2M2A25115 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2025

MST2C06 - Probability Theory - II

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A

Answer any four questions. Each question carries 2 weightages.

1. Define characteristics function. Is $\varphi(t)=1$, is a characteristic function? If so, identify the corresponding distribution function?
2. Establish how existence of moments and differentiability of a characteristic function are related?
3. Briefly explain law of large numbers, central limit theorem and law of iterated logarithm. How they differ?
4. Let {Xn} be a sequence of independent random variables such that $P(X_k=k^{1/2}) = P(X_k = -k^{1/2}) = 1/2$. Examine whether the sequence obey the weak law of large numbers?
5. State Lindberg- Feller central limit theorem. Use it to establish Lindberge -Levy theorem?
6. Define conditional expectation. Derive conditional probability from conditional expectation?
7. If $\{Zn, \mathbb{F}_n\}$ is a martingale show that,

   $P\{\text{Max}(z_1, z_2 \ldots\ldots z_n) \geq a\} \leq \frac{E(Zn)}{a}$

PART B

Answer any four questions. Each question carries 3 weightages.

8. Prove that a characteristic function is real if and only if corresponding distribution function is symmetric about origin?
9. Let φ_1 and φ_2 be two characteristic functions. Then which of the following are characteristic functions,

   a) $\phi_1 + \phi_2$

   b) $\phi_1 - \phi_2$

   c) $\phi_1 \phi_2$

   d) $\frac{\phi_1}{\phi_2}$

   e) $\frac{1}{2}(\phi_1 + \phi_2)$

10. State and prove Liapunov central limit theorem?

11. State and prove Kolmogorov's strong law of large numbers for iid sequence of random variables?

12. State and establish Khintchin's weak law of large numbers?

13. If X_1, X_2........ are iid random variables with $E|X_1|<\infty$, prove that,

a) $E(X_1/S_n) = \frac{S_n}{n}$

b) $E(\frac{S_n}{n}/S_{n+1}) = \frac{S_{n+1}}{n+1}$

14. When do you say that a distribution function is infinitely divisible. Show that normal distribution and poisson distribution are infinitely divisible?

PART C

Answer any 2 questions. Each questions carries five weightages.

15. a) State and prove Kolmogorove three series criterion?

b) Let $\{X_n\}$ be an iid sequence with $E(X_n)=0$, Var $(X_n)= \sigma^2$. Let $Y_n=\sum_{i<j} X_i X_j$.

Show that $\{Y_n\}$ obeys the WLLN in the form $\frac{Y_n}{nC_2} \xrightarrow{P} \sigma^2$

16. Establish inversion theorem on characteristic function. Use it to find the density function of a Cauchy distribution.

17. Determine whether CLT holds for the sequence of independent random variables $\{X_n\}$, n=1,2,........

a) $P(X_n=\frac{n}{\log n})=\frac{\log n}{2n}= P(X_n=\frac{-n}{\log n})$

$P(X_n=0) = 1 - \frac{\log n}{n}$

b) $P(X_n =1) = \frac{1-2^{-n}}{2} = P(X_n =-1)$

$P(X_n= 2^n) = 2^{-(n+1)} = P(X_n= -2^n)$

18. a) State up crossing inequality. Hence establish Doob's martingale convergence theorem?

b) Establish Doob – Meyer decomposition of a sub martingale?

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2025

MST2C07- Applied Regression Analysis

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A

Answer any four (2 weightages each)

1. What are the basic assumptions of a linear regression model?
2. Define projection matrix. State its properties
3. Define coefficient of determination. What is its importance?
4. Define serial correlation. What are the consequences of presence of serial correlation in regression?
5. What are splines?
6. What are kernel smoothers in non-parametric regression
7. What is Link function in a generalized linear model.

(2 x 4=8 weightages)

PART B

Answer any four (3 weightages each)

8. Obtain the least square estimates of parameters of simple linear regression model?
9. Let Y_1 and Y_2 be independent random variables with means α and 2α, respectively. Find the least squares estimate of α and the residual sum of squares.
10. Discuss Ridge Regression and obtain the expression for the ridge estimate.
11. Explain four methods of scaling the residual.
12. Explain the testing of significance of simple linear regression model.
13. Describe forward selection method for variable selection in linear regression.
14. Explain Poisson regression model with necessary assumptions

(3x 4=12 weightages)

PART C

Answer any two (5 weightages each)

15. Let $Y_1 = \theta_1 + \theta_2 + \varepsilon_1$, $Y_2 = 2\theta_2 + \varepsilon_2$, $Y_3 = -\theta_1 + \theta_2 + \varepsilon_3$ where $\varepsilon_i (i = 1,2,3)$ are independent $N(0, \sigma^2)$. Derive an F-statistic for testing the hypothesis $H: \theta_1 = 2\theta_2$.
16. What is meant by multicollinearity? Explain its consequences.Describe the method of detection of multicollinearity.
17. Define polynomial regression model. Explain the procedure of estimating parameters of polynomial regression model using orthogonal polynomials.
18. Explain generalized linear model. What are the three components specified in GLM. Describe the estimation procedure.

