1B6A25309 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2025

BMT6E03 – Mathematical Programming with Python and Latex

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks.

1. Explain how variables store data values.
2. Explain the difference between s[:5] and s[0:5]
3. What is **List** in Python?
4. What are the uses of **break** statement in python?
5. Write a python program that uses **while** statement.
6. Write a python program to find the LCM of two numbers.
7. How do you reverse a string using slicing?
8. How do you create subplots using subplot() in python.
9. How to create 3D plots in python.
10. Generate a triangular wave using Fourier series
11. How to create lists (bulleted, numbered) in latex.
12. Write a latex code to get the output$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sinnxdx$

(Ceiling… 20 marks)

Section B

All questions can be attended.
Each question carries 5 marks.

13. Explain the concept of operator overloading in python.
14. Give an example for the usage of break in a for loop?
15. Write a Python program to convert Fahrenheit to Celsius
16. Write a Python program to open a file and write 'hello world, welcome to gen beta' to it
17. Write a Python program to calculate sine function using series expansion and plot it.

18. Write a Python program to plot ellipse in python.

19. Write a Latex program to get output as a two-way table to show the number of boys and girls studying in a School.

(Ceiling... 30 marks)

Section C
Answer any One Question.

20 .(a) Create a 3 × 2 matrix and print the sum of its elements using for loops.

(b) How can meshgrid be used for plotting 2D functions?

21.(a) Prepare a sample index using Latex.

(b) Write a Latex file to produce the following output (without frame).

D'Alembert's criterion

Consider the series $\sum a_n$ with $a_n \neq 0$

If $\lim \sup_{x\to\infty} \frac{|a_{n+1}|}{|a_n|} < 1$ the series $\sum a_n$ converges absolutely

If $\lim \inf_{x\to\infty} \frac{|a_{n+1}|}{|a_n|} > 1$ the series $\sum a_n$ diverges

(1× 10 = 10 marks)

1B6A25305 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2025

BMT6B10 – Real Analysis

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks

1. Give an example of continuous function which is not bounded.
2. Give an example to show that the continuous image of an open interval need not be an open interval.
3. Show that $f(x) = x^2$is uniformly continuous on $[-1,1]$.
4. Define non-uniform continuity criteria
5. State Weierstrass Approximation Theorem.
6. Show that step function is Reimann integrable.
7. State Substitution Theorem.
8. Find the m^{th} partial sum of the series $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$.
9. Discuss the convergence of $g_n(x) = x^n, x \in \mathbb{R}, n \in \mathbb{N}$.
10. State the Theorem on interchange of limit and continuity.
11. Test the uniform convergence of the series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots.$$

12. Test for convergence of the integral $\int_0^{\infty} e^{-x^2} dx$.
13. Find the Cauchy Principal value of $\int_{-1}^{5} \frac{dx}{(x-1)^3} dx$
14. Prove that Beta function is symmetric.
15. Find $\int_0^1 x^7(1-x)^8 dx$.

Ceiling – 25 Marks

Section B
All questions can be attended
Each question carries 5 marks

16. State and prove Bolzano Intermediate Value Theorem.

17. If f : [a, b] $\rightarrow \mathbb{R}$ is continuous on [a, b], then prove that f$\in \mathcal{R}$ [a,b].

18. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on(0,1]

19. Let (f_n) be a sequence of continuous functions on A$\subset \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function f:A$\rightarrow \mathbb{R}$. Then prove that f is continuous on A.

20. If f$\in \mathcal{R}$ [a, b] then prove that f is bounded on [a, b].

21. Show that the $\int_0^\infty \frac{3}{e^x+5} dx$ converges.

22. Discuss the convergence of $\int_0^\pi \frac{sinx}{x^3} dx$.

23. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Ceiling – 35 Marks

Section C
Answer any two questions
Each question carries 10 marks

24. (a) State and prove Uniform Continuity Theorem.

(b) If f:A$\rightarrow \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R

25. State and prove Cauchy's criterion for Riemann integrable functions.

26. (a) If f$\in \mathcal{R}$ [a, b], then prove that the value of the integral is uniquely determined.

(b) Prove that a sequence (f_n) of bounded functions on A $\subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$ as n $\rightarrow \infty$.

27. (a) If n is a positive integer then prove that $\Gamma(n) = (n-1)!$

(b)Evaluate $\int_0^1 x^4(1-x)^3 dx$ using Beta and Gamma function.

2×10 = 20 Marks

1B6A25306 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2025

BMT6B11 – Complex Analysis

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

SECTION A

Answer the following questions. Each carries two marks (Ceiling 25).

