4M19196 (Pages : 3) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March /April 2019

BMAT4B04 – Theory of Equations , Matrices & Vector Calculus

(2017 Admission onwards)

ne: 3 hours Max. Marks: 80

PART – A

Answer *all* questions. Each question carries *one* mark

1. Give an equation whose roots are the negatives of the roots of $x^4 - 3x^3 + x^2 - x + 2 = 0$.
2. Find $\sum \frac{1}{\alpha}$ if α, β, γ are the roots of $x^3 + ax^2 + bx + c = 0$.
3. Find the real root of $x^3 - 8x^2 + 22x - 20 = 0$ if $3 + i$ is a root.
4. Give the minimum number of imaginary roots of $x^5 - x^4 - 4x - 1 = 0$, obtained by using Descartes rule of sign.
5. Define nullity of a Matrix.
6. Find the rank of $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix}$.
7. Find the Characteristic roots of $\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$.
8. Write the normal form of $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
9. Find the value of λ so that the vectors $\boldsymbol{a} = 2\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}, \boldsymbol{b} = 3\boldsymbol{i} + 2\boldsymbol{j} - \lambda\boldsymbol{k}$ are perpendicular.
10. Show that the vector $\mathbf{u}(t) = (\sin t)\boldsymbol{i} - (\cos t)\boldsymbol{j} + 5\boldsymbol{k}$ has constant length.
11. If $\mathbf{T} = (\cos t)\boldsymbol{i} - (\sin t)\boldsymbol{j}$, find the principal unit normal vector **N**.
12. Give a vector formula for curvature in terms of velocity and acceleration.

$(\mathbf{12 \times 1 = 12} \; \boldsymbol{Marks})$

PART – B

Answer *any nine* questions. Each question carries *two* marks

13. Find a rational cubic equation whose roots are $3, 2 - i$.
14. Remove the second term of the equation $x^4 + 8x^3 + x - 5 = 0$.
15. Transform the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ into one with integral coefficients and the leading coefficient unity.

16. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$.

17. If A is a non-singular matrix and A' its transpose, show that rank(A') = rank(A).

18. Reduce to Echelon form and find the rank of the matrix $\begin{bmatrix} 0 & 2 & 4 & 2 \\ 1 & 2 & 2 & 3 \\ 3 & 4 & 6 & -1 \end{bmatrix}$.

19. Solve the homogeneous system of equations $x - 2y + 3z = 0,\ 2x + 5y + 6z = 0$.

20. Find the inverse of $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$ using elementary column transformations.

21. Find a unit vector perpendicular to the plane of
$P = (2, -2, 1)$, Q$(3, -1, 2)$ and R$(3, -1, 1)$.

22. Find the parametric equation of the line passing through the points
$P = (-2, 0, 3)$ and Q$(3, 5, -2)$.

23. Find the unit tangent vector to the curve $\mathbf{r}(t) = \mathrm{t}\,\boldsymbol{i} + (\mathrm{t}^3)\boldsymbol{j}$ at the point $(1, 1, 0)$.

24. Find length of the Catenary $\mathbf{r}(t) = \mathrm{t}\,\boldsymbol{i} + (\cosh \mathrm{t})\boldsymbol{j}$ from $t = 0$ to $t = 1$.

$(9 \times 2 = 18\ Marks)$

PART – C

Answer *any six* questions. Each question carries *five* marks

25. Solve the equation $x^5 - x^4 - 4x^2 + 7x - 3 = 0$, given that it has multiple roots.

26. Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$. Given that the roots are in GP.

27. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\frac{1}{\alpha-1}, \frac{1}{\beta-1}, \frac{1}{\gamma-1}$ and hence find $\frac{1}{\alpha-1} + \frac{1}{\beta-1} + \frac{1}{\gamma-1}$.

28. Show that interchange of a pair of rows of a matrix does not change its rank.

29. If A is a square matrix of order n, prove that the sum of all the Eigen values of A is the trace of A.

30. Find the eigen values and the eigen vector corresponding to any one of the eigen values of $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.

31. Find the distance from the point $(2, -3, 4)$ to the plane $x + 2y + 2z = 13$.

32. Convert the spherical coordinates $\left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ in to rectangular coordinates and cylindrical coordinates .

33. Find the length of the curve $x = \sin t - t \cos t, y = \cos t + t \sin t, z = t^2$ from $(0, 1, 0)$ to $(-2\pi, 1, 4\pi^2)$.

