1B3N18225 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2018

BSTA3B03 – Theory of Estimation

(2017 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

Part A (Answer **all** questions. Each carries 1 mark)

Fill in the blanks (Questions 1-7)

1. The standard deviation of the sampling distribution of a statistic is known as ....
2. If $X_1, X_2, \ldots\ldots, X_n$ is a random sample of size 'n' from a population $N(\mu, \sigma^2)$ then the distribution of $\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2}$ is ....
3. If $X \to N(0,1)$ and $Y \to \chi^2_{(9)}$ and are independent then $\frac{3X}{\sqrt{Y}}$ is distributed as ....
4. An unbiased estimator whose variance tends to zero as the sample size increases is ....
5. If a sufficient estimator exists then it is a function of the .... estimator.
6. The estimator of σ^2 based on a random sample $X_1, X_2, \ldots, X_n$ from a population $N(0, \sigma^2)$ by the method of moments is ....
7. A 95% confidence interval for the population proportion based on a large sample is given by ......

Multiple Choice Questions (Questions 8 – 12)

8. The mode of t-distribution with n degrees of freedom is

   (a) n (b) $\sqrt{n}$ (c) 0 (d) None of these

9. If T_1 and T_2 are unbiased for θ with $V(T_1) = \frac{\sigma^2}{3}$ and $V(T_2) = \frac{\sigma^2}{5}$ then the efficiency of T_1 with respect to T_2 is

   $(a)\frac{5}{3}$ $(b)\frac{3}{5}$ $(c)\frac{3}{25}$ $(d)\frac{25}{3}$

10. The denominator of Cramer-Rao inequality is known as

    (a) Lower bound of the variance (c) Upper bound of the variance
    (b) Information limit (d) All of these

11. The set of equations obtained in the process of least square estimation are called:

    (a) Likelihood equations (c) Both (a) and (b)
    (b) Normal equations (d) neither (a) nor (b)

12. For a fixed confidence coefficient $1 - \alpha$, the most preferred confidence interval for the parameter θ is one:

    (a) With largest width (c) with shortest width
    (b) With an average width (d) none of these

(12 ×1 = 12 marks)

Part B

(Answer any seven questions. Each carries 2 marks)

13. Distinguish between an estimator and estimate.
14. Define student's t-distribution. What is its mean and variance.
15. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size 'n' from an exponential distribution with mean μ. Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is unbiased for μ.
16. Define a consistent estimator. State sufficient conditions for the existence of the consistent estimator.
17. State Fisher-Neyman factorization theorem for sufficiency.
18. State Cramer-Rao inequality.
19. Briefly describe the method of least squares.
20. Distinguish between point estimation and interval estimation.
21. Give the confidence limits for the ratio of variances of two normal populations.

(7 × 2 = 14 marks)

Part C

(Answer any six questions. Each carries 5 marks)

22. If $X_1, X_2, \ldots\ldots, X_n$ is a random sample of size 'n' from $N(\mu, \sigma^2)$ then find the distribution of $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
23. If $X_1, X_2, \ldots\ldots, X_n$ is a random sample from a normal population with means θ and variance 1 then show that $T = \frac{1}{n}\sum_{i=1}^{n} {X_i}^2$ is an unbiased estimator of $\theta^2 + 1$
24. Verify whether $T_n = \frac{n\overline{X}}{n+1}$ is a consistent estimator of θ, where $\overline{X}$ is the mean of a random sample of size 'n' taken from a Poisson distribution with parameter θ.
25. Let $X_1, X_2, \ldots X_n$ be random sample from of size 'n' from a Poisson distribution with mean θ. Find a sufficient statistic for θ.
26. Discuss the method of maximum likelihood estimation. State four properties of maximum likelihood estimators.
27. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ find the m.l.e of σ^2 when μ is known.
28. Explain the concept of Bayesian estimation.
29. Obtain confidence interval for the mean of a normal population $N(\mu, \sigma^2)$ with confidence coefficient $1 - \alpha$, when σ^2 is unknown and sample size 'n' is small.

