2M2A25089 (Pages : 4) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, April 2025

MMT2C10 – Operation Research

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all the questions. Each carries 1 weightage.

1. Solve graphically:

   Maximize $5x_1 + 3x_2$

   Subject to $4x_1 + 5x_2 \leq 10$

   $5x_1 + 2x_2 \leq 10$

   $3x_1 + 8x_2 \leq 12$

   $x_1 \geq 0, x_2 \geq 0$

2. Define a convex function. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semi definite, then prove that $f(X)$ is a convex function.

3. If the primal problem contains an equation as constraint, then what about its dual ? Explain with an example.

4. State and prove max- flow min- cut theorem

5. Explain Personnel Assignment Problem

6. Explain Caterer problem

7. Define the concept of arborescence with an example

8. Write a note on sensitivity analysis

Part B

Answer any two questions from each unit.
Each question carries 2 weightage.

Unit I

9. Let $f(X)$ be a convex differentiable function defined in a convex domain $K \subseteq E_n$. Then $f(X_0), X_0 \in K$, is a global minimum if and only if $(X - X_0)'\nabla f(X_0) \geq 0$ for all X in K.

10. Solve using Simplex method:
    Maximize $f(X) = 4x_1 + 5x_2$.
    Subject to $x_1 - 2x_2 \leq 2$,
    $2x_1 + x_2 \leq 6$,
    $x_1 + 2x_2 \leq 5$,
    $-x_1 + x_2 \leq 2$,
    $x_1, x_2 \geq 0$.

11. Let $K \subseteq E_n$ be a convex set, $X \in K$, and $f(X)$ a convex function. Then if $f(X)$ has a relative minimum, then prove that it is also a global minimum. Also if this minimum is attained at more than one point, then prove that the minimum is attained at the convex linear combination of all such points.

Unit II

12. Solve using Dual Simplex method:
    Minimize $x_1 + 3x_2 + 2x_3$
    Subject to $4x_1 - 5x_2 + 7x_3 \leq 8$,
    $2x_1 - 4x_2 + 2x_3 \geq 2$,
    $x_1 - 3x_2 + 2x_3 \leq 2$,
    $x_1, x_2, x_3 \geq 0$.

13. Prove that if the primal problem is feasible, then it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.

14. Solve the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table.

	D_1	D_2	D_3	
O_1	3	5	7	30
O_2	4	1	3	40
O_3	3	6	2	25
O_4	5	3	4	65
	45	60	55	

Unit III

15. A Project consists of activities $A, B, C, ...M$. In the following data $X - Y = c$ means Y can start after c days of the work on$X$. A, B, C start simultaneously and $K$and$M$ are the last activities and take 14 and 13 days respectively. $A - D = 4, B - F = 6, B - E = 3, C - E = 4, D - H = 5 D - F = 3, E - F = 10, F - G = 4, G - I = 12, H - I = 3, H - J = 3, J - K = 8, I - K = 7, I - L = 7, L - M = 9$

Find the least time of completion of the project.

16. Solve graphically the LP problem: maximize $f = 4x_1 + 8x_2$, subject to $x_1 + 2x_2 \geq 20$, $2x_1+2x_2 \leq 100$, $x_1-3x_2 \leq 0$, $4x_1-x_2 \geq 0$, $x_1 \geq 0$, $x_2 \geq 0$.. Also analyse graphically how the optimal solution is modified when the following changes are introduced in the problem,

i)right hand side of second constraint is changed to 50

ii) the coefficient of x_2 in the constraints are changed from $(2, 2, -3, -1)$ to $(2, 1, -2, -1)$

17. Solve graphically the game whose payoff matrix given by $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$

Part C

Answer any two questions.
Each question carries 5 weightage.

18. The manager of an agricultural farm of 80 hectares learns that for effective protection against insects, he should spray atleast 15 units of chemical A and 20 units of chemical B per hectare. Three brands of insecticides are available in the market which contain these chemicals. One brand contains 4 units of A and 8 units of B per kg and costs Rs 5 per kg, the second brand contains 12 and 8 units respectively and costs Rs 8 per kg, and the third contains 8 and 4 units respectively and costs Rs 6 per kg. It is also learnt that more than 2.5 kg per hectare of insecticides will be harmful to the crops. Determine the quantity of each insecticide he should buy to minimize the total cost for the whole farm.

