1M4A25205 (Pages : 2) Reg. No:....................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth SemesterM.Sc. Mathematics Degree Examination, April 2025

MMT4E09– Differential Geometry

(2022 Admission onwards)

Time: 3 hours Max. Weightage: 30

Part A

Answer all questions. Each question carries 1 weightage

1. Sketch the vector field $\mathbb{X}(p) = (p, X(p))$, on $\mathbb{R}^2$ where $X(p) = (x_2, x_1)$.
2. Show that the graph of any function $f: \mathbb{R}^n \to \mathbb{R}$ is a level set for some function $F: \mathbb{R}^{n+1} \to \mathbb{R}$.
3. Show that the gradient of$f$at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at P.
4. Find the velocity, the acceleration, and the speed of the parametrized curve $\alpha(t) = (cost, sint)$.
5. Let $f: U \to \mathbb{R}$ be asmooth function and let $\alpha: I \to U$ be an integral curve of ∇f. Show that $\left(\frac{d}{dt}\right)(f \circ \alpha)(t) = \|\nabla f(\alpha(t))\|^2$ for all $t \in I$.
6. Find the length of the given parameterized curve $\alpha: I \to \mathbb{R}^{n+1}$ given by $\alpha(t) = (t^2, t^3),\ I = [0,2],\ n = 1$.
7. Describe the spherical image of the hyperboloid$x_1^2 - x_2^2 - x_3^2 = 4, x_1 > 0$ oriented by $\frac{\nabla f}{\|\nabla f\|}$, where $f(x_1, x_2, x_3) = x_1^2 - x_2^2 - x_3^2$.
8. Compute$\nabla_v f$ where $f: \mathbb{R}^{n+1} \to \mathbb{R}$ and $v \in R_p^{n+1}, p \in R^{n+1}$ are given by $f(x_1, x_2) = {x_1}^2 - {x_2}^2, v = (1, 1, \cos\theta, \sin\theta)$.

(8x1=8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Let U be an open set in $\mathbb{R}^{n+1}$ and let $f: U \to \mathbb{R}$ be smooth. Let $p \in U$ be a regular point of f, and let $c = f(p)$. Then show that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$.
10. Find and sketch the gradient field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.
11. Let S$\subset \mathbb{R}^{n+1}$ be a connected n- surface in $\mathbb{R}^{n+1}$, then show that there exists on S exactly two smooth unit normal vector fields $\mathbb{N}_1 and \mathbb{N}_2$ and $\mathbb{N}_2(p) = -\mathbb{N}_1(p)$, for all $p \in S$.

Unit 2

12. Let Sbe an$n$-surface in $\mathbb{R}^{n+1}$, let $\alpha: I \to S$ be a parametrized curve, and let $\mathbb{X}$ and $\mathbb{Y}$ are vector fields tangent to S along α. Verify that

    (a)$(f\mathbb{X})' = f'\mathbb{X} + f\mathbb{X}'$,

    (b) $(\mathbb{X} + \mathbb{Y})' = \mathbb{X}' + \mathbb{Y}'$.

13. Let S be an n-surface in $\mathbb{R}^{n+1}$. Let $p, q \in S$ and α be a piecewise smooth parametrized curve from p to q. Then show that parallel transport $P_\alpha: S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.

14. Define Weingarten map. Show that the Weingarten map L_p is self - adjoint.

Unit 3

15. Prove that on each compact oriented n-surface S in $\mathbb{R}^{n+1}$, there exists a point $p$on Ssuch that the second fundamental form at p is definite.

16. Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$. Then show that the Gauss-Kronecker curvature $K(p)$ of S at p is non-zero for all $p \in S$if and only if the second fundamental form $\mathcal{S}_p$ of S at p is definite for all $p \in S$.

17. State and prove the Inverse function theorem for n-surfaces. **(6 x 2 = 12 weightage)**

Part C

Answer any two questions. Each carries 5 weightage.

18. Let S be an n-surface in$\mathbb{R}^{n+1}$, let $\mathbb{X}$ be a smooth tangent vector field on S, and let $p \in S$. Show that there exists an open interval I containing0 and a parametrized curve $\alpha: I \to S$ such that

    (i) $\alpha(0) = p$.

    (ii) $\dot{\alpha(t)} = \mathbb{X}(\alpha(t))$ for all $t \in I$

    (iii) $If \beta: \tilde{I} \to S$is any other parameterized curve in S satisfying (i) & (ii), then $\tilde{I} \subset I$ and $\beta(t) = \alpha(t) \forall t \in \tilde{I}$.

19. Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$exhibited as a level set $f^{-1}(c)$ of a smooth function $f: \mathbb{R}^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0$ for all $p \in S$. Then show that the Gauss map maps S onto the unit sphere S^n.

