2B1N23292

(Pages : 2)

Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2023

MMT1C01 – Algebra – I

(2022 Admission onwards)

Time : 3 hours

Max. Weightage : 30

Part A

Answer all questions. Each carries 1 weightage

1. Verify whether $\phi(x,y) = (-y,x)$ is an isometry of the Euclidean plane $\mathbb{R}^2$.
2. Find the order of the factor group $\dfrac{\mathbb{Z}_4 \times \mathbb{Z}_{12}}{(\langle 2\rangle \times \langle 2\rangle)}$.
3. State The Fundamental Homomorphism Theorem.
4. Find the reduced form and the inverse of the reduced form of the word $a^2b^{-1}b^3a^4c^4c^2a^{-1}$.
5. Define Solvable group.
6. Show that $(x,\ y : y^2x = y,\ yx^2y = x)$ is a presentaion of the trivial group of one element.
7. Let $\phi_\pi :\ \mathbb{Q}[x] \to \mathbb{R}$ be the evaluation homomorpism with $\phi_\pi(x) = \pi$. Find the Kernel of ϕ_π.
8. Find the inverse of $2i + j + k$ in the ring of quaternions.

($8 \times 1 = 8$ weightage)

Part B

Answer any two questions from each unit. Each carries 2 weightage

Unit I

9. Show that a subgroup M of a group G is a maximal normal subgroup of G if and only if G/M is simple.
10. Let X be a G–set. For $x \in X$ let $G_x = \{g \in G\ : gx = x\}$. Prove that G_x is a subgroup of G.

11. Find the order of the element $(2, 1) + \langle(1, 1)\rangle$ in the factor group $(\mathbb{Z}_3 \times \mathbb{Z}_6)/\langle(1, 1)\rangle$.

Unit II

12. Prove that Z has no composition series.

13. State and prove First Sylow Theorem.

14. Prove that for a prime number p, every group G of order p^2 is abelian.

Unit III

15. State and prove Eisenstein criterion for irreducibility of a polynomial.

16. Show that the multiplicative group $< F^*, . >$ of nonzero elements of a finite field F is cyclic.

17. Let R be a commutative ring and let $a \in R$. Show that $I_a = \{x \in R | ax = 0\}$ is an ideal of R.

($6 \times 2 = 12$ weightage)

Part C

Answer any two questions. Each carries 5 weightage

18. (a) Show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if gcd of m and n is 1.

    (b) Prove that $\prod_{i=1}^{n} \mathbb{Z}_{m_i}$ is cyclic and isomorphic to $\mathbb{Z}_{m_1 \ldots m_n}$ if and only if the numbers m_i for $i = 1, 2, \ldots n$ are mutually relatively prime.

19. (a) State and prove Burnside's Formula.

    (b) Let p be a prime. Let G be a finite group and let p divide $|G|$. Prove that G has an element of order p.

20. (a) Let H, K be normal subgroups of a group G such that $K \leq H$. Show that G/H is isomorphic to $(G/K)/(H/K)$.

    (b) Prove that the polynomial $\phi_p(x) = \dfrac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \ldots\ldots x + 1$ is irreducible over Q for any prime p.

21. (a) State and prove division algorithm for $F[x]$, where F is a field.

    (b) Show that the multplicative group of all non zero elements of a finite field is cyclic.

($2 \times 5 = 10$ weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2023

MMT1C02 – Linear Algebra

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A
Answer all questions.
Each carries 1weightage

1. Describe explicitly the linear transformation T from F^2 in to F^2 such that $T\varepsilon_1=(a,b)$, $T\varepsilon_2=(c,d)$.
2. Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$ which are continuous. Let T be the linear operator on V defined by $(Tf)(x)=\int_0^x f(t)dt$. Prove that T has no characteristic values.
3. Prove that the vector space of polynomials of degree less than or equal to three over R is isomorphic to R^4.
4. Let $\alpha_1=(1, 0, -1, 2)$ and $\alpha_2=(2, 3, 1, 1)$ and let W be the subspace of $\mathbb{R}^4$ spanned by α_1 and α_2, which linear functional of the form $f(x_1, x_2, x_3, x_4) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$ are in the annihilator of W?
5. Let W be an invariant subspace for T. Then prove that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T
6. Let W_1 and W_2 be subspaces of a finite dimensional vector space V. Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.
7. Prove that every finite dimensional inner product space has an orthonormal basis.
8. Prove that an orthogonal set of non zero vectors in an inner product space is linearly Independent.

