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Reg. No:.....

Name:

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Degree Examination, November 2018

BMAT1C01- Mathematics

(2017 Admission onwards)

Max. Time: 3 hours

Max. Marks : 80

Part -A

Answer all questions. Each carries one mark

1. If $n(A) = 2$ and $n(B) = 3$, then power set of $A \times B$ has number of elements.
2. $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4, 5, 6\}$ and $U = \{1, 2, \dots, 50\}$. Then $A \cap B^c = \dots$
3. Define relation on a set.
4. $\lim_{x \rightarrow 1} \frac{x^3 + 4x^2 - 3}{x^2 + 5}$
5. $F(x) = [x]$, the greatest integer function is continuous at
6. Find $dr/d\alpha$ at $\alpha = \sqrt{3}$ if $r = 1 - \frac{1}{\alpha}$.
7. Find the critical points of $f(x) = x^4 - x^3 + 10$.
8. Evaluate $g'(2)$ if $g(2) = \frac{1}{(t+2)^2}$
9. Explain linearization of $f(x)$ at $x \approx a$
10. $\lim_{x \rightarrow 2} f(x) = a$, then $\lim_{y \rightarrow 2} f(y) = \dots$
11. If $A = \emptyset$, then $(A \cap B)^c = \dots$
12. Vertical asymptotes of $y = \frac{x+1}{3x+2}$

(12 x 1 = 12 Marks)

PART -B

Answer any seven questions. Each carries two marks

3. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is transitive?
4. Prove that $(A \cap B)^c = A^c \cup B^c$
5. Evaluate $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$
6. Define continuity of a function at a point
7. Find dy/dx if $y = \frac{\sqrt{x}}{\sin x}$
8. Find the slope of the tangent of the curve $y = x^3 - 3x$ at $(3, 2)$
9. Find the rate of change of area of a circle with respect to the radius.
10. Find the critical points of the curve $y = x^{-1/3}(x+2)$
11. $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 3}{3x^2 - 2x}$

(7 x 2 = 14 Marks)

PART -C

Answer any six questions. Each carries five marks

22. Let A is the set of even numbers less than 10, B is the set of prime numbers less than 10 and U is the set of natural numbers less than 10. Find $(A-B) \times (B \cap A^c)$.
23. Define an equivalence relation with an example.
24. $F(x) = \frac{(x+3)[x+2]}{x+2}$, Where $[x]$ is the greatest integer function.
Find $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$.
25. If $f(x) = \sqrt{1+x}$, $L=1$, $x_0=0$, $\epsilon=0.1$. Find an open interval containing x_0 and a value of $\delta > 0$ such that $0 < |x - x_0|$ implies $|f(x) - L| < \epsilon$.
26. The curve $y = x^2 + ax - b$ and $y = cx + x^2$ have common tangent at the point $(-1, 0)$. Find a, b and c.
27. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
28. Discuss the continuity of the function $f(x) = x \sin(1/x)$ if $x \neq 0$, $f(0) = 0$.
29. Find dy/dx if $y = \frac{(x^2+1)(x+3)^{2/3}}{(x-2)^{3/2}(x^2-3)}$.

(6 x 5 = 30 Marks)

PART -D

Answer any three questions. Each carries eight marks

30. (a) State mean value theorem.
(b) Verify mean value theorem for $f(x) = \sqrt{x^2 - 4}$ in $[2, 3]$
(c) Find the interval in which $f(x) = x^3 - 6x^2 + 5x - 2$ increasing and decreasing.
31. Evaluate (a) $\lim_{x \rightarrow 0} (\frac{1}{x^2} - \cot^2 x)$ (b) $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$
32. Sketch the graph of the function $y = 6x/(3+x^2)$.
33. (a) Find the volume of the solid lies between the plane perpendicular to x-axis, $x = -1$ and $x = 1$. The cross sections perpendicular to x-axis are circular disks whose diameter run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.
(b) Find the derivative of $f(x) = (x-1)(x-2)(x-3)$.

34. Let R_1 and R_2 be relations on a set A represented by the matrices $MR_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and

$$MR_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Find the matrices which represent (a) } R_1 \cap R_2 \quad \text{(b) } R_1 \cup R_2$$

and $R_1 \circ R_2$.

Is this relations an equivalence relation?

