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1M1N25280 (Pages : 2) Reg. No:...............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2025

MST1C01 - Analytical tools for Statistics - I

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

(Answer *any four* questions. Each carries 2 weightage)

1. Define Riemann Stieltjes integral of a function with respect to another one.
2. Define a function of bounded variation and state the property related with its monotonicity.
3. Distinguish between point wise and uniform convergence of a series of functions.
4. Find the function to which the sequence $\{f_n(x)\}$ converges point wise, where

   $f_n(x) = \frac{x}{1+nx}$ for $x \in [0,1]$.
5. Define limit and continuity of a multivariable function.
6. Define implicit function and give an example with illustration.
7. Obtain the Laplace transform of the function F(t)= sin at, where a is a constant.

(4*2=8 weightage)

PART B

(Answer *any four* questions. Each carries 3 weightage)

8. If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a,b], then show that the $f_1 + f_2 \in R(\alpha)$, and

   $\int_a^b (f_1 + f_2)d(\alpha) = \int_a^b f_1 \, d(\alpha) + \int_a^b f_2 \, d(\alpha).$
9. (a) If f is continuous on [a,b] and α is of bounded variation on [a,b], prove that $f \in R(\alpha)$.
   (b) State Eulers summation formula.
10. Show that the series $x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots$ is not uniformly convergent at x=0.

11. (a) Define directional derivative of a multivariable function.

(b) Find the directional derivative of $u = 2x^3y - 3y^2$ at $P(1,2,-1)$ in a direction towards $Q(3,-1,5)$.

12. Explain the Lagrangian method of finding the optima of a multivariable function.

13. If the Laplace transform $L\{F(t)\} = f(s)$, then find $L\{F(at)$, and $L\{e^{at}F(t)\}$.

14. Obtain $L\{t^2 \cos at\}$, and $L\{(1-2t)e^{-2t}\}$.

(4*3= 12weightage)

PART C

(Answer *any two* questions. Each carries 5 weightage)

15. (a) Find $\int_0^{10} f(x)\, d\,\alpha(x)$, where $f(x) = x$, and $\alpha(x) = x + [x]$.

(b) State and prove the first mean value theorem of Riemann Stieltjes integral.

16. Prove that $\{f_n(x)\}$ where $f_n(x) = nxe^{-nx^2}$ converges point wise on the interval [0,1], but the sequence of integrals $\int_0^1 f_n(x)$ doesn't converge.

17. If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & if\ (x.y) =\neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$, show that both the partial derivatives exists at (0,0), but the function is not continuous at (0,0).

(b) Investigate the maxima and minima of the function $f(x,y) = x^2 + y^2 + x + y + xy$.

18. (a) Evaluate the inverse Laplace transform, $L^{-1}\left\{\frac{5}{p^2(p+5)^2}\right\}$.

(b) If the Laplace transform $L\{F(t)\} = f(s)$, then show that $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$.

(2*5=10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2025

MST1C02 - Analytical Tools for Statistics - II

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

(Use of scientific Calculator is permitted)

PART A

Answer any four (2 weightage each)

1. What is a nilpotent matrix? Give a non-trivial example.
2. Find a basis for the vector space of all 3×3 symmetric matrices.
3. Define the minimal polynomial of a matrix. Find the minimal polynomial of the matrix $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$.
4. Find the matrix representation of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x + y, x - y, 2x)$.
5. Define direct sum. Prove that if $V = U \oplus W$, then $dimV = dimU + dimW$.
6. State and prove Cayley-Hamilton theorem.
7. What is a quadratic form? Write the matrix of the quadratic form $x_1^2 + x_2^2 + 2x_3^2 + 4x_1x_2 + 2x_1x_3 + 6x_2x_3$.

(2 x 4=8 weightage)

PART B

Answer any four (3 weightage each)

8. Find a basis for the row space and column space of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.
9. Define rank of matrix. Prove that $rank(AB) \leq min(rank(A), rank(B))$.