$(6 \times 5 = 30\ Marks)$

PART – D

Answer *any two* questions. Each question carries *ten* marks

34. (i) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

(ii) Solve by Cardan's method: $x^3 - 18x - 35 = 0$.

35. (i) Test for consistency and solve $x + y + 2z = 4, 2x - y + 3z = 9, 3x - y - z = 2$.

(ii) Show that if λ is a non-zero characteristic root of a non-singular matrix A, then λ^{-1} is a characteristic root of A^{-1}.

36. (i) Find the length of the curve $r(t) = (\cos^3 t)j + (\sin^3 t)k$ from $t = 0$ to $t = \frac{\pi}{2}$.

(ii) Show that if f is a twice differentiable function of x, then the curvature of the graph of $y = f(x)$ is $\kappa(x) = \frac{|f''(x)|}{\{1+[f'(x)]^2\}^{3/2}}$.

$(2 \times 10 = 20\ Marks)$

||

B4M19197(A) (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March /April 2019

BMAT4C04(CH) – Mathematics

(2017 Admission onwards)

ime: 3 hours Max. Marks: 80

PART-A

Answer all the twelve questions Each question carries 1 mark

1. Find the Laplace transform of unit step function.
2. Find $L^{-1}\left(\frac{5s}{s^2+25}\right)$.
3. Examine whether $x|x|$ *is* odd, even or neither odd nor even.
4. Find the smallest positive period p of $\sin 2x$.
5. Show that $F = yzi + zxj + xyk$ is solenoidal.
6. Find a normal vector to the line $x - 2y + 2 = 0$.
7. Find the parametric representation of a straight line through a point (4,2,0) in the direction of the vector $i + j$.
8. Find the domain and the boundary of the domain of the function $f(x, y) = \sqrt{y - x}$.
9. The plane $x = 1$ intersect the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1,2,5).
10. Let $f(x, y) = 100 - x^2 - y^2$ find the level curve of $f(x, y) = 75$.
11. Compute $8 +_{10} 6$ using indicated modular addition.
12. Let $*$ be a binary operation on Z letting $a * b = a - b$ check whether $*$ is associative or not .

(12 x 1 = 12 marks)

PART-B

Answer any seven questions, Each question carries 2 marks

13. Find $L^{-1}\left(\frac{e^{-3s}}{(s-1)^3}\right)$.
14. Find $L\,(e^{2t} \sin 2t \sin 3t)$.
15. .Find a_0 in the Fourier series expansion $f(x) = \begin{cases} -2x & -\pi < x < 0 \\ 2x & 0 < x < \pi \end{cases}$ $p = 2\pi$.
16. Find the parametric representation of a straight line through a point (4,2,0) in the direction of the vector $i + j$.
17. Show that $\nabla^2\left(\frac{x}{r^2}\right) = \frac{-2x}{r^4}$.
18. If F is a differentiable vector function and $\emptyset$ is a differentiable scalar function then $div\,(\emptyset F) = (grad\ \emptyset) \cdot F + \emptyset\ div\ F$.
19. Using chain rule express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ in terms of u and v if $w = xy + yz + zx,\ x = u + v,\quad y = u - v,\ z = uv$.
20. Find all solutions in C of the equation $z^2 = i$.

21. Let + and · be the usual binary operation of addition and multiplication on the set Z an $H = \{n^2 | n \in Z^+\}$. Determine whether H is closed under addition and multiplication.

(7 x 2 = 14 marks)

PART-C

Answer any six questions, Each question carries 5 marks

22. Show that $L(sinat) = \frac{a}{s^2+a^2}$.
23. Find $L^{-1}\left(\frac{-s-10}{(S^2-s-2)}\right)$.
24. Find two half range expansions of the function $f(x) = x, 0 < x < 2$.
25. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, −1,
26. A particle moves so that its position vector is given by $r(t) = cos\omega t\, i + sin\omega t\, j$ wher a constant. Show that the velocity of the particle is perpendicular to r.
27. If $Z = x + f(u)$ where $u = xy$ show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x$.
28. Let G be a group with binary operation $*$ then prove that $a * b = a * c \Rightarrow b = c$ *and* $b * a = c * a \Rightarrow b = c$.
29. Complete the table so that $*$ is a commutative binary operation on the set $S = \{a, b, c, d\}$

*	a	b	c	d
a	b			
b	d	a		
c	a	c	d	
d	a	b	b	c

(6 x 5 = 30 marks)