19. Solve using Dual Simplex method:

Minimise $f = 3x_1 + 5x_2 + 2x_3$

Subject to $-x_1 + 2x_2 + 2x_3 \geq 3$,

$x_1 + 2x_2 + x_3 \geq 2$,

$-2x_1 - x_2 + 2x_3 \geq -4$

$x_1, x_2, x_3 \geq 0$

20. Find the maximum flow in the graph G with arcs and arc lengths given as follows:

Note that a and b are the initial and final vertices respectively and the arc lengths are the capacities.

Arcs	(a,1)	(a,2)	(a,3)	(1,4)	(1,5)	(1,6)	(2,4)	(2,6)
Length	3	2	1	1	4	2	2	1
Arcs	(3,5)	(3,6)	(4,3)	(4,b)	(5,2)	(5,b)	(6,b)	
Length	1	1	2	0	1	5	2	

21. (*a*) State and prove the Minimax theorem.

(*b*) Define a saddle point of a function. Let $f(X, Y)$ be such that both $\max_X \min_Y f(X, Y)$ and $\min_Y \max_X f(X, Y)$ exist. Then prove that the necessary and sufficient condition for the existence of a saddle point (X_0, Y_0) of $f(X, Y)$ is that $f(X_0, Y_0) = \max_X \min_Y f(X, Y) = \min_Y \max_X f(X, Y)$.

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Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, April 2025

MMT2C06 – Algebra II

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A

Answer all questions. Each carries 1 weightage

1. Let R be a finite commutative ring with unity. Show that every prime ideal in R is a maximal ideal.
2. Show that $\sqrt{\sqrt[3]{2} - i}$ is algebraic over Q.
3. State Fundamental theorem of Algebra.
4. Find the number of primitive 8th roots of unity in GF(9).
5. Let $\sigma : Q(\sqrt{2}) \to Q(\sqrt{2})$ be defined by $\sigma(a + b\sqrt{2}) = 1 + b\sqrt{2}$, where $a, b \in Q$. Verify whether σ is an automorphism of $Q(\sqrt{2})$.
6. Find all conjugates of $\sqrt{2} + i$ over R.
7. Is regular 7-gon is consructible?. Justify your answer.
8. Define finite normal extesion of F and give example.

(8 × 1 = 8 weightage)

Part B

Answer any two questions from each unit. Each carries 2 weightage

Unit I

9. Let E be a simple extension F(α) of a field F, and let α be algebraic over F. Let the degree of irr(α, F) be $n \geq 1$. Show that every element β of E = F(α) can be uniquely expressed in the form $\beta = b_0 + b_1\alpha + ... + b_{n-1}\alpha^{n-1}$, where the b_i are in F.

10. Prove that an ideal $\langle p(x) \rangle \neq \{0\}$ of F[x] is maximal if and only if $p(x)$ is irreducible over F.

11. Prove that trisecting the angle is impossible.

Unit II

12. Let p be a prime and let $n \in Z^+$. If E and E$'$ are fields of order p^n, then prove that E $\simeq$ E$'$.

13. State and prove the Conjugation Isomorphism theorem.

14. Find the spliting field of $\{x^3 - 2\}$ over Q.

Unit III

15. State Main Theorem of Galois Theory.

16. Prove that the Galois group of the p^{th} cyclotomic extension of Q for a prime p is cyclic of order $p - 1$.

17. Find $\Phi_8(x)$ over Z_3.

($6 \times 2 = 12$ weightage)

Part C

Answer any two questions. Each carries 5 weightage

18. (a) Show that if E is finite extension field of a field F and K is a finite extension field of E, then K is a finite extension of F, and $[K : F] = [K : E][E : F]$.

    (b) Find a basis of $Q(\sqrt{2}, \sqrt{3})$ over Q.

19. (a) Prove that every field of characteristic zero is perfect.

    (b) Prove that a finite extension of a field of characteristic zero is a simple extension.

20. State and prove Isomorhism Extension theorem.

21. Let K be the splitting field of $x^4 + 1$ over Q.

    (a) Prove that

    i. Show that $[K : Q] = 4$

    ii. G(K/Q) is isomorhic to Kline 4-group.

    (b) Find an intermediate field E with $Q \leq E \leq K$ such that $[E : Q] = 2$.

($2 \times 5 = 10$ weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, April 2025

MMT2C07 – Real Analysis II

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A

Answer ALL questions. Each question carries 1 weight.

1. Define a σ-algebra of sets. Give one example.

2. If $A \subseteq B$ are sets, prove that $m^*(A) \leq m^*(B)$.

3. Prove that the characteristic function of a measurable set is a measurable function.

4. Let $|f|$ be an integrable function on $[0,1]$. Prove/Disprove that f is integrable on $[0,1]$.

5. Define f^+ of an extended real valued function f on a set E.

   Find f^+ where $f(x) = \sin x$ on $[0,2\pi]$.