20. Let S be an n-surface in $\mathbb{R}^{n+1}$, let $p \in S$ and let $\mathbb{v} \in S_p$. Then prove there exists an open interval I containing 0 and a geodesic $\alpha: I \to S$ such that:

    (i) $\alpha(0) = p$ and $\dot{\alpha}(0) = \mathbb{v}$.

    (ii) If $\beta: \hat{I} \to S$ is any other geodesic in S with $\beta(0) = p$ and $\dot{\beta}(0) = \mathbb{v}$then $\hat{I} \subset I$ and $\beta(t) = \alpha(t)$ for all $t \in \hat{I}$.

21. Show for a parameterized n-surface $\varphi: U \to \mathbb{R}^{n+1}$ in $\mathbb{R}^{n+1}$ and for $p \in U$, there exists an open set $U_1 \subset U$ about p such that $\varphi(U_1)$ is an n-surface in $\mathbb{R}^{n+1}$.

(2 x 5 = 10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2025

MMT4E11– Graph Theory

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A

Answer all the questions. Each carries 1 weightage.

1. Prove that every nontrivial tree has at least two vertices of degree one.
2. Differentiate between an edge cut and a bond of a graph using an example.
3. Construct a graph with $K = 2, K' = 3$ and $\delta = 4$.
4. Let M be a matching and K be a covering such that $|M| = |K|$. Then prove that M is a maximum matching and K is a minimum covering.
5. Explain Personnel Assignment Problem.
6. If G is k-critical, then prove that $\delta \geq k - 1$.
7. Find the chromatic polynomial of C_4, cycle with four vertices.
8. Explain Stereographic Projection.

Part B

Answer any two questions from each unit.
Each question carries 2 weightage.

Unit I

9. Prove that any spanning tree $T^* = G[e_1, e_2, ..., e_{n-1}]$ constructed by Kruskal's algorithm is an optimal tree.
10. A graph G with $\nu > 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.
11. Prove that, a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

Unit II

12. Let G be a connected graph that is not an odd cycle.Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

13. Let M and N be djsjoint matchings of G with,$| M | > | N |$. Then prove that there are disjoint matchings M' and N' of G such that $| M' | = | M | -1$, $| N' | = | M | +1$ and $M' \cup N' = M \cup N$.

14. Prove that $r(k,k) \geq 2^{k/2}$

Unit III

15 Let G be a k-critical graph with a 2-vertex cut $\{u,v\}$. Then prove that $G = G_1 \cup G_2$, where $G_i$is a $\{u,v\}$-component of type $i(i = 1,2)$

16. Prove that, if two bridges overlap, then either they are skew or else they are equivalent 3-bridges

17 Prove that a loopless digraph D has an independent set S such that each vertex of D not in S is reachable from a vertex in S by a directed path of length at most two.

Part C

Answer any two questions.
Each question carries 5 weightage.

18. (a) Define the concept of contraction of an edge and prove that if e is a link of G, then $\tau(G) = \tau(G - e) + \tau(Ge)$.

    (b) Express K_5 by the above formula

19. (a)Explain The Chinese Postman Problem

    (b) Write Fleury's Algorithm and prove that If G is Eulerian, then any trail in G constructed by Fleury's algorithm is an Euler tour of G.

20. Prove that i) $\alpha + \beta = \nu$

    (ii) If $\delta > 0$, $\alpha' + \beta' = \nu$

    (iii) In a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum

    edge covering.

21. Prove that for any positive integer k, there exists a k-chromatic graph containing no triangle.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2025

MMT4E14– Computer Oriented Numerical Analysis

(2022 Admission onwards)

Time: 1.5 hours Max. Weightage : 15

Part A (Short Answer Questions)

(Answer all questions. Each question has weightage 1)

1. Write the output of

```
i =5
print i
i =i**2
print i
```

2. Explain *continue* statement with examples.
3. Differentiate between the data structures List and Tuple.
4. Explain try-except and try-finally statements in python.

(4x 1=4 weightage)

Part B

(*Answer any three from the following five questions. Each question has weightage 2*)

5. Write a python program to find the root of the equation $f(x) = x^2 - 3x + 2$ using Newton Raphson method with initial guess x=0 .
6. Write a python program to find the value of function using Lagranges interpolation, when the function value at *n* points are given.
7. Write a python program programme to find $\int_1^3 e^x dx$ using trapezoidal rule of integration for n =4.
8. Write a python program to estimate $y(0.4)$ when $y'(x) = x^2 + y^2$ with $y(0) = 0$ by taking h = 0 .2 by using Runge Kutta method of order 4.
9. Write a python programme to find the integral $\int_1^2 x^2 dx$ using the method of Simpson rule for n = 100

(3x2=6 weightage)

Part C

(Answer any one from the following two questions. Each question has weightage 5)

10. Write a python programme to find the root of the function $f(x) = x^2 - 2x - 3$ on $[1,4]$ by using bisection method.

