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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2025

MMT1C01 - Algebra - I

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer all questions. Each carries 1 weightage

1. Find the order of the element (8, 4, 10) in the direct product $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.
2. List all abelian groups upto isomorphism of order 16.
3. Define isometry of $\mathbb{R}^2$.
4. Let $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_3$ be the homomorphism such that $\phi(1) = 2$. Find Ker ϕ.
5. Prove that Z has no composition series.
6. Find the reduced form and the inverse of the reduced form of the word $a^2b^{-1}b^3a^4c^4c^2a^{-1}$.
7. Give a presentation of $\mathbb{Z}_4$ involving two generators.
8. Give an example for a factor ring of an integral domain may be a field.

(8 × 1 = 8 weightage)

Part B

Answer any two questions from each unit. Each carries 2 weightage

Unit I

9. Let M be a maximal normal subgroup of a group G. Show that G/M is simple.
10. Let X be a G-set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exist $g \in G$ such that $gx_1 = x_2$. Prove that $\sim$ is an equivalence relation on X.
11. State and prove Burnside's Formula.

Unit II

12. Find isomorphic refinements of $\{0\} < 10\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 25\mathbb{Z} < \mathbb{Z}$.

13. Prove that every group of prime power order is solvable.

14. Prove that every group is a homomorphic image of a free group.

Unit III

15. Prove that a nonzero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F.

16. State and prove Eisenstein criterion for irreducibility of a polynomial.

17. Define an ideal in a ring R. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and let N be the subring of all functions f such that $f(2) = 0$. Is N an ideal in F? why or why not?

(6 × 2 = 12 weightage)

Part C

Answer any two questions. Each carries 5 weightage

18. (a) Show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.

    (b) Prove that a factor group of a cyclic group is cyclic.

19. (a) Define decomposable group. Prove that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.

    (b) State and prove Third Isomorphism theorem.

20. (a) State and prove First Sylow Theorem.

    (b) If p and q are distinct primes with $p < q$, then show that every group G of order pq has a single subgroup of order q and this subgroup is normal in G. If q is not congruent to 1 modulo p, then show that G is abelian and cyclic.

21. (a) Let ϕ_π : Q[x] $\to \mathbb{R}$ be the evaluation homomorpism with $\phi_\pi(x) = \pi$. Find the Kernal of ϕ_π. Also show that ϕ_π is a one - to- one map.

    (b) Show that the multiplicative group $< F^*, . >$ of nonzero elements of a finite field F is cyclic.

(2 × 5 = 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2025

MMT1C02 - Linear Algebra

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A: Answer all questions. Each carry 1 weightage

1. Prove that the only subspaces of R^1 are R^1 and the zero subspace.
2. Express $\frac{1}{1-\sqrt{2}}+\frac{2}{\sqrt{12}-2}$ as a linear combination of 1, $\sqrt{2}$ and $\sqrt{3}$.
3. Let T be a linear operator on R^2 defined by $T(x_1,x_2)=(x_2,-x_1)$ describe T geometrically.
4. Let T be the linear operator on R^3 defined by

   $\mathrm{T}(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3)=(3\mathrm{x}_1,\mathrm{x}_1-\mathrm{x}_2,2\mathrm{x}_1+\mathrm{x}_2+\mathrm{x}_3)$ is T invertible? If so find its inverse.
5. Let T be a linear operator on R^2 defined by $T(x_1,x_2)=(-x_2,x_1)$. Find the matrix of T in the ordered basis, $B=\{\alpha_1,\alpha_2\}$ where $\alpha_1=(1,2)$ and $\alpha_2=(1,-1)$.
6. Prove that the similar matrices have the same characteristic polynomial.
7. State and prove the Cauchy Schwarz inequality in an inner product space.
8. Prove that an orthogonal set of non-zero vectors is linearly independent.

$(8\times 1=8\ weightage)$

Part B: Answer any two questions from each unit. Each carry 2 weightage

Unit I

9. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. suppose V is finite dimensional then prove that rank(T)+ nullity(T)= dim V.

