Spatial Soliton Propagation in Graded Index Kerr media

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In this paper, we study the propagation and amplification of spatial soliton in a graded index Kerr medium. The exact soliton solution of the nonlinear Schrodinger equation in graded index Kerr medium within in the integrable limit has been constructed using Backlund transformation technique. As the soliton propagates through the graded index Kerr medium, it gets amplified. This can be used for the amplification and focusing of spatial solitons to overcome inevitable energy losses.

I. INTRODUCTION

Optical solitons are localised pulse or beam of light that propagate without decay in a nonlinear medium with dispersion, diffraction or both. In nonlinear optics they are classified temporal or spatial solitons depending on whether their shape remains intact in time or in one space dimension [1, 2]. Mathematically, wave propagation in a Kerr medium is governed by the nonlinear Schrodinger equation(NLSE). The study of pulse propagation through inhomogeneous media demands special attention as it characterizes real physical systems.

In this paper, we study the propagation and amplification of spatial soliton in a graded index Kerr medium. The exact soliton solution of the nonlinear Schrodinger equation in graded index Kerr medium within in the integrable limit has been constructed using Backlund transformation technique. As the soliton propagates through the graded index Kerr medium, it gets amplified. This can be used for the amplification and focusing of spatial solitons to overcome inevitable energy losses.

II. MATHEMATICAL MODEL

Considering the propagation of a continuous wave optical beam inside a bulk, graded-index nonlinear Kerr medium, the refractive index has the form[3]:

$$n = n_0 + n_1 x^2 + n_2 I \tag{1}$$

n_0 and n_2 are the linear part of the refractive index and Kerr nonlinearity respectively. $n1 > 0$so that, in the low intensity limit, the graded-index waveguide acts as a linear defocusing lens. The normalised nonlinear wave equation governing beam propagation in such a waveguide can be written as

$$iU_z + U_{xx} + 2|U|^2 U + \alpha_1 x^2 U - i\alpha_2(z) U = 0 \tag{2}$$

where $U(z, x)$ represents the complex envelope amplitude, subscripts x and z respectively denote the partial derivative with respect to normalized distance and transverse dimension. α_2 is the gain coefficient.

The integrability conditions for equation (2) can be identified through the linear eigen value problem $\alpha_1 = -\alpha_2^2$, and putting $\alpha_2 = \beta$, equation (2) becomes

$$iU_z + U_{xx} + 2|U|^2 U + \beta^2 x^2 U + i\beta U = 0. \tag{3}$$

The complete integrability of equation (3) is confirmed by the existence of Lax pair for arbitrary values of β. A variable transformation given by

$$U(z, x) = \psi(z, x) \exp(\frac{i\beta x^2}{2}), \tag{4}$$

is introduced to construct Lax pairs, then equation (3) becomes

$$i\psi_z + \psi_{xx} + 2|\psi|^2\psi + 2i\beta\psi + 2i\beta x\psi_x = 0 \tag{5}$$

The ZS/AKNS inverse scattering problem is defined by

$$\chi_x = A\chi, \chi_z = B\chi, \tag{6}$$

where A and B are Lax pairs given by

$$A = \begin{bmatrix} -i\lambda & \psi \\ -\psi^* & i\lambda \end{bmatrix}, \tag{7}$$

$$B = \begin{bmatrix} -2i\lambda^2 + 2\lambda i\beta x + i|\psi|^2 & 2\lambda\psi + i\psi_x - 2\beta x\psi \\ -2\lambda\psi^* + i\psi_x + 2\beta x\psi^* & 2i\lambda^2 - 2\lambda i\beta x - i|\psi|^2 \end{bmatrix} \tag{8}$$

The compatibility condition given by

$$A_z - B_x + [AB] = 0, \tag{9}$$

requires the non isospectral parameter λ which is defined by

$$\lambda = \lambda_0 \exp(-2\beta_z), \tag{10}$$

the subscript z denotes partial derivative with respect to distance. The exact soliton solution for the nonlinear Schrodinger equation 5 in graded index Kerr media within the integrable limit has been constructed using Backlund transformation technique .