(8x1=8weightage)

Part B

Answer any two questions from each unit.
Each carries 2 weightage

Unit -1

9. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

10. Let V be a n- dimensional vector space over the field F, and let $B = \{\alpha_1, \dots \alpha_n\}$ and $B' = \{\alpha'_1 \dots \alpha'_n\}$ be two ordered bases of V. then prove that there is a unique, necessarily invertible, nxn matrix P with entries in F such that $[\alpha]_B = P[\alpha]_{B'}, [\alpha]_{B'} = P^{-1}[\alpha]_B$ for every vector $\alpha \in V$.

11. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. suppose V is finite dimensional then prove that rank (T) + nullity (T) = dim V.

Unit -2

12. If W is the subspace of $\mathbb{R}^5$ which is spanned by the vectors $\alpha_1 = (2, -2, 3, 4, -1)$, $\alpha_2 = (-1, 1, 2, 5, 2)$, $\alpha_3 = (0, 0, -1, -2, 3)$ and $\alpha_4 = (1, -1, 2, 3, 0)$ then find the annihilator of W.

13. Let T be a linear operator on the finite dimensional space V. Let $c_1, c_2, \dots c_k$ be the distinct characteristic vector of T and let W_i be the space of characteristic vector associated with the characteristic value c_i . If $W = W_1+W_2+\dots\dots\dots+W_k$ then prove that $\dim W = \dim W_1 + \dim W_2 + \dots\dots + \dim W_k$

14. Let V and W be vector spaces over the field F. Let B be the ordered basis for V with dual basis B^* and let B' be an ordered basis for W with dual basis B'^*. Let T be a linear transformation from V in to W: Let A=$[T]_{BB'}$ and let B=$[T^t]_{B'^*B^*}$. Then Prove that $B_{ij} = A_{ji}$.

Unit -3

15. (a) let T be a linear operator on a finite dimensional space V. if T is diagonalizable and $c_1, c_2, \dots c_k$ be the distinct characteristic vector of T then prove that there exist linear operators $E_1, E_2, \dots, E_k$ on V such that

    (i) $E_i E_j = 0, i \neq j$

    (ii) E_i is a projection

    (iii) The range of E_i is the characteristic space for T associated with c_i

16. Apply Gram-Schmidt process to the vectors β_1=(1,0,1), β_2=(1,0,-1), β_3=(0,3,4) to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

17. Let W be a subspace of an innerproduct space V and Let β be a vector in V then prove that the vector $\alpha \in W$ is a best approximation to β by vectors in W if and only if $\beta - \alpha$ is orthogonal to every vector in W.

(6x2=12 weightage)

Part C
Answer any two questions
Each carries 5 weightage

18. (a) Let V be a finite dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\beta_1, \beta_2, \dots, \beta_n$ be any vectors in W then prove that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j, j = 1,2, \dots, n$

(b) If W is a subspace of a finite dimensional vector space V, prove that every linearly independent subset of W is finite and is part of a basis for W.

19. (a) Let the linear operator on $\mathbb{R}^3$ which is representation the standard ordered basis by the matrix.

A= $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ check whether T is diagonalizable or not

(b) Let g, $f_1, \dots f_r$ be linear functional on a vector space V with respective null spaces N, $N_1, N_2, \dots, N_r$ then prove that g is a linear combination of $f_1, \dots f_r$ iff N contains the intersection $N \cap N_1 \cap N_2 \cap \dots \cap N_r$

20. Let T be a linear operator on a finite dimensional vectorspace V. If f is the characteristic polynomial for T, then prove that f(T)=0.

21. (a)Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then Prove that E is a Idempotent linear transformation of V onto W, $W^\perp$ is the null space of E and V=W$\oplus$ $W^\perp$

(b) State and prove Bessel's inequality.

(2x5=10 weightage)

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Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2023

MMT1C03 – Real Analysis – I

(2022 Admission onwards)

Time : 3 hours

Max. Weightage : 30

Part-A

Answer all questions. Each question carries 1 weightage

1) Prove that every infinite subset of a countable set is countable.
2) Let X be a set of cardinality 2 and A be the set of all sequences whose terms are elements of X. Prove that A is uncountable.
3) Give an example of a continuous bounded function on the segment $(1,2)$ which has no maximum. Justify your answer.
4) Check the differentiability of the function $f(x)=\begin{cases} x\sin\frac{1}{x} & (x\neq 0) \\ 0 & (x=0)\end{cases}$.
5) Let $f(x)=\begin{cases} 1 & (if\ x\ is\ rational) \\ 0 & (if\ x\ is\ irrational)\end{cases}$. Prove that $f\notin\mathcal{R}$ on $[0,1]$.
6) Let P be a partition of $[a,b]$ and P^* is a refinement of P. Prove that $L(P,f,\alpha)\leq L(P^*,f,\alpha)$.
7) Let $f_n(x)=\frac{x}{1+nx^2}$. Prove that $\{f_n\}$ converges uniformly on $[0,1]$.
8) Define equicontinuous family of functions and give one example.

