1B4A25200 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Mathematics Degree Examination, April 2025

BMT4B04 – Linear Algebra

(2022Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A
All Questions can be attended.
(Each Question carries 2 marks-Ceiling marks 25)

1. Define diagonal matrix with an example.
2. If A is any square matrix, show that $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric.
3. Find the values of x, y and z that satisfies the matrix relationship
$$\begin{bmatrix} x+3 & 2y+4 \\ z+4 & 4x+5 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -4 & 2x+1 \end{bmatrix}$$
4. If $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find A^3.
5. Define Vector Space and give one example.
6. Find $W(x, sinx)$.
7. What are the subspaces of $\mathbb{R}^2$?
8. What is the maximum possible rank of an $m \times n$ matrix A , which is not square?
9. Define basis and give one example.
10. Express the product $AX = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, as the linear combination of the column vectors of A.
11. Define Rank and Nullity of a matrix A.
12. Define Reflection about $X-$ axes.
13. Define Dilation and Contraction.
14. Find the Eigen values of the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.
15. Prove that if A is Orthogonal, then $\det A = \pm 1$.

Section B
All Questions can be attended.
(Each Question carries 5 marks-Ceiling marks 35)

16. Solve the given system of linear equations by inverting the coefficient matrix,
$$x + y + z = 5$$
$$x + y - 4z = 10$$
$$-4x + y + z = 0$$

17. If A, B, C are Square Matrices of the same order. Then Prove that $tr(ABC) = tr(BCA) = tr(CAB)$.

18. Prove that $det\ A = (a-b)(b-c)(c-a)$, where $A = \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$

19. Show that $W = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R} : a_1x_1 + a_2x_2 + a_3x_{3+}a_4x_4 + a_5x_5 = 0\}$is a Subspace.

20. Show that $S = \{(1,1), (-1,2),\}$ is a Basis for $\mathbb{R}^2$.

21. Find the coordinate vector of $x^2 + 2x - 1$ with respect to the Basis $B = \{x + 1, x^2 + x - 1, x^2 - x + 1\}$ of P_2.

22. Let $T_1(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$ and $T_2(x_1, x_2) = (3x_1, 2x_1 + 4x_2)$

    i) Find the Standard Matrices for T_1 *and* T_2.

    ii) Find the Standard Matrix for $T_1 \circ T_2$.

23. Find the characteristic polynomial and eigen values of the Matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 17 & 8 \end{pmatrix}$

Section C
Answer any Two Questions
(Each Question carries 10 marks)

24. Solve the following System of Equations by Gaussian-Jordan elimination

$$x + 2y + z = 2,$$
$$3x + y - 2z = 1,$$
$$4x - 3y - z = 3,$$
$$2x + 4y + 2z = 4$$

25. Evaluate the determinant of the matrix by suitable row or column reduction

$$A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

26. Find the rank and nullity of the Matrix $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ 2 & -1 & 1 & -1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$

    Also, verify that $rank\ A + nullity\ A = No\ of\ columns\ of\ A$

27. Find the eigen values and basis for the eigen space corresponding to the eigen values of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

Section C
Answer any one of the question.
The question carries 10 marks

20. (a) Find the orthogonal trajectories of the curve $y = cx^2$.

    (b) Find the Wronskians of the functions $f(x) = x^k \cos 2\ln x$, $g(x) = x^k \sin 2\ln x$.

21. Find the Fourier series of the function $f(x) = x^2, 0 < x < 2\pi$.

1B4A25202 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Computer Science Degree Examination, April 2025

BMT4C04(CS) – Mathematics – 4

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks. Overall ceiling 20

1. Solve the initial value problem $\frac{dy}{dx} = -2xy,\ y(0) = 1.$
2. Determine the degree and order of the differential equation $y'' - [1 + (y')^2]^{\frac{3}{2}} = 0.$
3. Give example of a homogeneous linear ODE.
4. Verify that $y = \frac{1}{(x^2+C)}$ is a one-parameter family of solutions of the ODE $y' + 2xy^2 = 0.$ Determine from y the particular solution satisfying the initial condition $y(0) = -1.$
5. Find the Laplace transform of $cosh\ at.$
6. What is a Partial Differential Equation? Give Example.
7. Apply the operator $(D - I)^2$ to the function e^x
8. Verify that $u = \cos 2y\ \sinh 2x$ is a solution of the Laplace Equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$
9. Find the wronskian of the functions $y_1 = \cos \omega x$ and $y_2 = \sin \omega x.$
10. Solve $y'' - 3y' - 40 = 0$
11. Write the formula for Simpson's rule of integration.
12. Write the formula for one dimensional heat equation.

Section B

All questions can be attended.
Each question carries 5 marks. Overall ceiling 30.

13. Find the inverse Laplace Transform of $\frac{3s+7}{s^2-2s-3}$.

14. Evaluate $\int_0^1 x^2 dx$ using Trapezoidal rule with n = 10.

15. Find the Laplace Transform of the following piece-wise defined function

$$f(t) = \begin{cases} 2, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi. \\ \sin t, & t \ge 2\pi \end{cases}$$

16. What is an exact Differential Equation? Test for exactness and solve $\cos(x+y)\ dx + (3y^2 + 2y + \cos(x+y))\ dy = 0.$

17. Find a basis of solutions of the ODE $x^2y'' - xy' + y = 0$

18. Are the functions $5 \sin x \cos x$ and $3 \sin 2x$ linearly independent on the interval $x > 0$. Justify.

19. Find the Fourier series of the function $f(x) = x^2 (-\pi < x < \pi)$, which is assumed to have the period 2π.

Section C

Answer any one of the question.
The question carries 10 marks.

20. Solve the initial value problem $y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$, using Laplace Transform

21. Find an Integrating factor and solve the initial value problem $(e^{x+y} + ye^x)dx + (xe^y - 1)dy = 0,\ y(0) = -1.$