

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Degree Examination, November 2019

BMT1C01- Mathematics - I

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

A maximum of 20 marks can be earned from this section

Answer all question. Each question carries 2 marks

1. The position of a bus at a time x is $y = 3x^2 + 8x$, for $x > 0$. At what time is the instantaneous velocity 11m/s?
2. Evaluate $\lim_{x \rightarrow 2} \frac{(x^2+x-6)}{(x^2+2x-8)}$.
3. If $h(x) = x^3 f(2x^2)$, find $h'(x)$.
4. Find $\int \frac{dx}{(3x+1)^5}$.
5. Prove the constant multiple rule for anti-differentiation.
6. Show that there is a number x_0 such that $x_0^5 - x_0 = 3$
7. A spherical balloon is being inflated. Find a general formula for the instantaneous rate of change of the volume V with respect to the radius .
8. Verify Rolle's Theorem for $f(x) = \cos x$ on $[\frac{\pi}{2}, \frac{3\pi}{2}]$.
9. Find $\sum_{j=3}^{102} (j - 2)$.
10. If $f(x) = \begin{cases} 2, & 1 \leq x < 4 \\ 5, & 4 \leq x < 7 \\ 1, & 7 \leq x \leq 10 \end{cases}$ find $\int_1^{10} f(x) dx$.
11. Let $F(x) = \int_0^{x^2} \frac{dt}{1+t^2}$. Find $F'(x)$.
12. Find the average value of $f(x) = x^2 + 1$ on $[1,2]$.

(Maximum Marks 20)

Section B

**A maximum of 30 marks can be earned from this section
Answer all question. Each question carries 5 marks**

13. Calculate an approximate value for $\frac{2}{\sqrt{0.99+(0.99)^2}}$.
14. Find the equation of the tangent line to the curve $2x^6 + y^4 = 9xy$ at the point (1,2).
15. Find the rate of change of $y = (x^2 + 1)^{27}(x^4 + 3x + 1)^8$ with respect to x .
16. Find $\lim_{x \rightarrow \pi} (x - \pi)^{\cot x}$.
17. Prove that, for any positive numbers a and b , $\frac{a+b}{2} \geq \sqrt{ab}$.
18. A ball of radius r is cut into three pieces by parallel planes at a distance of $r/3$ on each side of the center. Find the volume of each piece.
19. An object on the x -axis has velocity $v = 2t - t^2$ at time t . If it starts out at $x = -1$, at time $t = 0$, where is it at time $t = 3$? How far has it travelled?

(Maximum Marks 30)

Section C

(Answer any One Question. Each carries ten Marks)

20. Sketch the graph of $f(x) = \frac{x}{1+x^2}$.
21. Find the area between the graphs $x = y^2 - 2$ and $y = x$.

(1 x 10 = 10 Marks)

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Mathematics Degree Examination, November 2019

BMT1B01- Basic Logic and Calculus - I

(2019 Admission onwards)

Time: 2.5 hours

Max. Marks : 80

Section A

A maximum of 25 marks can be earned from this section

Each question carries 2 marks

Write the converse and contrapositive of the implication 'If the radio is working, then the battery is good'.

Prove that $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Determine the truth value of each proposition, where $P(x, y) : x^2 > y$, and x and y are real numbers. (a) $(\exists x)(\exists y) P(x, y)$ (b) $(\forall y)(\exists x) P(x, y)$

Find the domain of the function $f(x) = \frac{2x + \sqrt{x+1}}{3x-1}$

Find $\lim_{x \rightarrow 0} \frac{\sin 5x}{2x}$

Find the differential of the function $f(x) = x \tan x$, at $x = \frac{\pi}{4}$

Find the linearization of $f(x) = 4x^3$ at $x = 3$.

State the mean value theorem for continuous functions.

Find the interval in which $f(x) = x^2 - 6x + 5$ is increasing.

Determine where the graph of the function $f(x) = x^3 - 6x$, is concave upward and where it is concave downward.

Find the equation of tangent line to the graph of the function $f(x) = 2x^4$ at the point (1,2).

Evaluate $\sum_{k=1}^3 \frac{k-3}{k}$

Graph the function $f(x) = -x^2$ in $[0, 2]$

Find the average value of $f(x) = 3x^2 - 1$ in $[0, 4]$.

Find the derivative of the function $f(x) = \int_1^x \frac{1}{1+x^2} dx$

Section B

A maximum of 35 marks can be earned from this section

Each question carries 5 marks

16. Define a tautology and determine whether $(p \rightarrow \sim q) \leftrightarrow (q \rightarrow \sim p)$ is a tautology.
17. State and prove distributive laws of logic.
18. Show that the function $f(x) = |x|$, is differentiable everywhere except at $x = 0$.
19. Using the precise definition of limit, prove that $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$
20. Find the dimensions of a rectangle of area 144 m^2 that has the smallest possible perimeter.
21. Find all asymptotes of the graph of the function $f(x) = \frac{x^2 - 3}{x^2 - 1}$
22. Find the value of 'c' whose existence is guaranteed by the mean value theorem for integrals for the function $f(x) = \sqrt{x + 3}$ on $[1, 6]$.
23. A car moves along a straight road with velocity function $v(t) = t^3 + 2t - 3$, $0 \leq t \leq 8$, where $v(t)$ is measured in feet per second. Find the displacement of the car between $t = 1$ and $t = 4$.

Section C

Answer any two questions

Each question carries 10 marks

24. (a) Prove by contradiction: $\sqrt{2}$ is an irrational number.
(b) Prove that $n^4 - n^2$ is divisible by 3.

25. (a) Let $f(x) = \begin{cases} ax + b, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$

Find the values of a and b which will make the function continuous on $(-\infty, \infty)$

(b) Show that the equation $f(x) = x^3 + x - 1$, has exactly one zero in $(0, 1)$

26. (a) Find all relative extreme values and absolute extreme values of the function

$$f(x) = -x^3 + 12x + 5, \quad \text{where } -3 \leq x \leq 3.$$

(b) Find the points of inflection of $f(x) = (x - 1)^{\frac{1}{3}}$

27. (a) State and prove fundamental theorem of integral calculus (First form).
(b) Find the area of the region under the graph of $f(x) = 4 - x^2$ bounded by X-axis.