10. Show that a linear operator T on a finite dimensional vector space is one - one if and only if it is onto. Give examples to show that this is not true for infinite dimensional vector spaces.

11. Let T be a linear transformation from a vector space V into another vector space W, then prove that T is non singular if and only if T carries each linearly independent subset of V into a linearly independent subset of W.

Unit II

12. If W_1 and W_2 are subspaces of a finite dimensional vector space, then show that $W_1 = W_2$ if and only if $W_1^0 = W_2^0$

13. Let g ,f_1 ..,... f_r be linear functional on a vector space V with respective null spaces N,N_1,N_2,....,N_r then prove that g is a linear combination of f_1,....f_r if and only if N contains the intersection $N_1 \cap N_2 \cap .... \cap N_r$

14. let T be the linear operator in R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ prove that T is diagonalizable by exhibiting a basis for R^3, each vector of which is a characteristic vector of T.

Unit III

15. Apply Gram-Schmidt process to the vectors β_1=(2,0,-3), β_2=(1,2, 1), β_3=(0,3,3) to obtain an orthonormal basis for R^3 with the standard inner product.

16. Let V be a real vector space and E an idempotent linear operator on V. Prove that (I+E) is invertible also find $(I + E)^{-1}$.

17. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W, then prove that (i) E is an idempotent linear transformation of V on to W. (ii) $V = W \oplus W^{\perp}$, where $W^{\perp}$ is the null space of E.

(6 x 2 = 12 weightage)

Part C: Answer any two questions. Each carry 5 weightage

18. (a) If W is a subspace of a finite dimensional vector space V, prove that every linearly independent subset of W is finite and is part of a basis for W.

(b)) Let V be a finite dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same fieldF and let $\beta_1, \beta_2, \dots, \beta_n$ be any vectors in W then prove that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j, j = 1,2, \dots, n$

19. Let T is a linear operator on a finite dimensional vector space V. Then prove that the minimal polynomial divides the characteristic polynomial for T

20. (a) If W is a k –dimensional subspace of an n dimensional vector space V, then prove that W is the intersection of (n-k) hyperspaces in V

(b) If f is a non zero linear functional on the vector space V then prove that the null space of f is a hyperspace in V. conversely , every hyperspace in V is the null space of a (not unique) non zero linear functional on V

21. (a)Verify that the standard inner product on F^n is an inner product

(b) let V be a real or complex vector space with an inner product. Show that the quadratic form determined by the inner product satisfies the parallelogram law

$||\alpha + \beta||^2 + ||\alpha - \beta||^2 = 2||\alpha||^2 + 2||\beta||^2$

(2 x 5 = 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2025

MMT1C03 - Real Analysis -I

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part-A

Answer all questions. Each question carries 1 weightage

1) Let E' be the set of all limit points of a set E. Prove that E' is closed.
2) For $x, y \in \mathbb{R}$, defined $d(x,y) = |x^2 - y^2|$. Determine whether d is metric or not in $\mathbb{R}$.
3) Let E be a set where every point is an isolated point. Prove that any function defined on E is continuous on E.
4) Suppose f is differentiable and $f' \geq 0$ on the segment $(0,1)$. Prove that f is increasing on $(0,1)$.
5) If $f \in \mathcal{R}(\alpha)$and $g \in \mathcal{R}(\alpha)$, prove that $f^2 \in \mathcal{R}(\alpha)$ and $fg \in \mathcal{R}(\alpha)$.
6) Let $\alpha(x) = x$ and $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$. Prove that $f \notin \mathcal{R}(\alpha)$ on $[0,1]$
7) Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{x}{1+nx^2}$, is uniformly convergent on $[0,1]$.
8) Define equicontinuous family of functions.

Part-B

Answer **any two** questions from each unit. Each question carries **2 weightage**

Unit - I

9) Prove that a finite set has no limit points
10) Prove that Compact subsets of metric spaces are closed.
11) Prove that, if f is a continuous mapping of a metric space X into a metric space Y, and E is a connected subset of X, then $f(E)$ is connected in Y.