6. State the General Lebesgue Dominated Convergence Theorem.

7. Define the total variation of a real valued function on an interval. Give one example.

8. Prove that a Lipschitz function is absoltely continuos on $[a,b]$.

$8 \times 1 = 8$ Weights.

Part B

Answer any TWO questions from each UNIT. Each question carries 2 weights.

Unit I

9. Prove that a set having zero outer measure is a measurable set.

10. Let $\{B_k\}_{k=1}^{\infty}$ be a descending sequence of measurable sets.

    Is it true that $m\left(\cap_{k=1}^{\infty} B_k\right) = \lim_{k\to\infty} m(B_k)$? Justify your answer.

11. If f and g are measurable functions that are finite almost everywhere on a measurable set E, then prove that fg is measurable on E.

Unit II

12. State and prove Fatou's lemma. Give example for strict inequality.

13. Let f be integrable over E. If $\{E_n\}_{n=1}^{\infty}$ is an increasing sequence of measurable subsets of E, then prove that $\int_{\bigcup_{n=1}^{\infty} E_n} f = \lim_{n\to\infty} \int_{E_n} f$.

14. State and prove Vitali Convergence theorem.

Unit III

15. State and prove Jordan's theorem.

16. Let f be an absolutely continuous function on $[a,b]$.

Prove that f is differentiable almost everywhere on (a,b).

Also prove that the derivative f' is interable on $[a,b]$ and $\int_a^b f' = f(b) - f(a)$.

17. State and prove Jensen's inequality.

$6\times2 = 12$ Weights.

Part C

Answer any TWO questions. Each question carries 5 weights.

18. Prove that there exists non-measurable subsets of $\mathbb{R}$.

19. State and prove Egoroff's theorem.

20. Let f and g be bounded measurable functions on a set E of finite measure and let α and β be real numbers. Prove that $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.

If $f \leq g$ on E, then prove that $\int_E f \leq \int_E$.

21. If the function f is monotone on the open interval (a,b), then it is differentiable almost everywhere on (a,b).

$2\times5 = 10$ Weights.

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, April 2025

MMT2C08 – Topology

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer **all** *questions. Each question carries* **1** *weightage.*

1. Let (X, d) be a metric space. Show that given distinct points $x, y \in X$ there exists opens sets U, V such that $x \in U, y \in V$ and $U \cap V = \emptyset$.
2. Define usual topology, semi-open interval topology and their open sets on $\mathbb{R}$.
3. Prove that a subset A of a space X is dense in X iff for every nonempty open subset B of $X, A \cap B \neq \emptyset$.
4. Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.
5. Prove that every quotient space of a discrete space is discrete.
6. Let $f: X \to Y$be a continuous surjection. If X is connected then prove that Y is also connected.
7. Prove that the intersection of any family of boxes is a box and the intersection of finite number of large boxes is a large box.
8. Prove that the projection functions are open.

(8x1 = 8 weightage)

Part B

Answer any **two** *questions from each unit. Each question carries* **2** *weightages.*

UNIT- I

9. Let $(X, \mathcal{T})$ be a topological space and $\mathfrak{B} \subset \mathcal{T}$. Then prove that $\mathfrak{B}$ is a base for iff for any $x \in X$ and any open set G containing x, there exist $B \in \mathfrak{B}$ such that $x \in B$ and $B \subset G$.
10. Define hereditary property. Prove that second countability is a hereditary property.
11. For a subset A of a space X, prove that $\bar{A} = A \cup A'$.

UNIT- II

12. Prove that the product topology is the weak topology determined by the projection functions.

13. Let(X, d) be a compact metric space and let $\mathcal{U}$ be an open cover X. Then show that there exists a positive real number r such that for any $x \in X$ there exists$V \in \mathcal{U}$ such that $B(x, r) \subset V$.

14. Show that Lindelöf property is a weakly hereditary property.

UNIT- III

15. Prove that in a Hausdorff space, limits of sequences are unique.

16. Define regular space and prove that regularity is a hereditary property.

17. Let $X = \prod_{i \in I} X_i$, each X_i being a topological space. Suppose $\{x_n\}$ is a sequence in X and that $x \in X$. Then prove that $\{x_n\}$ converges to x iff for each $i \in I$, the sequence $\{\pi_i(x_n)\}$ converges to $\pi_i(x)$ in X_i.

(6x2 = 12 weightages)

Part C

Answer any* two *questions. Each question carries* 5*weightages.

