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1B3N20198 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2020

BMT3B03 - Theory of Equations and Number Theory

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A(*Short Answer type)*
Ceiling (Maximum marks – 25Marks
Each question carries 2 marks

1. Find the quotient and reminder when $\mathbf{x^5 - 3x^2 + 6x - 1}$ **divided by** $\mathbf{x^2 + x + 1}$
2. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two of its root is 1.
3. Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$
4. Define Symmetric and sigma function ?
5. Evaluate the sum $\sum_{k=1}^{50}(k^3 + 2)$.
6. Let a function $f(x)$ on $\mathbb{W}$ be recursively defined as

   $(x) = \begin{cases} x - 10 & if\ x > 100 \\ f(f(x+11)) & if\ 0 \le x \le 100 \end{cases}$. Then find $f(98)$.
7. Evaluate $\sum_{d|18}(\frac{18}{d})$, where d is a positive integer.
8. Determine whether 1729 is prime or composite.
9. Express the gcd of the pair of numbers 24, 28 as a linear combination of the numbers.
10. State the Fundamental Theorem of Arithmetic.
11. Prove that Two positive integers a and b are relatively prime if and only if $[a, b] = ab$.
12. Determine the number of incongruent solutions of the linear congruence

    $91y \equiv 119 \ (mod\ 28)$
13. Define Euler's phi function. Compute $\varphi(18)$ and $\varphi(21)$.
14. Let a be a solution of the congruence $x^2 \equiv 1(mod\ m)$. Show that $m - a$ is also a solution.
15. Compute $\tau(36)$ and $\sigma(36)$.

(Ceiling =25Marks)

Section B(*Paragraph type)*
Ceiling (Maximum marks) – 35Marks
Each question carries 5 marks

16. (a) Define Taylor's Formula

(b) Calculate the values of $f(x) = -x^4 + 6x^3 + x - 1$, and their Derivatives at $x = 1$

17. Solve $x^3 - 7x^2 + 16x - 12 = 0$, which has multiple roots

18. Prove that there are infinitely many primes.

19. Let a function $f(x)$ on $\mathbb{W}$ be recursively defined as

$(x) = \begin{cases} x - 10 & if\ x > 100 \\ f(f(x + 11)) & if\ 0 \leq x \leq 100 \end{cases}$. Then compute $f(99)$ and $f(f(99)$.

20. Using the Euclidean algorithm, find $(2024, 1024)$

21. Let a and b be any two positive integers, and r be the remainder when a is divided by b. Let $d = (a, b)$ and $d' = (b, r)$. Then prove that $d'|d$.

22. Solve the linear congruence $15x \equiv 7\ (mod\ 13)$.

23. State and prove Fermat's Little Theorem.

(Ceiling =35Marks)

Section C (*Essay type)*
Answer any two questions
Each question carries 10 marks

24. (a) State Rolle's Theorem

(b) Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$

(c) How many real roots has the equation $x^4 - 4ax + b = 0$

25. State and prove Division algorithm for integers.

26. (a) Two positive integers, a and b are relatively prime if and only if there are integers α and β such that $\alpha a + \beta b = 1$.

(b) If $a|c$ and $b|c$, and $(a, b) = 1$, then prove that $ab|c$.

State and prove the Fundamental Theorem of Arithmetic.

27. Define a multiplicative function. Prove that the Euler's phi function φ is multiplicative.

(2 x 10 =20Marks)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2020

BMT3C03 - Mathematics – 3

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

A maximum of 20 marks can be earned from this section.

Each question carries 2 marks.

1. Graph the curve traced by the vector function$r(t) = 2\cos t\,\mathbf{i} + \mathbf{2sin}\,t\,\mathbf{j} + 3\mathbf{k}$, $t \geq 0$.
2. If $r(t) = i + tj + t^2k$, is the position vector of a moving particle. Find the tangential and normal components of the acceleration at any t.
3. Compute $\nabla F(x, y, z)$for $F(x, y, z) = \frac{xy^2}{z^3}$.
4. Define the Curl of a vector field.
5. Find the divergence of a vector field F.
6. Evaluate $\oint_C xdx$, where C is the circle $x = cost\ y = sint$, $0 \leq t \leq 2\pi$.
7. Find Complex Number satisfying $z^2 = i$.
8. Write the complex number $\frac{i}{1+i}$ in the form $a + ib$.
9. Sketch the graph of the equation $Im(\bar{z} + 3i) = 6$.
10. Find the image of the line x = 0 under the mapping $f(z) = z$^2.
11. Define interior point of a set.
12. Find all values of z satisfying the equation $\cos z = i\ sin\ z$.

Section B
A maximum of 30 marks can be earned from this section.
Each question carries 5 marks.

13. Evaluate $\oint_C ydx + xdy + zdz$, where C is the helix.

    $x = cost, y = sint,\ z = t, 0 \leq t \leq 2\pi.$

14. Evaluate the double integral $\iint_R e^{x+3y} dA$ over the region bounded by the graphs of $y = 1, y = 2, y = x$ and $y = -x + 5$.

15. Define length of a space curve. Find the length of the space curve traced by the vector function $\mathrm{r(t)} = \mathrm{acost}\,\mathbf{i} + \mathrm{asint}\,\mathbf{j} + \mathrm{at}\mathbf{k}, 0 \leq t \leq 2\pi.$

16. Find the level surface of $\mathrm{f(x, y, z) = x^2 + y^2 + z^2}$ passing through (1, 1, 1). Graph the gradient at the point.

17. Find a complex number z satisfying the equation $\bar{z}^2 = 4z$.

18. State and prove fundamental theorem for contour integrals.

19. Prove that $\cos^2 z + \sin^2 z = 1$.

Section C
Answer any One question. Each question carries 10 marks

20. (a) Find the first partial derivatives of $\mathrm{f(x, y)} = xe^{x^3y}$.

    (b) Verify that the given function $z = \ln(x^2 + y^2)$satisfies Laplace's equation

    $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$

21. Evaluate the double integral $\iint_R xe^{y^2} dA$ over the region bounded by the graphs of $y = x^2, x = 0, y = 4$.