Unit - II

12) Suppose f is continuous on $[a,b]$, $f'(x)$ exists at some point $x \in [a,b]$, g is defned on an interral I which contains the range of f and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$; $(a < t < b)$, prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$.
13) Prove that L'Hospital's Rule fails in the case of complex valued function.
14) Prove that $f \in \mathcal{R}(\alpha)$ on $[a,b]$ if and only if for every $\epsilon > 0$ there exists a partition P of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \epsilon$

Unit - III

15) If $\mathbf{f}$ maps $[a,b]$ inot $\mathbb{R}^k$ and $\mathbf{f} \in \mathcal{R}(\alpha)$ for some monotonically increasing function α, prove that $|\mathbf{f}| \in \mathcal{R}(\alpha)$ and

$$\left|\int_a^b \mathbf{f} d\alpha\right| \leq \int_a^b |\mathbf{f}| d\alpha$$

16) If K is a compact metric space, $f_n \in \mathcal{C}(K)$ for $n = 1,2,3,\ldots$,and if $\{f_n\}$ converges uniornlly on K, then prove that $\{f_n\}$ is equicontinuous on K.

17) State and prove Cauchy criterion for uniform convergence of sequence of function.

Part-C

Answer **any two** questions. Each question carries **5 weightage**

18) a) Give an example of a noncompact subset of $\mathbb{R}$. Justify your answer.

b) Let $a_1, a_2, \ldots a_k$ and $b_1, b_2, \ldots, b_k$ are real numbers with $a_i < b_i; \forall i$. Let

$$W = \{(x_1, x_2, \ldots, \quad x_k) \in \mathbb{R}^k;\ a_i \leq x_i \leq b_i\ (1 \leq i \leq k)\}$$

Prove that W is compact in $\mathbb{R}^k$.s

19) a) Define uniformly continuous function.

b) Give an example of a continuous function which is not uniornlly continuous. Justify your answer

c) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

20) a) Let $f \in \mathcal{R}$ on $[a,b]$. For $a < x < b$, put $F(x) = \int_a^x f(t)dt$. Prove that F is continuous on $[a,b]$ furthermore, if f is continuous at a point x_0 of $[a,b]$, then F is differentiable at x_0 , and $F'(x_0) = f(x_0)$.

b) State and prove the fundamental theorem of calculus.

21) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0 \in [a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, prove that $\{f_n\}$ converges uniformly on $[a,b]$, to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x) \quad ; \quad a \leq x \leq b$

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2025

MMT1C04 - Discrete Mathematics

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer **all** *questions. Each question carries* **1** *weightage.*

1. Define a total order. Give an example of a partial order which is not a total order.
2. Prove that every nonzero element of a finite Boolean algebra contains at least one atom.
3. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.
4. Prove that a tree with at least two vertices contains at least two pendant vertices.
5. If $\{x, y\}$ is a 2-edge cut of a graph G, show that every cycle of G that contains x must also contain y.
6. Prove that every nonzero element of a finite Boolean algebra contains at least one atom.
7. Give a grammar for the set of all regular expressions in Pascal.
8. Design a dfa which accepts all binary sequences that ends with the digits 101.

(8 × 1 = 8 weightage)

Part B

Answer any questions from each unit. Each question carries 2 weightage

Unit I

9. Let X be a set and let $\leq$ be a binary relation defined on X which is reflexive and transitive. Define a binary relation R on X by xRy if $x \leq y$ and $y \leq x$. Prove that R is an equivalence relation.
10. Let $(X, +, \,.\,, ')$ be a finite Boolean algebra. Prove that every element of $x \in X$ can be uniquely expressed as a sum of atoms contained in x.
11. Write the Boolean function $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$ in its disjunctive normal form.

Unit II

12. Prove that a graph G is bipartite if and only if it does not contain any odd cycle.
13. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G if and only if there exist vertices u and w of G distinct from v such that v is in every $u - w$ path in G.
14. For a connected plane graph G, prove that $n - m + f = 2$, where n, m, and f denote the number of vertices, edges, and faces of G, respectively.