PART-D

Answer any three questions, Each question carries 8 marks

30. Evaluate the Laplace Transforms of t^n using definition of Laplace transforms.
31. Find the Fourier series expansion of the function $f(x)$, which is periodic with the period which in $-\pi < x < \pi$ given by $(x) = \begin{cases} -x+1, -\pi < x \le 0 \\ x+1, 0 < x \le \pi \end{cases}$.
Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} = \frac{\pi^2}{8}$.
32. Prove that $curl\,(curl\,F) = grad\,div\,F - \nabla^2 F$.
33. (a) If $f(u, v, w)$ is differentiable and $u = x - y,\ v = y - z,\ w = z - x$ then show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$.
(b) Show that $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the Laplace equation.
34. (a) Define an abelian group.
(b) Show that $\langle nZ, + \rangle$ is a group and $\langle nZ, + \rangle \cong \langle Z, + \rangle$.

(3 x8 = 24 marks)

12

1B4M19197(B) (Pages : 3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March /April 2019

BMAT4C04(CS) – Mathematics

(2017 Admission onwards)

Time: 3 hours Max. Marks: 80

Part-A

Answer all Questions.
Each question carries 1 Mark

Laplace transform of 1 is ......

Give a periodic function with period 1.

Give an example for even function and odd function.

If $\vec{r} = \cos(x+y)i + x^2 j + 2xyk$, find $\frac{\partial \vec{r}}{\partial x}$ and $\frac{\partial \vec{r}}{\partial y}$.

Find $grad\ f$ if $f(x,y,z) = xyz$ at (1,1,1).

Find the tangent vector to the circle $x = \cos t$, $y = \sin t$, $z = 0$ at $t = \frac{\pi}{2}$.

The domain of the function $w = \sqrt{y - x^2}$ is ....

Find $\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y + 1}{x+1}$.

Define the level surface of a function $f : R^3 \to R$.

Prove that $p \vee \neg p$ is a tautology.

Translate the statement "$\forall x \in R\ (x^2 \geq 0)$" into English.

What is the truth value of $\neg p \to q$, if p: "$n^2 < 0$ *for integer n.*" and q: "*5 is a prime*".

($12 \times 1 = 12$ *Marks*)

Part-B

Answer any seven Questions.
Each question carries 2 Marks

Find the inverse Laplace transform of $\frac{s-3}{s^2-4}$.

Find the Fourier coefficient a_0 for the function $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$

Find the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

16. Find the length of the circular helix $\overrightarrow{r(t)} = cost\, i + \sin t\, j + tk$ from $(1,0,0)$ to $(1,0,2\pi)$.

17. Find the directional derivative of $f(x,y,z) = x^2 + y^2 x$ in the direction of $i + j + k$ at the point (1

18. Find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin xy = 0$.

19. Solve the partial differential equation $u_{yy} = u$.

20. State the converse and inverse of the proposition "*If Jhon is a poet, then he is poor*".

21. Express the statement "*Every student in this class has visited either Ooty or kodaikanal*" using pre and quantifiers.

$(7 \times 2 = 14$ M

Part-C
Answer any six Questions.
Each question carries 5 Marks

22. Find the $\mathcal{L}\{f(t)\}$ where $f(t) = \sinh t \cos t$.

23. Find the half range Fourier sine series of the function $f(x) = \pi - x \quad 0 < x < \pi$.

24. If $\vec{V}$ is a differentiable vector, prove that $div(curl\,\vec{V}) = 0$.

25. Find the value of a if $\vec{V} = (x + 3y)i + (y - 2z)j + (x + az)k$ is incompressible.

26. Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is not continuous at the origin.

27. Find all the second order partial derivatives of $f(x,y) = x\cos y + e^{xy}$.

28. Using logical equivalence show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

29. Construct the truth table for the proposition $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$.

$(6 \times 5 = 30$ M

Part-D
Answer any 3 Questions.
Each question carries 8 Marks

30. (a) Find the inverse Laplace transform of $\frac{s-2}{s^2 - 2s + 5}$.

(b) Find the Laplace transform of the function $f(x) = \begin{cases} t & 0 \le t \le 1 \\ 1 & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$

31. Find the Fourier series expansion of $f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$, $f(x + 2\pi) = f(x)$.

path of a moving particle at time t is given by the vector

.1 $$\vec{r} = (\sin t - t\cos t)\ i + (\cos t + t\sin t)\ j + t^2 k.$$

Find the speed and acceleration of the particle.

) Find the tangential and normal acceleration.

Let $w = f(x,y)$, $x = u(r,s)$ and $y = v(r,s)$. Write down the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$.

ca) $w = (x+y)^2$, $x = r+s$ and $y = rs$. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$.