18. (a) Le t X be a set and $\mathfrak{D}$ a family of subsets of X. Then prove that there exists a unique topology $\mathcal{T}$ on X, such that it is the smallest topology on X containing $\mathfrak{D}$.

    (b) Prove that metrisability is a hereditary property.

19. (a)Let $(X, \mathcal{T}), (Y, \mathcal{U})$ be spaces and $f: X \to Y$ a function and $x_0 \in X$. Then show that the following are equivalent. (i)f is continuous at x_0. (ii) the inverse image of everyneighbourhood of $f(x_0)$ in Y is a neighbourhood of x_0 in X .

    (iii) For every subset $A \subset X, x_0 \in \bar{A} \Rightarrow f(x_0) \in \overline{f(A)}$.

    (b) Prove that composition of continuous functions is continuous.

20. (a) Prove that every closed and bounded interval is compact.

    (b) Let X be a space and C be a connected subset of X. Suppose $C \subset A \cup B$ where A, B are mutually separated subsets of X. Then show that either $C \subset A$ or $C \subset B$.

21. (a) Prove that a topological space X is T_1 if and only if for any $x \in X$, the singleton set $\{x\}$ is closed.

    (b) If X is a regular topological space then prove that for any $x \in X$ and any open set G containing x there exists an open set H containing x such that $\overline{H} \subset G$.

(2x5 = 10 weightages)

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Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, April 2025

MMT2C09 – ODE and Calculus of Variations

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A

Answer all questions. Each question carries one weightage

1. Locate and classify the singular points of the equation $x(x-1)^2y'' - xy' + y = 0$ on the x axis.
2. Find the indicial equation and roots of the equation $x(1-x)y''+[c-(a+b+1)x]y'-aby = 0$.
3. Show that $xF(1,1,2;-x) = log(1+x)$, where $F(a,b,c,x)$ is the hypergeometric series.
4. Show that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$. Here $J_n(x)$ is the Bessel function of the first kind of order n.
5. Describe the phase portrait of the system

$$\begin{cases} \frac{dx}{dt} = -2, \\ \frac{dy}{dt} = 2. \end{cases}$$

6. Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$, where $J_n(x)$ is the Bessel function of the first kind of order n.
7. Show that any nontrivial solution of equation $y'' + q(x)y = 0$, (where $q(x)$ is a positive function) on $[a,b]$ has atmost a finite number of zeros in this interval.
8. State Picard's theorem.

($8 \times 1 = 8$ weightage)

Part B

Answer any two questions from each unit. Each question carries 2 weightage

Unit I

9. Obtain the power series solution of the Legendre equation $(1-x^2)y''-2xy'+p(p+1)y = 0$, where p is a constant.
10. Find the general solution of Gauss's Hypergeometric equation near its singular point $x = 0$.
11. State and prove the orthogonality property of Legendre polynomials.

Unit II

12. Find the general solution of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ when p is not an integer .

13. Find the general solution of the following system:

$$\begin{cases} \frac{dx}{dt} = 5x + 2y, \\ \frac{dy}{dt} = -x + y. \end{cases}$$

14. Verify that $(0, 0)$ is a simple critical point for the system and determine its nature and stability properties:

$$\begin{cases} \frac{dx}{dt} = -x - y - 3x^2y, \\ \frac{dy}{dt} = -2x - 4y + y \sin x. \end{cases}$$

Unit III

15. State and prove Sturm Comparison theorem.

16. Find the extremal for the integral function of $\int_{x_1}^{x_2} y^2 - (y')^2 \, dx$

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17. Check whether $f(x, y) = x^2|y|$ satisfies a Lipshitz condition on the rectangle $|x| \leq 1$ and $|y| \leq 1$. Also check if $\frac{\partial f}{\partial y}$ exist at all points on this rectangle.

(6 × 2 = 12 weightage)

Part C

Answer any two questions. Each question carries 5 weightage

18. Obtain the power series solution of the equation $y'' + y' - xy = 0$.

19. Find two independent Frobenius series solutions of the equation $xy'' - y' + 4x^3y = 0$.

20. Define Liapunov function. Suppose there exist a Liapunov function $E(x, y)$ for the system

$$\begin{cases} \frac{dx}{dt} = F(x, y) \\ \frac{dy}{dt} = G(x, y) \end{cases}$$

then show that the critical point $(0, 0)$ is stable. Furthermore, if $\frac{dE}{dt}$ is negative definite, then show that the critical point $(0, 0)$ is asymptotically stable.

21. Obtain Euler's differential equation for an extremal.

(2 × 5 = 10 weightage)