text{subject to: } & x_1 + x_2 \geq 1 \\ & 100x_1 + 10x_2 \geq 50 \\ & 10x_1 + 100x_2 \geq 10 \\ & x_1, x_2 \geq 0 \end{aligned}$$

27. Show that the set of all feasible solutions to an LPP is a convex set.

28. Explain Big-M method.

29. Write the LPP in the standard form, Maximaze $z = 3x_1 + 2x_2 + x_3$ subject to $2x_1 + 5x_2 + x_3 = 12, 3x_1 + 4x_3 = 11, x_2, x_3 \geq 0$ and x_1 is unrestricted

30. Prove that every loop in a transportation problem has an even number of cells.

31. Explain Vogel's approximation method.

32. Find an initial basic feasible solution by matrix minima method:

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

33. Solve the cost-minimizing assignment problem:

	I	II	II	IV	V
A	45	30	65	40	55
B	50	30	25	60	30
C	25	20	15	20	·40
D	35	25	30	30	20
E	80	60	60	70	50

(6 x 5= 30 marks)

SECTION D

Answer any **two** *out of three questions.*
Each question carries 10 mark.

34. Prove that there is a one-to-one correspondence between the optimum solutions to the general LPP and its reformulated LPP.

35. Solve by simplex method,

Minimize $z = 2x_1 + x_2$

subject to: $3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0$

36. Find an optimal solution to minimize the cost:

	D	E	F	G	H	Capacity
A	5	8	6	6	3	800
B	4	7	7	6	5	500
C	8	4	6	6	4	900
Requirement	400	400	500	400	800	

(2 x 10= 20 marks)