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M18079 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, March 2018

BMAT2B02 - Calculus

(2017 Admission onwards)

x. Time: 3 hours Max. Marks: 80

PART-A

(Answer *all* questions. Each question carries *one* mark)

Define critical point of a function.

Give an example of a function which is increasing on $(0, \infty)$.

When is a curve said to be concave up at or near a point on the curve?

A line $x = a$ is a vertical asymptote of the graph of a function $f(x)$ if .....

The production level (if any) at which average cost is smallest is a level at which the average cost equals ......

Find dy if $y = \cos 5x$.

Define the norm of a partition of a closed bounded interval.

If $\int_{-1}^{1} f(x)dx = 3$ and $\int_{-1}^{1} g(x)dx = 4$, then $\int_{-1}^{1}[5f(x) - 3g(x)]dx = \cdots$

If f is integrable on $[a, b]$, then its average value on $[a, b]$ is given by.....

Find $\frac{dy}{dx}$ if $y = \int_0^x \frac{2t}{1+t^2}\, dt$.

$\int_{-1}^{1} t^3(1 + t^4)^3\, dt = \cdots$

If f is smooth on $[a, b]$, then length of the curve $y = f(x)$ from $x = a$ to $x = b$ is .....

(12 x 1= 12 Marks)

PART-B

(Answer any *seven* questions. Each question carries *two* marks)

Find the absolute extrema values of the function $f(x) = 8x - x^3$ on $[-2, 1]$.

State the first derivative test for increasing and decreasing functions.

Using Sandwich theorem, find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

Find the linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$.

Using an area, evaluate the definite integral $\int_a^b x\, dx, 0 < a < b$.

Find the average value of $f(x) = x^2 - 6x + 8$ on $[0, 3]$.

Find $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\, dt$.

20. Find the area between the curves $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$.

21. Show that the centre of mass of a straight, thin strip or rod of constant density lies half way between its two ends.

(7 x 2 = 14 Marks)

PART-C

(Answer any *six* questions. Each question carries *five* marks)

22. Verify Rolle's Theorem for the function $f(x) = (x-a)^m(x-b)^n$, where m and n being positive integers and $x \in [a,b]$.

23. Find two positive numbers whose sum is 20 and whose product is as large as possible.

24. Show that the points of inflection of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$.

25. Find the area of the region between the curve $y = 4 - x^2, 0 \leq x \leq 3$, and the x-axis.

26. Find the volume of the solid that lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$, where the cross sections perpendicular to the x-axis on the interval $0 \leq x \leq 4$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$.

27. Find the volume of the solid formed by the revolution about the major axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

28. Find the length of the curve $y = \log \sec x$ between the points given by $x = 0$ and $x = \frac{\pi}{4}$.

29. The line segment $x = 1 - y, 0 \leq y \leq 1$, is revolved about the y-axis to generate a cone. Find its lateral surface area.

(6 x 5 = 30 Marks)

PART-D

(Answer any *three* questions. Each question carries *eight* marks)

30. Using the algorithm for graphing, graph the function $y = x^5 - 5x^4$. Include the coordinates of any local extreme points and inflection points.

31. A square piece of tin of side 12cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

32. Using limits of Riemann sums, establish the equation $\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$.

33. Show that the length of the arc of the curve $27y^2 = 4x^3$ measured between the origin and the point $(6, 4\sqrt{2})$ is equal to $2(3\sqrt{3} - 1)$.