(3 x 8 = 24 Marks)

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PART-A

Answer all questions. Each question carries *one* mark

1. A relation R on a set A is said to be anti symmetric if -----
2. Write the dual of $(A \cup B)^c \cap A = \emptyset$
3. $(A \cap B)^c =$ -----
4. The graph of $y = f(x-a)$ is the graph of $y = f(x)$ shifted -----
5. $\lim_{x \rightarrow 2} \sqrt{4x^2 - 3}$.
6. $\lim_{x \rightarrow 0^-} \frac{1}{x} =$ -----
7. The truth value of the statement $\exists x(x^3 = -1)$ the domain consists of all real numbers is -----
8. Write the converse of the statement, 'If today is Thursday then I have a test Today'.
9. The contra positive of $p \rightarrow q$ is-----
10. If $f(x) = \frac{x^2 - 1}{x - 1}$, then $f(1) =$ -----
11. $-10 \pmod{3} =$ -----
12. The graph of an odd function is symmetric about-----

(12 x 1 = 12)

PART-B

Answer any *seven* questions. Each question carries *two* marks

13. If $A_i = \{1, 2, 3, \dots, i\}$, $i = 1, 2, 3, \dots$ find $\bigcup_{i=1}^n A_i$ and $\bigcap_{i=1}^n A_i$.
14. Define a partial order relation on a set and give an example.
15. If $A = \{a, b, c\}$, find the power set of A.
16. Find the domain and range of the function $g(z) = \sqrt{4 - z^2}$.
17. Find the center and radius of the circle $x^2 + y^2 + 4x - 4y + 4 = 0$.
18. Find $\lim_{h \rightarrow 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.
19. If p is the proposition "You drive over 65 miles per hour" and q is the proposition "You get a speeding ticket", write the proposition "You drive over 65 miles per hour, but You do not get a speeding ticket" using p, q and logical connectives.
20. Let $Q(x, y)$ denote the statement " $x = y + 2$ ". What are the truth values of propositions $Q(1, 2)$ and $Q(2, 0)$
21. Express the statement "Every student in this class has studied Calculus" as a universal quantification.

(7 x 2 = 14)

PART- C

Answer any *six* questions. Each question carries *five* marks.

22. Let A be a set of nonzero integers and let ' \approx ' be the relation on $A \times A$ defined as $(a,b) \approx (c,d)$ whenever $ad = bc$. Prove that ' \approx ' is an equivalence relation.
23. Let $A = \{1,2,3,\dots,14,15\}$. Let R be the equivalence relation on A defined by congruence modulo 4. Then
- Find the equivalence classes determined by R .
 - Find a system B of equivalence class representatives which are multiples of 3.
24. For each $m \in P$, where P is the set of positive integers, let $A_m = \{m, 2m, 3m, \dots\}$. Then find
- $A_2 \cap A_7$
 - $A_5 \cap A_8$
 - $A_3 \cup A_{12}$
 - $A_3 \cap A_{12}$
 - $A_3 \cap A_5$.
25. Given $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$, Prove that $\lim_{x \rightarrow c} [f(x) + g(x)] = L + M$.
26. State Sandwich theorem for limit of functions. If $\lim_{x \rightarrow c} |f(x)| = 0$, show that $\lim_{x \rightarrow c} f(x) = 0$.
27. Show that $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ is not continuous at $x=2$, but has a continuous extension to $x=2$ and find that extension.
28. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.
29. Show that $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology using truth table.

(6 x 5 = 30)

PART D

Answer any *three* questions. Each question carries *eight* marks

30. Consider the function $f: A \rightarrow B$ and $g: B \rightarrow C$. Prove the following
- If f and g are one to one then so is $g \circ f$.
 - If f and g are onto then so is $g \circ f$.
31. Let B and $\{A_i\}$ with $i \in I$ be subsets of a universal set U . Then prove the following,
- $B \cap (\cup \{A_i\}) = \cup \{B \cap A_i\}$
 - $(\cup \{A_i\})^c = \cap \{A_i^c\}$ and $(\cap \{A_i\})^c = \cup \{A_i^c\}$.
32. Let R be the relation on the set P of positive integers defined by the equation $x + 3y = 12$
- Write R as a set of ordered pairs
 - Find the domain and the range of R
 - Find R^{-1}
 - Find the relation $R \circ R$
33. Show that the set Q of rational numbers is denumerable
34.
 - $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
 - $\neg p \rightarrow (p \rightarrow q)$ is a Tautology
 - $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

(3 x 8 = 24)