1B4M17087 (Pages : 2) Reg. No:...........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Mathematics Degree Examination, March 2017

MAT4B04 – Theory of Equations , Matrices & Vector Calculus

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART – A

Answer ***all*** questions. Each question carries ***one*** mark

1. Reduce $3x^3 + 6x^2 + 5x + 7 = 0$ to the standard form
2. Form an equation whose roots 2 times those of are $5x^2 - 11x + 3 = 0$
3. If α, β, γ are the roots of $x^3 + ax^2 + bx + c = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} =$ ---------
4. Find the least number of imaginary roots of the equation $x^7 - x^4 + 10x^3 - 28 = 0$
5. The product of all characteristic roots of a square matrix A is
   (a) $|A|$ (b) 1 (c) 0
6. Find the rank of $\begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$
7. Define Nullity of a matrix
8. Find the Rank of the unit matrix of order n.
9. Define the angle between two intersecting planes.
10. Get the spherical coordinates of the point whose Cartesian coordinates is $(1, 0, 0)$
11. Evaluate $\int_0^{\pi} cost\mathbf{i}\, dt$
12. Define the curvature function of a smooth curve.

(12 × 1 = 12 *Marks*)

PART – B

Answer ***any nine*** questions. Each question carries ***two*** marks

13. Form an equation of the lowest degree with real coefficients having $2 + \sqrt{-3}$ and $3 + \sqrt{-5}$ as two of its roots.
14. If the roots of $x^3 + px^2 + qx + r = 0$, are in AP, show that $2p^3 - 9pq + 27r = 0$
15. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\alpha}$
16. Explain Descartes' rule of signs.
17. Prove that zero is an eigen value of a matrix A if and only of A is non-singular.
18. Find the value of k, if the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & k \end{bmatrix}$ is 2.
19. Show that no skew-symmetric matrix can be of rank 1.
20. Find the eigen values of $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$
21. Examine whether the system of equations $2x - 4y = 3, -3x + 6y = -4$ has a solution.
22. Find the parametric equation of the line passing through the points $(-2, 0, 3)$ and $(3, 5, -2)$.
23. Find the angle between the planes $x + y = 1$ and $2x + y - 2z = 2$
24. Find the tangent vector to the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(2, 4, 8)$

(9 × 2 = 18 *Marks*)

PART – C

Answer ***any six*** questions. Each question carries ***five*** marks

25. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ whose roots are in geometric progression.
26. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$
27. Solve by Cardon's method: $x^3 - 18x - 35 = 0$
28. Show that multiplication of the elements of a row by a non-zero number does not change its rank.
29. If A is a square matrix and P is a non-singular matrix of the same order, show that the matrices A and $P^{-1}AP$ have the same characteristic roots.
30. Solve the system of equations $x + y + z = 6,\ \ x - y + z = 2,\ \ 2x + y - z = 1$
31. If A is a non-singular matrix prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A
32. Solve the initial value problem: $\frac{dr}{dt} = -t\mathbf{i} - t\mathbf{j} - t\mathbf{k}, \mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
33. Find T, N and κ for the space curve $\mathbf{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + 2\mathbf{k}$

(6 × 5 = 30 *Marks*)

PART – D

Answer ***any two*** questions. Each question carries ***ten*** marks

34. (i) Solve $x^3 + x^2 - 16x + 20 = 0$, given that it has a double root.
    (ii) If α, β, γ are the roots of the equation $x^3 - 6x + 7 = 0$, form an equation whose roots are $\alpha^3 + 2\alpha + 3,\ \ \beta^3 + 2\beta + 3,\ \ \gamma^3 + 2\gamma + 3$

35. (i) State Cayley Hamilton theorem
    (ii) Using Cayley Hamilton Theorem, find A^{-1} and A^3 if $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

36. (i) Find the equation of the plane passing through the $(1, 1, -1), (2, 0, 2)$ *and* $(0, -2, 1)$
    (ii) Find the rectangular coordinates of the centre of the sphere:
    $r^2 + z^2 = 4r\cos\theta + 6r\sin\theta + 2z$

(2 × 10 = 20 *Marks*)

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MAT4C04 – Mathematics

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

Part A

(Answer all 12 questions)

1. Give an example of Linear and Non-linear second order differential equations.
2. Find the general solution of $y'' - 2y = 0$.
3. Check the linear dependence of the functions e^{2x} and e^{3x}.
4. Evaluate $\int_0^\infty e^{-st} \sin 2t \, dt$.
5. State the existence theorem for Laplace Transform.
6. Find $L^{-1}\left[\frac{(s+1)^2}{s^3}\right]$.
7. Give a function of period 2.
8. Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} - u = 0$.
9. Check whether the function $f(x) = x^3 + \sin x$ is odd or even.
10. Write the formula for Euler Method to solve differential equations numerically.
11. Give the formula for finding the error in the Simpson's rule of integration.
12. Define unit step function.

(12 × 1 = 12 marks)

Part B

(Answer any 9 questions)

13. Reduce to first order and solve $2xy'' = 3y'$.
14. Solve $(D^2 + 3D + 2)y = 0$.
15. Find the particular solution of the differential equation $y'' + 4y = e^x$.
16. State and prove the first shifting theorem for Laplace transform.
17. Prove that $L(e^{at}) = \frac{1}{s-a}$.
18. Find the inverse Laplace transform of $\frac{s}{(s-1)^2+4}$.
19. Find $L(t \sin t)$.
20. Obtain the Fourier coefficient a_0 for the function $f(x) = e^{-x}$ in $(0,2\pi)$.
21. Define and plot rectangular wave function.
22. Verify $u = e^{-4t}\cos x$ is solution of one dimensional heat equation.
23. Using Picard's method, find an approximate solution of $y' = x + y$; $y(0) = 1$ in 3 steps.
24. Find an upper bound for the error incurred in estimating $\int_0^\pi x \sin x \, dx$ with trapezoidal rule with n=4.

(9 × 2 = 18 marks)

Part C
(Answer any 6 questions)

25. Using method of variation of parameters, solve $y'' + y = \sec\ x$.
26. Solve the initial value problem $x^2y'' - 3xy' + 4y = 0,\ y(1) = 0,\ y'(1) = 3$.
27. Using convolution find $L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$.
28. Solve the integral equation $y(t) = 2t - 4\int_0^t y(u)(t-u)du$.
29. Find $L(f(t))$ if $f(t) = \begin{cases} 2 & 0 < t < \pi \\ 0 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$.
30. Solve the partial differential equation $u_x - u_y = 0$ by separating the variables.
31. Find the Fourier series expansion of

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases};\quad f(x + 2\pi) = f(x).$$

32. Using Simpson's rule with n=4, evaluate the approximate value $\int_0^2 \frac{1}{1+x} dx$.
33. Apply Runge-Kutta method with $h = 0.1$, to the initial value problem $y' = x + y^2,\ y(0) = 1$, in 2 steps.

(6 × 5 = 30 marks)

Part D
(Answer any 2 questions)

34. Solve $y'' - 4y' + 5y = e^{2x} cosec\ x$.
35. Using Laplace transform solve $y'' - 3y' + 2y = 4e^{2t}$; $y(0) = -3, y'(0) = 5$.
36. Find the two half range expansion of the function

$$f(x) = \begin{cases} \frac{1}{2}(\pi + x) & -\pi \le x < 0 \\ \frac{1}{2}(\pi - x) & 0 \le x < \pi \end{cases}$$

(2 × 10 = 20 marks)