Part-B

Answer any two questions from each unit. Each question carries 2 weightage

Unit - I

9) Prove that a subset E of a metric space X is open if and only if E^c is closed.
10) Suppose $K\subset Y\subset X$. Prove that K is compact relative to X if and only if K is compact relative to Y.
11) Prove that continuous image of a compact set is compact.

Unit - II

12) Suppose f is a continuous mapping of $[a,b]$ into $\mathbb{R}^k$ and f is differentiable in (a,b). Prove that there exists $x\in(a,b)$ such that $|\mathbf{f}(b)-\mathbf{f}(a)|\leq(b-a)|\mathbf{f}'(x)|$.

13) Prove that if f is continuous on $[a, b]$ and α is monotonically increasing on $[a, b]$, then f is integrable with respect to α in the Riemann sense.

14) If $f_1 \in \mathcal{R}(\alpha)$ and $f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$, prove that $f_1 + f_2 \in \mathcal{R}(\alpha)$ and

$$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$$

Unit - III

15) If f maps $[a, b]$ inot $\mathbb{R}^k$ and $\mathbf{f} \in \mathcal{R}(\alpha)$ for some monotonically increasing function α, prove that $|\mathbf{f}| \in \mathcal{R}(\alpha)$ and

$$\left| \int_a^b \mathbf{f} d\alpha \right| \leq \int_a^b |\mathbf{f}| d\alpha$$

16) Suppose $f_n \rightarrow f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that for each n, $\lim_{t\to x} f_n(t) = A_n$. Prove that $\{A_n\}$ converges and

$$\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$$

17) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

Part-C

Answer **any two** questions. Each question carries **5 weightage**

18) a) If f is a continuous mapping of a compact metric space X into a metric space Y, prove that f is uniformly continuous on X.

b) Let f be monotonic on (a, b). Prove that the set of points in (a, b) at which f is discontinuous is at most countable.

19) a) State and prove Taylor's theorem

b) Suppose $f \in \mathcal{R}(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Prove that $h \in \mathcal{R}(\alpha)$ on $[a, b]]$

20) a) Prove that if f is continuous on $[a, b]$ and α is monotonically increasing on $[a, b]$, then $f \in \mathcal{R}(\alpha)$.

b) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0 \in [a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$; $a \leq x \leq b$

21) State and prove the Stone-Weierstrass theorem.

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2023

MMT1C04 – Discrete Mathematics

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

(Short Answer Questions)

(Answer all questions. Each question has weightage 1)

1. Prove that the intersection of two chains is a chain but that their union need not be a chain.
2. Let (X, + , . ,′) be a Boolean algebra. Prove that x + 1 = 1 for all x∈X.
3. Let $A = \{1, 2, ..., 12\}$ and a ≤ b if a|b . Find a maximal and minimal elements of the set $B = \{2,3\}$.
4. Prove that every graph has an even number of vertices of odd degree.
5. Define identity graph and give an example.
6. Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of G.
7. Find a dfa that accepts all strings on {0,1} starting with prefix 01.
8. Define a language L and L*, the star closure of L. Give an example.

(8x1=8 weightage)

Part B

(Answer any two question from each unit.Each question carries weightage 2)

Unit I

9. Let X = RU{*} where * is some point not on the real line. Define ≤ on X as {(x,y)∈ ℝ X ℝ, x ≤ y in the usual order }U{(*,*)}. Prove that ≤ is a partial order on X.
10. State and prove Stone representation theorem for finite Boolean algebra.
11. Write the following Boolean function in the disjunctive normal form

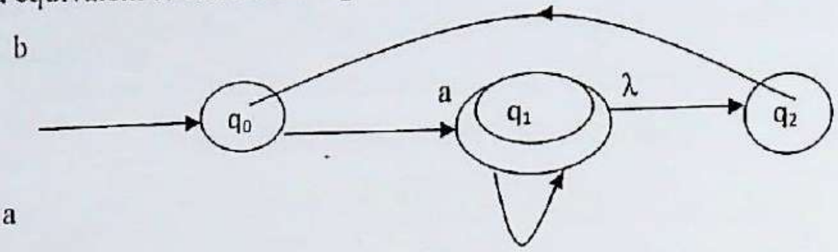
$$F(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$$

Unit II

12. If G is a simple graph , then prove that $\kappa(G) \le \lambda(G) \le \delta(G)$.
13. Prove that an edge is a cut edge if and only if it belongs to no cycle.
14. Prove that K_5 is non-planar.