10. Define linear independence and dependence of sets of vectors. Check for linear independence and dependence of the following vectors $V_1 = (1, 0, -1, 1)$, $V_2 = (3, 0, 2, 2)$ and $V_3 = (4, 0, 0, 1)$.

11. Define orthogonal vector and orthonormal vector. Find a vector that is orthogonal to both (1,2,3) and (4,5,6).

12. State and prove the Gram-Schmidt orthogonalization theorem. Obtain an orthonormal basis for the set

$S = \{\mu_1\ \mu_2\ \mu_3\}$ *where* $\mu_1 = (1\ -2\ 0\ 1)\ \ \mu_2 = (-1\ 0\ \ 0\ -1)\ ,\ \mu_3 = (1\ 1\ \ 0\ 0)$

13. What is the singular value decomposition (SVD) of a matrix? Find the singular value decomposition of the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

14. Define the rank and signature of a quadratic form. Find the rank and signature of the quadratic form $Q(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 3x_3^2 + 2x_1x_2 + 6x_2x_3$.

(3x 4=12 weightage)

PART C

Answer any two (5 weightage each)

15. Define subspace of a vector space. For any two subspaces U and V, prove that $\dim(U+V) = \dim(U) + \dim(V) - \dim(U \cap V)$

16. State and prove Rank-Nullity Theorem. Find the rank and nullity of the matrix

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 5 & 1 \\ 7 & 8 & 5 \end{pmatrix}.$$

17. a) Determine the geometric and algebraic multiplicities of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$

b) For a real symmetric matrix, show that the characteristic vectors corresponding to distinct characteristics roots are orthogonal.

18. a) Explain different classification of a quadratic form

b)Prove that Moore-Penrose inverse is exist and is unique.

c)Find the Moore- Penrose inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(5x2=10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2025

MST1C03 - Probability Theory - I

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A
(Answer any 4 questions. Weightage 2 for each question)

1. Define Limsup, Liminf, and limit of a sequence of sets. How they are related?
2. Explain the concept of probability measure induced by a random variable.
3. Define a sigma field. Show by an example that intersection of two sigma field is a sigma field.
4. Define the distribution function of a random variable. Show that it is right continuous.
5. Explain Lebesgue measure using caratheodory extension theorem.
6. State correspondence theorem on distribution function. What is its significance?
7. State and prove Cr-inequality.

Part B
(Answer any 4 questions. Each question carries weightage 3)

8. Show by an example that union of two sigma fields need not be a sigma field.
9. Define sigma field generated by open intervals and closed intervals on the real line. Show that they coincide.
10. Show that a distribution function has at the most a countable number of discontinuity points.
11. If X is a continuous nonnegative random variable with distribution function F show that $E(X) = \int_0^\infty F(x)dx$.
12. State and prove Jensen's inequality.
13. If $\{X_n\}$ converges in probability to X and f is a continuous show that $f(X_n\}$ converges in probability to f(X).
14. State and prove Kolmogorov 0-1 law.

Part C

(Answer any 2questions. Each question has weightage 5)

15. State and prove Jordan decomposition theorem. Obtain the Jordan decomposition of the distribution function $F(x) = 0, x<0$

$$= 1-p+p(1-e^{-x}), 0 \leq x < \infty.$$

16. Define independence of a class of events. State and prove Borel 0-1 laws.
17. State and prove basic inequality. Deduce Markov inequality from it.
18. Define convergence in probability and almost sure convergence. Establish the mutual relationship between them. Also establish the mutual relationship between rth mean convergence and convergence in probability.

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2025

MST1C04 - Distribution Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

(Answer any four 2 weightages each)

1. Write down the p.m.f. of the trinomial distribution. Also obtain
   (a) the marginal distributions; and (b) the expression for correlation coefficient.
2. Define probability generating function. Can $P(s) = \frac{2}{1+s}$ be the pgf of a random variable X. Give reasons.
3. Find the distribution of $Y = \frac{1}{X}$ where $X \sim Cauchy\ (\alpha, \theta)$.
4. Find the characteristic function of standard Laplace distribution and obtain the cumulants.
5. Let $f(x,y) = \begin{cases} 8xy, 0 < x < y < 1 \\ 0, otherwise \end{cases}$ be the joint probability density function of X & Y. Find E(Y |X).
6. Define mixture distributions. Discuss the practical situations where mixture distributions are appropriate.
7. Define non central F- distribution. When will this reduces to central - F distribution.