(2 x 5 = 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2025

MST2C08- Estimation Theory

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A

(Answer any 4 questions. Weightage 2 for each question)

1. Define Ancillary statistic with an example.
2. Explain method of moment estimation. If $f(x)=\theta x^{\theta-1}, 0<x<1, \theta>0$. Find the moment estimator of θ.
3. a) Describe the method of construction of confidence intervals using pivots.

   b) What is meant by coverage probability of a confidence interval?
4. Explain a) Prior distribution b) Posterior distribution.
5. Find the Cramer-Rao lower bound of the variance of the unbiased estimator of θ with $f(x;\theta) = \theta(1-\theta)^x, x = 0,1,2 \ldots$ and $0 < \theta < 1$.
6. If $f(x) = \theta e^{-\theta x}, x > 0, \theta > 0$. Find Fisher information $I_X(\theta)$.
7. Define unbiased confidence interval.

(4*2=8 Weightage)

PART B

(Answer any 4 questions. Weightage 3 for each question)

8. State and Prove Fisher- Neyman Factorization theorem.
9. a) Show that for distribution belonging to One parameter exponential family the MLE$\hat{\theta}$ is CAN for θ with asymptotic variance $\frac{1}{nI(\theta)}$.

   b) Give an example of a family which is not an exponential family but is a member of Cramer family of distributions.

10. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from a Poisson distribution with parameter λ. Check the consistency and unbiasedness of the estimator $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} X_i$ of λ.

11. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a geometric distribution with $P(X = x) = \theta(1-\theta)^x, x = 0,1,2, \ldots; 0 < \theta < 1$. Find MVUE of θ.

12. State and prove Basu's theorem.

13. Show that under certain regularity conditions , MLE is consistent.

14. a) State and prove Cramer-Rao inequality.

    b) Explain the procedure of obtaining UMVUE in the presence of complete sufficient statistic.

(4*3=12 Weightage)

PART C

(Answer any 2 questions. Weightage 5 for each question)

15. Let $x_1, x_2, \ldots x_n$ be iid Bernoulli (p) random variables and $y = \sum_{i-1}^{n} X$. On assuming prior distribution of $p \to Beta(\alpha, \beta)$; Identify

    i)Marginal pdf of Y

    ii)Baye's estimate of p

16. a) State and prove Invariance property of CAN estimator.

    b) Define shortest length confidence interval and explain the role of sufficient statistic in determining the same.

17. State and Prove Lehman-Scheffe Theorem.

18. Let $X \sim N(\theta, 1)$ and prior PDF of θ be $N(0,1)$. Find the Baye's estimator of θ under squared error loss function.

(2*5=10 Weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2025

MST2C09 - Stochastic Processes

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

(Use of Scientific Calculator is permitted)

PART A

Answer any four (2weightage each)

1. For an ergodic chain stationary distribution exits uniquely. Prove
2. Explain renewal reward process and regenerative process.
3. Explain Hidden Markov Chain with the help of an example
4. Explain Geometric Brownian motion.
5. With suitable example explain counting process.
6. Explain Open queueing network and Closed queueing network.
7. A fair die is tossed repeatedly. If Xn denote the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain {Xn}.

(4 x 2 =8weightage)

Part B

Answer any four (3 weightage each)

8. Derive the steady state probabilities of M/M/1 model
9. State and prove central limit theorem on renewal process
10. State and prove elementary renewal theorem
11. Derive the Chapman - Kolmogorov equation for continuous time Markov chain.
12. Derive decomposition of a Poisson process
13. Explain stationary and weakly stationary processes with the help of examples.
14. Obtain the stationary distribution for the Markov chain with TPM

$$P = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} \end{bmatrix}$$

Also obtain mean recurrence time for each state.

(4x3=12weightage)

PART C

Answer any two (5 weightage each)

15. (i) Explain classification of a Stochastic Process with example.

    (ii)Show that a Markov Chain is completely specified by initial probabilities and one-step transition probabilities.

    (iii)Explain Martingales.

16. Derive the differential equation satisfied by a Poisson process.

17 (a)What is a renewal process? Show that the mean renewal function m(t) satisfies the renewal equation: $m(t) = F(t) + \int_0^t m(t-x) f(x) dx$.

    (b) Using renewal argument show that: $E(X_1+....+X_{N(t)+1})=E(X).[1+M(t)]$

    (c) With probability 1, show that $\frac{N(t)}{t} \rightarrow \frac{1}{\mu} as\, t \rightarrow \infty$

18. Describe an M/G/1 model in detail

(2 x 5 =10 weightage)