Check whether $(p \to r) \wedge (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent or not.

) State and prove absorption law of logic.

$(3 \times 8 = 24\ Marks)$

13

1B4M19197(D) (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March /April 2019

BMAT4C04(ST) – Mathematics

(2017 Admission onwards)

Time: 3 hours Max. Marks: 80

Section A

Answer all the twelve questions.
Each question carries 1 mark.

1. By definition Laplace transform of $f(t)$, $\mathcal{L}[f(t)] =$......
2. $\mathcal{L}[t^a], a > 0$ is ......
3. Define curl of a vector valued function.
4. Find the vector (in components) whose initial point is $(-1, 5, 8)$ and terminal point is $(1, 4, 6)$.
5. Write Cauchy-Schwarz inequality.
6. Find the parametric representation of the straight line $y = 2x + 3, z = 7x$.
7. Find a vector normal to the surface $ax + by + cz = d$, where a, b, c and d are constants.
8. Define level curves.
9. Write the domain of the function $\frac{1}{\sqrt{16-x^2-y^2}}$.
10. Express $\frac{4+i}{2-3i}$ in the form $a + ib$.
11. Write $Im\, z$ in terms of $\bar{z}$.
12. Define interior point of a set S in the complex plane.

(12 × 1 = 12 *Marks*)

Section B

Answer any seven out of Nine questions.
Each question carries 2 marks.

13. Show that $\mathcal{L}[cosh at] = \frac{s}{s^2-a^2}$.

14. Find $\mathcal{L}[\sin^2 4t]$.

15. Define periodic function. Give one example.

16. Find the resultant (in components) and its magnitude

$\boldsymbol{p} = [1,2,0]$ $\boldsymbol{q} = [0,4,-1]$ $\boldsymbol{u} = [4,0,-3]$ $\boldsymbol{v} = [6,2,4]$.

17. Find the moment vector $\boldsymbol{m}$ and the moment m of a force $\boldsymbol{p} = [1,2,3]$ about a point $Q(0,1,1)$ when $\boldsymbol{p}$ acts on a line through $A(1,0,3)$.

18. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$.

19. Find f_y if $f(x,y) = \frac{2y}{y+cosx}$.

20. Solve the partial differential equation $u_y = (coshx)yu$.

21. Find $Arg\left(\frac{i}{-2-2i}\right)$.

($7 \times 2 = 14$ *Mark*

Section C

Answer any six out of Eight questions.
Each question carries 5 marks.

22. Prove that $\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}, s > 0$ when $n=1, 2, 3...$

23. Find $\mathcal{L}^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$.

24. Expand $f(x) = sinx$ $0 < x < \pi$ as a half range cosine series.

25. Find the total length of the hypocycloid $\boldsymbol{r}(t) = [a\cos^3 t, a\sin^3 t]$.

26. *Let $\boldsymbol{v} = [x,-y,z]$ be the velocity vector of a steady fluid flow. Is the flow irrotational? Incompressible? Find the streamlines.*

27. Show that the function $f(x,y) = \frac{2x^3y}{x^6+y^2}$ has no limit as $(x,y) \to (0,0)$.

28. Prove that

    a) z is real if and only if $\bar{z} = z$

    b) *z is either real or pure imaginary if and only if* $\bar{z}^2 = z^2$.

29. Find all values of $(-1)^{\frac{1}{3}}$.

($6 \times 5 = 30$ *Marks*

Section D

Answer any three out of five questions.
Each question carries 8 marks.

30. Find the Fourier series representing x in the interval $[-\pi, \pi]$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots = \frac{\pi}{4}$.

31. Find the Fourier series of the function $f(x) = \begin{cases} 1 + x, if -1 < x < 0 \\ 1 - x, \ if\ 0 \leq x < 1 \end{cases}$ of period $p = 2L = 2$.

32. a) Find the directional derivative of $f = x^2 + y^2 + z^2$ at $P: (2, -2, 1)$ in the direction of $[-1, -1, 0]$.

    b) Find the divergence of the function $(x^2 + y^2 + z^2)^{-\frac{3}{2}}[x, y, z]$.

33. a) Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 = 0$ at (2,3, 6).

    b) Evaluate $\frac{\partial z}{\partial u}$ in terms of u and v where

    $z = 4e^x lny, \ x = \ln(u \ cosv), \quad y = u \ sinv.$

34. Prove that for any two complex numbers z_1, z_2

    $|z_1 + z_2| \leq |z_1| + |z_2|$. Using this also prove $|z_1 + z_2| \geq ||z_1| - |z_2||$.