(2 x 4=8 weightages)

PART B

(Answer any four 3 weightages each)

8. Define the Hyper geometric distribution. Show that under conditions the hypergeometric distribution tends to the binomial distribution.
9. Let X_1 and X_2 be independent and identically distributed geometric (p) random variable. Find the conditional probability $P(X_1 = t | X_1 + X_2 = n)$.
10. Describe log normal distribution. Obtain its moment generating function and moments.

11. If X and Y are independent gamma random variables then show that $U = X + Y$ and $V = \frac{X}{X+Y}$ are independent.

12. Let $X_1, X_2, \dots, X_n$ be independent and identically distributed random variables with the uniform distribution on [0,1]. Find the pdf of k^{th} order statistic and identify the distribution.

13. Define the multivariate normal distribution in p dimensions. Derive its mean and variance.

14. Derive the central moments of t distribution with n degrees of freedom.

(3x 4=12 weightages)

PART C

(Answer any two 5 weightages each)

15. a) Define power series distribution. Identify the members of the family.

    b) State and prove the recurrence relation for central moments in Binomial distribution.

16. a) If $X_1, X_2, \dots, X_n$ are independent random variables having an exponential distribution with parameter θ_i, $i = 1, 2, \dots, n$; then $Z = \min(X_1, X_2, \dots, X_n)$ has exponential with parameter $\sum_{i=1}^{n} \theta_i$.

    b) Derive the raw and central moments of Normal Distribution.

17. a) State. and prove Cauchy – Schwartz inequality.

    b) If (X, Y) has a bivariate normal distribution, find $E(X|Y)$ and $E(Y|X)$.

18. In sampling from a normal population, show that the sample mean and sample variance are independently distributed.

(5x 2=10 weightages)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2025

MST1C05 - Sampling Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A: Answer any *four* questions. Each carries *two* weightage

1. Explain probability sampling and non-probability sampling.
2. What are non sampling errors? Explain .
3. What are the different types of ratio estimators in stratified random sampling?
4. What is multi-phase sampling? Why it is differ from multistage sampling?
5. Explain Stratified sampling and its advantages.
6. Explain Murthy's unordered estimator.
7. Cluster sampling will be efficient only when the variation between clusters is as small as possible; Prove.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage

8. Prove that in PPS sampling without replacement, Desraj ordered estimator is unbiased for population total. Derive its sampling variance.
9. Find an unbiased estimator of population mean in SRSWR. Find the variance of the estimate.
10. Differentiate between Hansen & Hurwitz method and Des-Raj ordered estimator.
11. Explain ratio and regression method of estimation in stratified population.
12. Carry out a comparison between the mean per unit and ratio estimator with regression estimator.
13. Show that for a population with linear trend, $V_{st} : V_{sy} : V_{ran} = \frac{1}{n} : 1 : n$
14. Differentiate between linear and circular systematic sampling. Explain them with the help of examples.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage

15. Explain the methods of allocation in stratified sampling and find efficiency of variances and compare them.

16. (a) What are the principle steps in sampling?

    (b) Show that Var $(\overline{y_{sys}}) = \frac{N-1}{Nn}(1+(n\text{-}1)\,\rho)S^2$, where ρ is the interclass correlation between the units of the same systematic sample.

17. (a) Explain πps Sampling.

    (b)Derive Yates-Grundy form of estimated variance of Hurvitz-Thomson estimator of population mean upon a PPS sample without replacement.

18. Give any three estimators of population mean in cluster sampling where clusters are of unequal size and discuss their properties.

(2 × 5 = 10 weightage)