III. SOLITON SOLUTION

A one soliton solution is generated by using Backlund transformation. Defining a pseudo potential,

$$\Gamma(n) = \frac{\chi_1(n)}{\chi_2(n)}. \tag{11}$$

Differentiating(11)

$$\Gamma(x) = -2i\lambda\Gamma + \psi + \psi^*\Gamma^2 \tag{12}$$

The n soliton solution $\psi(n)$ can be expressed in terms of the $(n-1)$ soliton solution.

$$\psi(n) = -\psi(n-1) - \frac{4\Gamma(n-1)\nu_n}{1 + |\Gamma(n-1)|^2}. \tag{13}$$

For the trivial case $n = 1$, $\psi(0) = 0$ and from equations (6), (7) and (8) we get the eigen functions $\psi_1(0)$ and $\psi_2(0)$ as

$$\psi_1(0) = \alpha(0) \exp\left[-i(2\lambda x + \int 2\lambda^2 dz)\right], \tag{14}$$

$$\psi_2(0) = \gamma(0)\left[i(2\lambda x + \int 2\lambda^2 dz)\right]. \tag{15}$$

Substituting (14) and (15) in (11) and putting $\lambda = \mu_1 + i\nu_1$, we obtain $\Gamma(0)$ as

$$\Gamma(0) = \exp\left[-i4\mu_1 x - i4\int(\mu_1^2 - \nu_1^2)dz\right] \tag{16}$$
$$\exp\left[+2\delta_1 + 4\nu_1 x + 8\int \mu_1\nu_1 dz + 2\Delta_1\right]$$

where we have used
$\frac{\alpha(0)}{\gamma(0)} = \exp[-2i(\delta_1 + i\Delta_1)]$. Substituting (16) in (13), we obtain one soliton solution of (5).

$$\psi(1) = -2\nu_1 \exp[-i4\mu_1 x + 4\int(\mu_1^2 - \nu_1^2)dz + 2\delta_1] \tag{17}$$
$$\text{sech}[4\nu_1 x + 8\int \mu_1\nu_1 dz + 2\Delta_1],$$

and $U(1)$ is given as

$$U(1) = \exp(\frac{i\beta x^2}{2})\text{sech}[4\nu_1 x + 8\int \mu_1\nu_1 dz + 2\Delta_1] \tag{18}$$
$$(-2\nu_1 \exp[-i4\mu_1 x + 4\int(\mu_1^2 - \nu_1^2)dz + 2\delta_1]),$$

The figures(1-2) show the plots of the solution for propagation distance $z = 3$ and $z = 5$. The plots reveal that when the beam propagates through the graded index Kerr medium, it gets amplified.

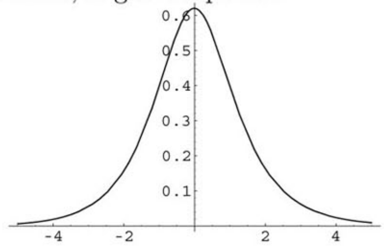

FIG. 1. Soliton beam propagation for z=3, $|U(z,x)|$ along y-axis and beamwidth along x-axis. $\alpha_2 = 0.8$

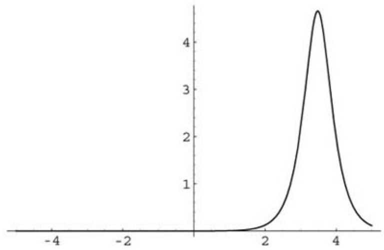

FIG. 2. Soliton beam propagation for z=5.$|U(z,x)|$ along y-axis and beamwidth along x-axis. $\alpha_2 = 0.8$

IV. CONCLUSION

The propagation and amplification of spatial solitons in graded index Kerr media has been studied in this work. The exact soliton solution of the nonlinear Schrodinger equation in graded index Kerr medium within in the integrable limit has been constructed using Backlund transformation technique. As the soliton propagates through the graded index Kerr medium, it gets amplified. This can be used for the amplification and focusing of spatial solitons to overcome inevitable energy losses.

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[1] A. Hasegawa, Y. Kodama, Solitons in Optical Communications, Clarendon Press, Oxford, 1995.
[2] G.P. Agrawal, Nonlinear Fiber Optics, 2nd ed., Academic Press, San Diego, 1995.
[3] S. A. Ponomarenko1,G. P. Agrawal Do Solitonlike Self-Similar Waves Exist in Nonlinear Optical Media? PRL 97, 013901 (2006)