Unit III

15. Find a dfa equivalent to the following nfa



16. Find a grammer that generate the language $L = \{a^{n+2}b^{2n} : n \geq 1\}$.

17. Show that the language L={awa:w∈ {a,b}*} is regular.

(6x2=12 weightage)

Part C

Answer any two from the following four questions. Each question has weightage 4

18. (a)Prove that every Boolean functions of n variables $x_1, x_2, \ldots x_n$ can be uniquely expressed as a sum of terms of the form $x_1^{\varepsilon_1} x_2^{\varepsilon_2} \ldots x_n^{\varepsilon_n}$ where each $x_i^{\varepsilon_i}$ is x_i or x_i'.

    (b) Prove that the set of all symmetric Boolean functions of n Boolean variables $x_1, x_2 \ldots, x_n$ is a sub algebra of the Boolean algebra of all Boolean functions of these variables . Also prove it is isomorphic to the power set Boolean algebra of the set {0,1,….,n}.

19. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

    (b)Prove that for a cubic graph vertex connectivity and edge connectivity are equal.

20. (a)Prove that every tree with at least two vertices has at least two pendant vertices.

    (b)Let G be a connected graph. Prove that the following statements are equivalent

    (i) G is Eulerian.

    (ii) The degree of each vertex of G is an even positive integer.

    (iii) G is an edge –disjoint union of cycles.

21. (a) Are the grammars G1 = ({S}, {a,b}, S, { S→SS, S→aSb, S→ λ , S→bSa}) and G2 =({S}, {a,b}, S, { S→SS, S→SSS, S→aSb, S→bSa, S→ λ }) are equivalent.

    (b) Let L be the language accepted by a non deterministic finite accepter $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Then prove that there exist a dfa $M_D=(Q_D, \Sigma, \delta_D, \{q_0\}, F_D\}$ such that $L = L(M_D)$.

(2x5=10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2023

MMT1C05 – Number Theory

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Answer *all* questions. Each carries 1 weightage

1. Show that $\sum_{d/_n} \mu(d) = \left[\frac{1}{n}\right] = \begin{cases} 1 & if\ n = 1 \\ 0 & if n > 1 \end{cases}$
2. Define Mangolt function $\Lambda(n)$ and show that for $n \geq 1$, $\log n = \sum_{d/_n} \Lambda(d)$.
3. Give an example of a multiplicative function which is not completely multiplicative.
4. Derive Selberg identity
5. Calculate the highest power of 10 that divides 1000!
6. Determine whether 888is a quadratic residue or non residue modulo of the prime 1999..
7. Prove that product of two shift enciphering transformations is also shift enciphering transformations .
8. Write a short note on cryptosystem.

($8 \times 1 = 8$ weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Show that if f is an arithmetical function with $f(1) = 0$, then there is a unique arithmetical function f^{-1} such that $(f * f^{-1})$=$(f^{-1} * f) = 1$
10. State and prove the Mobius inversion formula.
11. State and prove the Euler's Summation formulae.

Unit 2

12. Show that for $x \geq 2$; $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p \;=\; x \log x - x + O(\log x)$.

13. Show that for $n \geq 1, the\ n^{th}\ prime\ \ P_n$ satisfies the inequalities

$$\frac{1}{6}\ nlogn < P_n \ < 12\left(n \log n + n \log \frac{12}{e}\right)$$

14. State and prove the Abel's identity

Unit 3

15. State and prove Gauss lemma

16. Explain the advantages and disadvantages of public key cryptosystems as compared to classical cryptosystems.

17. Solve the following system of simultaneous congruences $17x + 11y \equiv 7(mod\ 29)$ $13x + 10y \equiv 8(mod\ 29)$.

(6 × 2 = 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage

18. (a) State and Prove Chinese Reminder Theorem.

    (b) Show that $\frac{1}{6}\frac{n}{\log n} < \pi(n) < \frac{6n}{\log n}\ , for\ every\ integer\ n \geq 2$.

19. (a) State and Prove Quadratic reciprocity law.

    (b) Prove that 5 is a quadratic residue of an odd prime p if $p \equiv \pm 1(mod\ 10)$.

20. Let P_n denotes the n^{th} prime. Prove the following are equivalent:

    (a). $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$

    (b). $\lim_{x\to\infty} \frac{\pi(x)\log \pi(x)}{x} = 1$

    (c). $\lim_{n\to\infty} \frac{P_n}{n \log n} = 1$

21. (a) Find the inverse of $A = \begin{bmatrix} 15 & 17 \\ 4 & 9 \end{bmatrix} \in M_2\left(\frac{Z}{26\,Z}\right)$

    (b)Describe algorithm for (finding the discrete logs in the finite fields

(2 × 5 = 10 weightage)