1. Define a differentiable complex function and Show that $f(z) = z^n, n \geq 1$, is an integer is differentiable for all z.
2. Define analytic function and singular point of a function with an example.
3. Suppose that $f(z)$ is analytic in a domain D, if $f'(z) = 0\ \forall z \in D$, prove that $f(z)$ is constant in D
4. If $f(z) = u + iv$ is analytic in a domain D then prove that its component functions u and v are harmonic in D
5. Evaluate $ln(-1-i)$ and $ln(-2i)$.
6. Evaluate $\int_C xydx + x^2dy$ where C is the graph of $y = x^3, -1 \leq x \leq 2$
7. Define complex valued functions and evaluate $\int_0^1 (1+it)^2 dt$
8. If f is an analytic function in a simply connected domain D and C is a contour in D then prove that $\int_C f(z)dz$ is independent of the path C.
9. Integrate $\int_C f(z)dz$ in the counter clockwise direction where $f(z) = \frac{e^z}{((z-i)^2(z^2+4)}dz$ where $C : \mid z \mid = 3/2$
10. Prove that the series $\Sigma_{k=1}^{\infty} z_k$ where $z_k = i^k/k^2$ converges.
11. Obtain the Maclaurin series representation of $cosz$.
12. Identify the type of singularity of $\frac{sinz}{z}$.

13. Determine whether $z = 0$ is an essential singularity of $f(z) = e^{(z+1)/z}$.

14. Determine the zeros and their order of the function

$$f(z) = (z + 2 - i)^2.$$

15. Find the residues at the singular point (a) $\frac{4}{1-z}$ (b) $\frac{\sin z}{z^4}$

SECTION B

Answer the following questions. Each carries five marks (Ceiling 35).

16. Show that $f(z) = u + iv$ where $u(x, y) = \frac{x}{x^2+y^2}, v(x, y) = \frac{-y}{x^2+y^2}$ is analytic in any domain D that does not contain the point $z = 0$.

17. Explain Cauchy Riemann Equation in polar co-ordinates.

18. Evaluate $\int_C (6x^2 + 2y^2)dx + 4xydy$ where C is given by $x = \sqrt{t},\ y = t, 4 \leq t \leq 9$

19. Show that if z_0 is any constant complex number interior to any simple closed contour C , then prove that $\oint_C \frac{1}{(z-z_0)^n} dz = 2\pi i$ if $n = 1$, and 0 otherwise .

20. Find all Maclaurin and Laurent series representation of the function $f(z) = \frac{-1}{(z-1)(z-2)}$ about $z = 0$.

21. Expand the Laurent series representation for $f(z) = \frac{1}{z(z-1)}$ in the region $1 < \mid z - 2 \mid < 2$

22. Find the residue at at the singular point (a) $f(z) = \frac{1}{z(e^z-1)^2}$ (b) $g(z) = \frac{1}{z^4+1}$.

23. Evaluate $\int_0^{2\pi} \frac{dz}{13-5\sin z}$.

SECTION C

Answer any two questions (2 x 10 = 20 Marks).

24. (a)If f is a analytic in a simply connected domain D, then prove that f has an antiderivative in D, that is there exist a function F such that $F'(z) = f(z)$.
(b) Evaluate $\int_0^{1+i} e^{\pi z} dz$.

25. Let f be analytic in a simply connected domain D and let C be a simple closed contour lying entirely in D. If z_0 is any point interior to C then prove that for $n = 1, 2, 3, \ldots$ the derivative of $f(z)$ exist and $f^{(n)}(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-z_0)^{n+1}} dz$.

26. State and prove Taylor's Theorem.

27. (a)State and prove Cachy's Residue Theorem.
(b)Evaluate $\oint_C \frac{dz}{z^3(z-1)}$ where C is the circle $\mid z \mid = 2$ by using Residue Theorem .

1B6A25307 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2025

BMT6B12 – Calculus of Multivariable – 2

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A

All questions can be attended. Each question carries 2 marks.

1. Find the equation of tangent and normal lines to the curve $x^4 - x^2 + y^2 = 0$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$.
2. Show that the point (0,0) is a critical point of $f(x,y) = y^2 - x^2$ but that it does not give rise to a relative extremum of f.
3. Evaluate $\int_0^1 \int_0^2 xy(x-y)dx\,dy$.
4. Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} dx\,dy$.
5. Evaluate $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2} dy\,dx$.
6. Find the mass of the lamina occupying a region R bounded by the graphs of $y = e^x, y = 0, x = 0$ and $x = 1$, if its mass density is $\rho(x,y) = 2xy$.
7. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dz\,dy\,dx$.
8. Find the image of the region $S = \{(u,v): 0 \le u \le 2, 0 \le v \le 1\}$ under the transformation $x = u - v, y = v$.
9. Sketch several vectors associated with the vector field $F = i + j$.
10. If $r = xi + yj + zk$, then prove that curl r = 0.
11. Prove that the vector field $3y^4z^2i + 4x^3z^3j - 3x^2y^2k$ is solenoidal.
12. State the fundamental theorem for line integrals.
13. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
14. Find the parametric representation for the cone $x = \sqrt{y^2+z^2}$.
15. State divergence theorem.