11. Write a python programme to solve the system of equations $3x + 6y + z = 16$ $2x + 4y + 3z = 13$ $x + 3y + 2z = 9$ using Gauss elimination method.

(1x5=5 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2025

MMT4C15 – Advanced Functional Analysis

(2022 Admission onwards)

Time: 3hours Max. Weightage : 30

Part A

Answer all questions from1to 8. Each question carries 1 weightage.

1. Let X be a normed space Xover $\mathbb{C}$ and $A \in BL(X)$ and p be a polynomial in one variable. Then prove that $\sigma(p(A)) = \{p(z): z \in \sigma(A)\}$.
2. Define week convergence of a sequence in a normed space X.Give an example of week convergence does not imply convergence.
3. Let X be a reflexive normed space .Then prove that if X is a Banach space then it is reflexive in any equivalent norm
4. Let A be a compact operator on a normed space Xover $\mathbb{C}$. Then prove that $\sigma_a(A) \neq \emptyset$.
5. Let X be an inner product space, $\{u_1, u_2, \cdots\}$ be an orthonormal set in X and $f \in X'$. Then prove that $\sum_n |f(u_n)|^2 \leq \|f\|^2$.
6. Give an example of a normal operator which is neither unitary nor self-adjoint.
7. Let A and B be self-adjoint. Then prove that AB is self- adjoint if and only if A and B commute.
8. Show, by an example, that if $k \in \sigma_e(A)$ does not follow that $\bar{k} \in \sigma_e(A^*)$.

Part B

Answer two questions from each unit. Each question carries 2weightage.

Unit I

9. Let $X = \ell^p$ with norm$\| \ \|_p, 1 \leq p \leq \infty$andconsider the right shift operator C on X given by $C(x) = (0, x(1), x(2), \cdots)$. Then prove that $\sigma_e(C) = \emptyset$, $\sigma_a(C) = \{k \in K: |k| = 1\}$ and $\sigma(C) = \{k \in K: |k| \leq 1\}$.
10. Let X be a Banach space, $A \in BL(X)$ and $|A^p| < 1$ for some positive integer p. Then prove that the operator $I - A$ is invertible
11. Let X be a Banach space over K and $A \in BL(X)$. Then prove that $\sigma(A)$ is a bounded subset of K.Then also prove that $\sigma(A)$ is a compact subset of K.

Unit II

12. Let X and Y be normed spaces and $F: X \to Y$ be linear. Then prove that if F is a compact map if and only if for every bounded sequence (x_n)in X, $F(x_n)$ has a subsequence which converges in Y.

13. Let X be a normed space and $A \in CL(X)$. Then prove that the spectrum of A, $\sigma(A)$, is a countable set.

14. Let H be a Hilbert space, G be a subspace of H and g be a continuous linear functional on G. Then prove that there is a unique continuous linear functional f on H such that $f|_G = g$ and $\|f\| = \|g\|$.

Unit III

15. State and prove generalized Schwarz inequality.

16. Let H be a Hilbert space and $A \in BL(H)$. Then prove that $\sigma_e(A) \subset \sigma_a(A)$ and $(A) = \sigma_a(A) \cup \{k: \bar{k} \in \sigma_e(A^*)\}$.

17. Let H be a Hilbert space and $A \in BL(H)$. Let A be a self-adjoint operator, then prove that $\|A\| = \{|\langle A(x), x\rangle|: x \in H, \|x\| \leq 1\}$.

Part B

Answer any two questions. Each question carries 5weightage.

18. Let X be a normed space and $A \in BL(X)$ be of finite rank
    a).Prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.
    b).Show that $\sigma(A) \not\subset \sigma_a(A)$, in general.

19. Let $1 \leq p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then prove that the dual of ℓ^p is ℓ^q.

20. State and prove Riesz representation theorem.

21. Let $H \neq \{0\}$ be a Hilbert space and $A \in BL(H)$be self-adjoint.
    (i). Then prove that$\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A]$.
    (ii). Let $F_n = \text{span}\{x_1, x_2, \cdots, x_n\}, n = 1,2,3, \cdots$
    $\alpha_n = inf_{x \in F_{n,\|x\|=1}} \langle Ax, x\rangle$, and $\beta_n = \sup_{x \in F_{n,\|x\|=1}} \langle Ax, x\rangle$. Then prove that $m_A \leq \alpha_{n+1} \leq \alpha_n \leq \cdots \leq \alpha_1 \leq \beta_1 \leq \cdots \leq \beta_n \leq \beta_{n+1} \leq M_A$. Also prove that, if span of $\{x_1, x_2, \cdots\}$ is dense in H, then $m_A = \lim_{n\to\infty} \alpha_n$ and $M_A = \lim_{n\to\infty} \beta_n$.