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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc. Mathematics Degree Examination, November 2019

BMAT3B03 – Calculus and Analytic Geometry

(2018 Admission onwards)

Time: 3 hours   Max. Marks: 80

PART- A
Answer *all* Questions. Each carries *one* mark

1. Find $\frac{dy}{dx}$ if $y = x^x$, $x > 0$.

2. If $\cos hx = \frac{5}{4}$, then $\sin hx =$ ....

3. Find $\frac{dy}{dx}$ if $y = x \sin hx - \cos hx$.

4. Range of the function $y = \cos hx$ is ....

5. Define hyperbolic sin function in terms of exponential function

6. Evaluate $\lim_{x \to \frac{\pi}{2}^-} \frac{\sec x}{1 + \tan x}$.

7. The nth term of the sequence $2, 1, 4, 3, 6, 5, 8, 7, \ldots$

8. The least upper bound of $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots$ is

9. Examine the convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$.

10. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{n-1} - 1}{6^{n-1}}$.

11. Graph the polar region $r \leq 0$ and $\theta = \frac{\pi}{4}$.

12. Find the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

(12 x1=12 marks)

PART B
Answer any *nine* questions. Each question carries *two* marks

13. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4 \cos \theta}{3 + 2 \sin \theta} d\theta$.

14. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

15. Show that the series $\frac{1}{1} + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \ldots \ldots$ Converges.

16. Show that the series $\sum_{n=1}^{\infty} n^2$ diverges.

17. Find the Taylor series expansion of $f(x) = e^{-x}$ at $x = 0$.

18. Find the polar equation of the ellipse $9x^2 + 4y^2 = 36$.

19. For what values of x does the power series $\sum_{n=1}^{\infty} n! \, x^n$ converges?

20. Sketch the curve $r = 1 - \cos\theta$.

21. Find the limit of the sequence, whose n^{th} term is $a_n = \dfrac{(^{10}/_{11})^n}{(^9/_{10})^n + (^{11}/_{12})^n}$.

22. Find the limit of the sequence whose n^{th} term is $a_n = \dfrac{n^2}{2n-1} \sin {^1/_n}$.

23. Test the convergence of the series $\sum_{n=1}^{\infty} n \sin {^1/_n}$.

24. Identify the conic and find its directrix $r = \dfrac{12}{3 + 3\sin\theta}$.   (9 x 2 = 18 marks)

PART-C
Answer any *six* questions. Each question carry *five* marks

25. Show that $\lim_{x \to \infty} (1 + \frac{1}{x})^x = e^x$.

26. Find the entire length of $x^{2/3} + y^{2/3} = 1$.

27. Test the following series for convergence $\dfrac{1.2}{3.4.5} + \dfrac{2.3}{4.5.6} + \dfrac{3.4}{5.6.7} \cdots \cdots$

28. Show that $\cos h^{-1} x = In(x + \sqrt{x^2 - 1}), x \geq 1$.

29. Find $\lim_{x \to \infty} x^{1/x}$.

30. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

31. Calculate e with an error of less than 10^{-6}.

32. Find he area shared by the curves $r = a\sqrt{2}$ and $r = 2a \cos\theta$.

33. Find the coordinates of the center, eccentricity , foci and the equation to the directrix of

$x^2 - y^2 - 2x + 4y = 4$.

(6 x 5 = 30 marks)

PART- D
Answer any *two* questions. Each question carry *ten* marks

34.   (a) Graph the curve $r^2 = \sin 2\theta$.

       (b) Find the points of intersection of the curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

35.   (a) Solve for x : $3^{\log_3 7} + 2^{\log_2 5} = 5^{\log_5 x}$.

       (b) Discuss the convergence of the series $\sum_{n=2}^{\infty} \dfrac{1}{n(Inn)^p}$, $P > 0$.

36.   State and prove ratio test.   (2 x 10 = 20 marks)

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                                                    Name: ....

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Third Semester B.Sc. Degree Examination, November 2019

BMAT3C03 – Mathematics

(2018 Admission onwards)

Time: 3 hours   Max. Marks: 80

Part A: Answer *all* questions. Each question carries 1 Mark.

1. Verify that the function $x^2 + y^2 = 1$ is a solution of the differential equation $x + y\frac{dy}{dx} = 0$.

2. What is a general solution of differential equation $\frac{dy}{dx} + p(x)y = 0$.

3. Define a perticular solution of an ordinay differential equation.

4. Check for exactness: $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$.

5. The general solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ is ....

6. State whether the functions x and $logx$ are independent or not?

7. Find the row reduced echelon form of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & 2 \end{bmatrix}$

8. Define rank of a non zero matrix.

9. The system of linear equations $AX = 0$ has a non trivial solution if the deteminent A is ....

10. The eigen values of a Triangular marix are....

11. In Trapezoidal rule, find an upper bound for error estimate $|E_T|$.

12. Simpsons rule give exact value of the integral in ... number of steps.

                                                    (12×1=12 marks)

Part B: Answer any *SEVEN* questions. Each question carries 2 Marks.

13. Give a singular solution of $(\frac{dy}{dx})^2 - x\frac{dy}{dx} + y = 0$.

14. Find an integrating factor of the differential equation $2siny^2 dx + xycosy^2 dy = 0$.

15. Solve the differential equation $(1 + x^2)\frac{dy}{dx} = 1 + y^2$.

16. The ODE $y'' + ay' + by = 0$ has solutions $y = e^{\lambda x}$, where λ is a root of $\lambda^2 + a\lambda + b = 0$, then what are general solutions of the ODE for three different cases of λ.

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17. Write the Euler Cauchy equation of second order ODE and its auxilary equation.

18. Show that a square matrix A and itstranspose A^T have same characteristic roots.

19. Find the characteristic equations of the matrix $A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$

20. Explain the Trapezoidal rule, to find the approximate value of the given integral $\int_a^b f(x)dx$.

21. Using Picard's iteration method, solve $y' = x + y$, $y(0) = 1$. Find the first two approximated value of y at $x = 0.1$.

(7×2 = 14 Marks)

Part C: Answer any *SIX* questions. Each question carries 5 Marks.

22. Solve $x^3 y' + 3x^2 y = \cos x$.

23. Find the orthogonal trajectories of $x^2 - y^2 = c^2$.

24. Find the curve through the origin in the XY-plane which satisfies $y'' = 2y'$ and whose tangent at the origin has slope 1.

25. Solve the initial value problem $y'' + 3y' + 2.25y = -10e^{-1.5x}$, $y(0) = 1$ and $y'(0) = 0$

26. Under what condition the rank of the matrix $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

27. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to its canonical form.

28. Use Trapezoidal rule, evaluate with $n = 4$ to estimate $\int_0^2 \frac{1}{1+x} dx$.

29. Find the minimum number of subintervals needed to approximate the integral $\int_0^3 \sqrt{1+x}\,dx$, using simpson's rule with an error of magnitude less than 10^{-4}.

(6×5=30 Marks)

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Part D: Answer any *THREE* questions. Each question carries 8 Marks.

30. Solve $y' + x\sin 2y = x^3\cos^2 y$.

31. Using the method of reduction of order solve the differential equation $x^2 y'' - 5xy' + 9y = 0$, given that $y = x^3$ is a solution.

32. Find all the eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and an eigen vector corresponding to any one of the eigen value.

33. State Cayley Hamilton theorem. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence obtain A^{-1}

34. Find approximate solutions to the initial value problem $y' = 1 + y^2$, $y(0) = 0$, using Picard's iteration method, Compare it with the actual solution.

$(3 \times 8 = 24 \text{ Marks})$