34. Show that the centre of mass of a thin plate of constant density covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis. **(3 x 8 = 24 Marks)**

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2M18080 (Pages : 3) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, March 2018

BMAT2C02 - Mathematics

(2017Admission onwards)

ax. Time: 3 hours Max. Marks: 80

Answer all questions. Each question has ONE mark

1. $\sum_{k=1}^{n}\left(\frac{1}{n}\right) =$ ....
2. Find the norm of the partition $P = \{0, 1.2, 1.5, \ 2.3, 2.6, 3\}$
3. Express the limit $\lim_{\|P\|\to 0}\sum_{k=1}^{n}\frac{1}{1-c_k}\Delta x_k$ as a definite integral where P is a partition of [2,3].
4. Give a parametric representation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$
5. Define a hyperbolic sine function.
6. Express $\sinh^{-1} x$ using the logarithmic function for all real values of x.
7. Find a formula for the n^{th} term of the sequence 2,6,10,14,18, ...
8. Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \cdots = 9$.
9. Find the Taylor polynomial of order 2 for $f(x) = \frac{1}{x}$ at 2.
10. Replace the polar equation $r\cos\theta = 2$ by equivalent Cartesian equation.
11. What is the polar equation of the circle with centre $(-1, \frac{\pi}{2})$ and radius 2?
12. Write the polar equation of an ellipse whose eccentricity is e and semi-major axis is a.

(12 × 1 = 12 Marks)

II. Answer any SEVEN questions. Each question has TWO marks

13. Show that if f is continuous on $[a, b]$ with $a \neq b$ and $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.
14. Find a function $y = f(x)$ on the domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with derivative $\frac{dy}{dx} = \tan x$ that satisfies the condition $f(3) = 5$.
15. Define a smooth curve.

16. State Pappu's theorem for volumes.

17. Evaluate $\frac{d}{dt}\left(\tanh\sqrt{1+t^2}\right)$

18. State the Sandwich theorem for sequences.

19. Find the sum of the series $\frac{1}{2\,.\,3}+\frac{1}{3\,.\,4}+\frac{1}{4\,.\,5}+\cdots+\frac{1}{(n+1)(n+2)}+\cdots$

20. Show that the point $(2,\frac{\pi}{2})$ lies on the curve $r = 2\cos 2\theta$.

21. Find the area of the region in the plane enclosed by the cardiod $r = 1-\cos\theta$.

($7 \times 2 = 14$ Marks)

II. Answer any SIX questions. Each question has FIVE marks

22. What values of a and b minimize the value of $\int_a^b (x^4-2x^2)\,dx$?

23. Find the volume of the solid that lies between planes perpendicular to the x-axis at $x=-1$ and $x=1$. The cross sections perpendicular to the x-axis between these planes are squares whose diagonals run from the semi circle $y=-\sqrt{1-x^2}$ to the semi circle $y=\sqrt{1-x^2}$.

24. Using integration find the lateral surface area of the cone generated by revolving the line segment $y=\frac{x}{2}$, $1\le x\le 4$ about the y- axis. Check the answer with the geometric formula, Lateral Surface Area= $\frac{1}{2}\times$Base Circumference$\times$Slant Height.

25. Evaluate $\int_0^{\ln 2} 4e^x \sinh x\,dx$

26. For any number x, prove that $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$

27. Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{4^n n!n!}{(2n)!}$

28. Find the length of the cardiod $r=a(1-\cos\theta)$.

29. Find the area of the surface generated by revolving the right-hand loop of the lemniscate $r^2=\cos 2\theta$ about the y-axis.

($6\times 5 = 30$ Marks)

nswer any THREE questions. Each question has EIGHT marks

(i). Suppose that a company's marginal revenue from the manufacture and sale of egg beaters is $\frac{dr}{dx} = 2 - \frac{2}{(x+1)^2}$, where r is measured in thousands of dollars and x is in thousands of units. How much money should the company expect from a production run of $x = 3$ egg beaters?

(ii). Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(i). Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

(ii). Find the area of the surface generated by revolving the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$, $1 \le y \le 2$, about the x-axis.

Discuss the convergence of the p-series $\sum \frac{1}{n^p}$.

For what values of x does the series $1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 - \frac{1}{8}(x-3)^3 + \ldots\ldots\ldots$ converge? What is its sum? What series do you get if you differentiate the given series term by term? For what values of x does the new series converge? What is its sum?

Find the area shared by the circle $r = 2$ and the cardiod $r = 2(1 - \cos\theta)$.

($3 \times 8 = 24$ Marks)