(Ceiling 25 Marks)

Section B

All questions can be attended. Each question carries 5 marks.

16. Find all relative maxima, relative minima and saddle points of the function

$$f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$$

17. Find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

18. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2}\, dy\, dx$

19. Find the area of the part of the plane $y + z = 2$ inside the cylinder $x^2 + z^2 = 1$

20. Compute the divergence and curl of the vector $F = xyzi + 3x^2yj + (xz^2 - yz^2)k$ at (1, 2, -1).

21. Evaluate $\int_C ydx + zdy + xdz$ where C consists of the line segment from (0, 0, 0) to (2, 3, 4) and from (2, 3, 4) to (6, 8, 5).

22. Find the flux of the vector filed $F(x,y,z) = y\,i + x\,j + 2z\,k$ across the unit sphere $x^2 + y^2 + z^2 = 1$.

23. Evaluate $\oint_C F.dr$, where $F(x,y,z) = \cos z\, i + x^2\, j + 2y\, k$ and C is the curve of intersection of the plane $x + z = 2$ and the cylinder $x^2 + y^2 = 1$.

(Ceiling 35 Marks)

Section C

Answer any Two Questions. Each question carries 10 marks.

24. Find positive numbers x, y, z such that$x + y + z = 18$ and xyz is a maximum.

25. Find the centre of mass of the solid T of uniform density bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = z$.

26. $F(x,y) = 2xy\,i + (1 + x^2 - y^2)\,j$. Show that Fis conservative, and find a potential function. Also evaluate the work done by F in moving a particle along any path from $(1,0)$ to (2,3).

27. State and Prove Green's theorem in the plane.

(2 x 10 = 20 Marks)

1B6A25308 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2025

BMT6B13 – Differential Equations

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A

All questions can be attended. Each question carries 2 marks.

1. Determine the values of r for which the differential equation $y' + 2y = 0$ has solutions of the form $y = e^{rt}$.
2. Solve $y' = \frac{x^2}{y}$.
3. Find an integrating factor for the equation $(2x + 3)y' + y = x$.
4. Find the value of b for which the equation $(xy^2 + bx^2y)dx + (x + y)x^2dy = 0$ is exact.
5. Find the general solution of $y'' + 5y' + 6y = 0$.
6. Write the standard form of second-order linear nonhomogeneous differential equation and state the existence and uniqueness theorem for second-order linear differential equations.
7. State Abel's theorem.
8. Find the Wronskian of $y_1 = e^{-2t}, y_2 = t^2e^{-2t}$.
9. Find the general solution of $y'' + 8y' + 16y = 0$.
10. Define unit step function and write its Laplace transform.
11. Find $L^{-1}\left[\frac{2\pi}{s+\pi}\right]$
12. Find the Laplace transform of $\cosh(2t) + t^2 + t - 1$
13. Find the Laplace transform of $\{e^{ct}\cos bt\}$.
14. Show that the product of two even function is even
15. Write the formulae for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period $2L$.

(Ceiling 25 Marks)

PARTB

***All* the questions can be attended. Each question carries 5 marks.**

16. Solve the IVP $y' = (1-2x)y^2$, $y(0) = -\frac{1}{6}$ and determine the interval in which the solution exists.
17. Solve the differential equation $(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$.
18. Use the method of reduction of order to find a second solution of the differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0, t > 0$, $y_1(t) = t$.
19. Solve the differential equation $y'' + 4y' + 4y = \frac{e^{-2t}}{t^2}$ by method of variation of parameters.
20. Using the definition of Laplace transform, find $L[e^{iat}]$.
21. Using convolution property, find $L^{-1}\left[\frac{1}{(s+1)(s-2)}\right]$.
22. Express the function $f(x) = x$, when $-1 < x < 1$ as a Fourier series with period 2.
23. Replace the PDE $u_{xx} + u_{yy} = 0$ by a pair of ordinary differential equations using the method of separation of variables.

(Ceiling 35 Marks)

PART C

Answer any *two* questions. Each question carries 10 marks.

24. (a) Solve the IVP $y' - y = 2te^{2t}, y(0) = 1$.

    (b) Solve the differential equation $ydx + (x^2y - x)dy = 0$.
25. Find the general solution of the differential equation $y'' - 4y = te^t + \cos 2t$.
26. Using Laplace transformation solve $y'' + 2y = r(t)$, $y(0) = 0$, $y'(0) = 0$

    where $r(t) = \begin{cases} 1 \text{ if } t \geq 1 \\ 0 \text{ if } t < 1 \end{cases}$
27. Find the Fourier series for the function $f(x) = \begin{cases} 2 + x, \text{ when } -2 \leq x \leq 0 \\ 2 - x, \text{ when } 0 < x \leq 2 \end{cases}$ and $f(x+4) = f(x)$.

(2 x 10 = 20 Marks)