(6 × 5=30 marks)

Part D

(Answer any three questions. Each carries 8 marks)

30. State the relations between normal, chi-square, student's t and snedecor's F-distributions.
31. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size 'n' from $N(\mu, \sigma^2)$ show that (i) the sample mean $\overline{X}$ is a sufficient estimator for μ when σ^2 is known (ii) the sample variance S^2 is not sufficient for σ^2 when μ is known.
32. Show by an example that the maximum likelihood estimator need not be unbiased.
33. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size 'n' from poisson distribution with mean θ. Obtain the Cramer-Rao lower bound for the variance of the unbiased estimator for θ.
34. Two random samples of sizes 10 and 12 from two normal populations having equal variances have means 12 and 10 and variances 2 and 5 respectively. Obtain 95% and 98% confidence limits for the difference between two population means.

(3 × 8 =24 marks)

1B3N18226 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2018

BSTA3C03 - Statistical Inference

(2017 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART A (Answer all the questions. Each carries 1 mark.)

Fill in the blanks (Questions 1-6)

1. The variance of a chi-square distribution with 'n' degrees of freedom is ....
2. The standard deviation of any statistic is called its......
3. A Student's 't' distribution was discovered by ....
4. Value of an estimator is called an..................
5. Critical region is a region of ....
6. Paired t test is applicable only when the observations are ....

Choose the correct answer (Questions 7-12)

7. The range of F variate is ....

   (a) 0 to 1 (b) $-\infty$ to $+\infty$ (c) 0 to ∞ (d) None of these

8. Degrees of freedom for chi –square in the case of 3X2 contingency table is...........

   (a) 6 (b) 2 (c) 4 (d) None of these

9. If $t1$ and $t2$ are two unbiased estimators such that $V(t1) < Var(t2)$, then

   (a) $t1$ is more efficient than $t2$ (b) $t2$ is more efficient than $t1$

   (c) Both are equally efficient (d) None of these

10. Power of a test is related to

    (a) type I error (b) type II error (c) type I and type II error (d) None of these

11. For a two tailed test with $\alpha = 0.05$ the best critical region of a Z test is.................

    (a) $Z < -1.96$ (b) $Z > 1.96$ (c) $|Z| \geq 1.96$ (d) None of these

12. If $F \rightarrow F(n_1, n_2)$ then $1 / F \rightarrow$............

    (a) $F(n_2, n_1)$ (b) $F(n_1, n_2)$ (c) $t(n_1 + n_2 - 1)$ (d) none of these

(12 x 1 = 12 Marks)

PART B

(Answer any SEVEN questions. Each carries two marks.)

13. State and prove the additive property of chi-square distribution.
14. Define statistical inference.
15. Distinguish between type I and type II errors in testing of hypothesis.
16. What do you mean by sampling distribution?
17. What is the difference between a parameter and a statistic?

18. Write four properties of maximum likelihood estimator?

19. Write a short note on the method of moments for estimating the parameter of a population.

20. Obtain the confidence interval for the mean of a population when variance is known?

21. Distinguish between null and alternative hypothesis.

(7 x 2 = 14 Marks)

PART C

(Answer any SIX questions. Each carries 5marks.)

22 Obtain the distribution of the mean of a sample from a normal population.

23. Derive the distribution of sample variance from a normal population.

24. Obtain 90% confidence interval forthe mean of a Normal distribution $N(\mu, \sigma)$.

25. Obtain sufficient estimate of μ of a normal population $N(\mu, \sigma)$.

26. Estimate parameter p of a Binomial distribution $B(n, p)$using method of moments.

27. Explain the chi square test for independence of attributes.

28. Explain the steps involved in testing of hypothesis with example?

29. Describe the test procedure for testing equality of population proportions based on large samples

(6 x 5 = 30 Marks)

PART D

(Answer any THREE questions. Each carries 8 marks.)

30. Let $X_1, X_2, ...X_n$ is a random sample from a normal population with mean μ and variance σ^2. obtain maximum likelihood estimates of μ and σ^2 of a normal population.

31. Explain point estimation and interval estimation with examples.

32. The sample 1.3,0.6,1.7,2.2,0.3,1.1 was drawn from a population with pdf,

$f(x) = 1/\theta.\ 0<x<\theta$

Obtain maximum likelihood estimates for the mean and variance.

33. Four coins are tossed 80 times. The distribution of the heads are given below

No. Of heads:	0	1	2	3	4	Total
Frequency :	4	20	32	18	6	80

Test 1% level of level of significance if the coin is unbiased.

34. What are the desirable properties of a good estimator? Discuss with examples.

(3 x 8 = 24 Marks)