$(3 \times 8 = 24 \ Marks)$

14

19197(C) (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March /April 2019

BMAT4C04(PH) – Mathematics

(2017 Admission onwards)

3 hours Max. Marks: 80

Part A

Answer all questions.

Each question carries 1 Mark.

1. When n is a positive integer, what is the reduction formula for the laplace transform $L(t^n)$.
2. State the first shifting theorem; shifting on x-axis.
3. What is the volume of a parallelepiped whose co terminal edges are $\bar{a}, \bar{b}$ *and* $\bar{c}$.
4. Find the resultant of the given vectors $\bar{a} = [4, -2, -3], \bar{b} = [8, 8, 1]$ *and* $\bar{c} = [-12, -6, -2]$.
5. Find a unit vector perpendicular to the plane $4x + 2y + 4z = -7$.
6. Let $f(x)$ be a function defined on $[0, \pi]$, then what is the half range Fourier sine series for $f(x)$.
7. Find the fundamental period of the function e^x.
8. What is the order of the semi linear partial differential equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$.
9. Let $f(x) = x \sin x$ be a function defined on $[-\pi, \pi]$, then show that $f(x)$ is even.
10. Describe the domain of the function $f(x, y) = \sqrt{y - x^2}$.
11. Find the partial derivative of $f(x, y) = x^2 + 3xy + y - 1$ with respect to x at a point (4,-5).
12. Define a level curve.

(12 x 1=12 marks)

Part B

Answer any SEVEN questions.

Each question carries 2 Marks.

13. Find the Laplace transform $L\left(e^{-2t} \cos t\right)$.
14. Find the inverse Laplace transform $L^{-1}\left[\frac{1}{(s-1)^3}\right]$.
15. Find the Laplace transform $L(\sin^2 t)$ using Laplace transform of derivatives.

16. Find the Fourier sine series for the function $f(x) = x$ in $[0, \pi]$

17. Show that [3,5,9]; [73,-56,76] and [-4,7,-1] are linearly independent.

18. Show that $F = yz\bar{i} + xz\bar{j} + xy\bar{k}$ is solenoidal.

19. Define a conservative field.

20. Show that the function $u = x^2 - y^2$ is a solution of the two dimensional Laplaces equation

21. State the one dimensional wave equation?

(7 x 2 = 14 Mar

Part C
Answer any SIX questions.
Each question carries 5 Marks.

22. Solve the initial value problem $y'' + 4y' + 3y = 0$, given $y(0) = 3$ *and* $y'(0) = 1$.

23. Find the inverse Laplace transform of $L^{-1}\left[\frac{e^{-3s}}{(s-1)^4}\right]$.

24. Find the unit tangent vector to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at the point t = 2

25. Prove that the points $4\bar{i} + 8\bar{j} + 12\bar{k};\ 2\bar{i} + 4\bar{j} + 6\bar{k};\ 3\bar{i} + 5\bar{j} + 4\bar{k}$ *and* $5\bar{i} + 8\bar{j} + 5\bar{k}$ are coplanar.

26. Find the directional derivative of $f = x^2 + y^2$ at (1,1) in the direction of $2\bar{i} - 4\bar{j}$.

27. If $f(x) = -1 + x$ when $\pi < x < 0$ *and* $f(x) = 1 + x$ *when* $0 < x < \pi$ and $f(x + 2\pi) = f(x)$ then find the Fourier series.

28. Find the partial derivative of f with respect to x and y where $f(x, y) = x.\cos(xy)$.

29. Solve the partial differential equation $u_x - 4 = 0$.

(6 x 5=30 Marl

Part D
Answer any THREE questions.
Each question carries 8 Marks.

30. Find the Fourier series for the function $f(x) = e^x$ *in* $[-\pi, \pi]$ *and* $f(x + 2\pi) = f(x)$.

31. If $\bar{a}$ is a constant vector, then find the divergent and curl of $\bar{a} \times \bar{r}$. *where* $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$.

32. Solve $y'' + 2y' + 2y = \delta(t - \pi)$, given $y(0) = 1$ *and* $y'(0) = 0$.

33. Write the chain rule and find the partial derivative of $\omega = x + 2y + z^2$ with respect to r and s in terms of r and s, where $x = \frac{r}{s}$, $y = r^2 + Ins$ *and* $z = 2r$.

34. Applying the limit definition find $\lim_{(x,y)\to(0,0)}\left[\frac{4xy^2}{x^2 + y^2}\right]$ if it exist. **(3x8 = 24 Marks**