Unit III

15. Find a grammar that generates the language $L = \{a^n b^{n+1}: n \geq 0\}$.
16. Find a dfa that accepts all the strings on $\{0,1\}$, except those containing the substring 001.
17. Show that the language $L = \{awa: w \in \{a, b\}^*\}$ is regular.

($6 \times 2 = 12$ weightage)

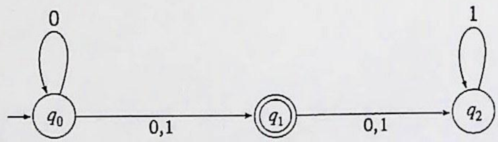
Part C

Answer any two questions. Each question carries 5 weightage

18. Let $(X, +, ., ')$ be a Boolean algebra. Then prove that the corresponding lattice $(X, \leq)$ is complemented and distributive. Conversely prove that if $(X, \leq)$ is bounded, complemented and distributive lattice then there exists a Boolean algebra structure on X, $(X, +, ., ')$ such that the partial order relation defined by this structure coincides with the given relation $\leq$.
19. Prove that a graph G with at least three vertices is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.
20. (a) Prove that K_5 is non-planar.

    (b) Define dual of a planar graph. Draw a planar representation of K_4 and its dual.
21. (a). Let L be the language accepted by a non-deterministic finite accepter. Then prove that there exists a deterministic finite accepter M_D such that $L = L(M_D)$.

    (b) Find a dfa equivalent to the following nfa



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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2025

MMT1C05 - Number Theory

(2022 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A

Answer *all* questions. Each carry 1 weightage

1. Define Mobius function and find it's Dirichlet inverse.
2. Prove that $\sum_{d|n} \mu(d) \frac{n}{d} = \phi(n), n \geq 1$.
3. Show that $[3x] - 3[x] = 0,1$ or 2.
4. Prove that $\psi(x) = \sum_{m \leq \log_2 x} \vartheta\left(x^{\frac{1}{m}}\right)$.
5. Prove that $\vartheta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$
6. Determine those odd prime p for which $(-3|p) = 1$.
7. If P and Q are odd integers, prove that $(m|P)(n|P) = (mn|P)$.
8. Explain Affine transformation.

Part B

Answer any *two* questions from each unit. Each carry 2 weightages

Unit 1

9. Prove that $\sum_{d|n} \Lambda(d) = \log n,\ n \geq 1$.
10. Prove or disprove: "Mangoldt function is multiplicative"
11. State and prove Legendre's identity.

Unit 2

12. Prove that $\lim\limits_{x \to \infty} \frac{\pi(x) \log x}{x} = \Leftrightarrow \lim\limits_{x \to \infty} \frac{\vartheta(x)}{x} = 1$.
13. Prove that for $n \geq 1, \frac{1}{6} n \log n < P_n$, where P_n is the n^{th} prime.
14. Prove that $\sum_{p \leq x} \frac{1}{p} = \log(\log x) + A + O\left(\frac{1}{\log x}\right), x \geq 2, A$ is a constant.

Unit 3

15. State and prove quadratic reciprocity law for odd primes.

16. Let $A = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$, encipher the word NOANSWER and decipher the word FWMDIQ, in 26 letter alphabets.

17. Single letter system with alphabets A – Z with numerical equivalents $0 - 25, ? = 26$, blank $= 27$. If B, ? are the cipher texts of space and E. Find the encryption formula.

Part C

Answer any *two* questions. Each carry 5 weightage

18. Prove that the class of arithmetical functions f with $f(1) \neq 0$ form a commutative group under the Dirichlet multiplication.

19. (a). State and prove Euler's summation formula.

    (b). Prove that $\sum_{n>x} \frac{1}{n^s} = O(x^{1-s}), s > 1$.

20. Prove that $\frac{1}{6}\frac{n}{\log n} < \pi(n) < \frac{6n}{\log n}, n \geq 2$.

21. State and prove Gauss Lemma. Also prove that $m = \left\{\sum_{t=1}^{\frac{p-1}{2}} \left[\frac{tn}{p}\right] + \frac{(n-1)(p^2-1)}{8}\right\} mod\ 2$.