58

1M1N24069 (Pages : 2) Reg. No:........................

Name: ....

FARSOOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2024

MST1C01 – Analytical Tools for Statistics – I

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

(Answer any four questions. Weightage 2 for each question)

1. Explain Linearity Properties of Reimann-Stieltjes Integral.
2. Define pointwise convergence. Give example.
3. How do you define the limit of a multivariable function at a given point?
4. Define Total Derivative.
5. What is meant by inverse Laplace Transform of a function?
6. Find Laplace transform of $f(t) = e^{5t}$
7. Define Riemann - Stieltjes integral.

(2x4=8 weightage)

Part B

(Answer any four questions. Weightage 3 for each question)

8. State and prove mean value theorem on Rieman-Stieltjes integral.
9. Let $f_n(x) = \frac{x}{n}$ for $x \in [0,1]$. Determine whether the sequence converges pointwise and uniformly. Justify your answer.
10. If f is a monotonic function on $[a, b]$ and g is continuous on $[a, b]$ show that $\int_a^b f dg$ exists.
11. Prove that if a sequence of functions f_n converges uniformly to f on a closed interval $[a, b]$ and each f_n is continuous, then f is also continuous on $[a, b]$.
12. Discuss the continuity of $f(x, y) = \begin{cases} \frac{x^2-y^2}{x+y} & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$
13. State and prove Taylor's theorem.
14. Find inverse Laplace transform of $\frac{12}{(s-3)^4}$.

(3x4=12 weightage)

Part C

(Answer any two questions. Weightage 5 for each question)

15. State the necessary and sufficient conditions for the existence of the Riemann-Stieltjes integral and provide an example to illustrate.

16. Distinguish between pointwise convergence and uniform convergence. Explain the consequences of uniform convergence in differentiability and integrability.

17. Explain Lagrange multiplier method for constrained optimization. Find extreme value of $u = 8x^2 - xy + 12y^2$ subject to $x + y = 42$

18. Solve the initial value problems using Laplace transform

    $i)\ y' + 2y = e^{-t}, y\,(0) = 1$ $\qquad$ $ii)\ y' + 3y = 10sint, y(0) = 0$

(2x5=10 weightage)

59

1M1N24070 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2024

MST1C02 – Analytical Tools for Statistics – II

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer any four (2 weightages for each) 1-7

1. Define a vector space and list its eight properties.
2. Differentiate between the sum and direct sum of subspaces, and provide examples to illustrate each concept.
3. How do you define Rank of product of matrices provide an example
4. Define Hermitian, Skew- Hermitian matrices and Unitary matrix and write the specialities using eigen values of them
5. Find minimal and characteristic polynomial of the matrix $\begin{pmatrix} 5 & 3 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{pmatrix}$
6. Explain rank and signature of a quadratic form X'AX.
7. Prove that the system $Ax = b$ is consistent if and only if b lies in the column space of$A$.

(4 x 2 =8 weightages)

Part B

Answer any four (3 weightages each) 8-14

8. Explain with an example for the following i) Quotient space ii) Inner product iii) orthogonality
9. State and prove Rank- nullity theorem
10. Define rank of a matrix. If $\rho(A)$ is the rank of A, then show that
$$\rho(AB) \leq min[\rho(A), \rho(B)].$$
11. Show that the characteristic roots of a Hermitian matrix are all real.
12. Show that F be a linear mapping from a polynomial spaceP^3 to P^2 such that $F(x) = \frac{d}{dx}F(x)$. Then find the rank and nullity of F.
13. Prove that the singular values of a matrix are unique, even though the SVD decomposition itself may not be unique.
14. Classify the quadratic form $X' \begin{pmatrix} 1 & 2 & 4 \\ 2 & 4 & 3 \\ 4 & 3 & 2 \end{pmatrix} X$. Also find the signature.

(4 x 3 =12 weightages)

Part C

Answer any two(5 weightages each) 15- 18

15. A) Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R^4 spanned by the vectors

$v_1 = (1,1,1,1)$, $v_2 = (1,2,4,5)$ *and* $v_3 = (1,-3,-4,-2)$.

B) Check for linear independence and dependence of the following vectors

$V_1 = (1,1,1,1)$, $V_2 = (1,1,1,0)$ *and* $V3 = (1,1,0,0)$.

16. Let A and B are two idempotent matrix of order m, and I_m be identity matrix of order m. Then show

A)$I_m - A$ is idempotent.

B) Each eigen values of A is 0 or 1.

C) $A + B$ is idempotent if and only if $AB = BA = 0$

D) AB is idempotent if and only if $AB = BA$

17. A) Show that Moore-Penrose inverse is unique

B) State and prove the necessary and sufficient condition for a real quadratic to be positive definite.

18. A) Find the Moore-Penrose inverse of the matrix: $A = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$

B) Prove that a quadratic form $x^T Ax$ is positive definite if and only if all the eigenvalues of A are positive.

(2 x 5 = 10 weightages)

60

1M1N24071 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2024

MST1C03 – Probability Theory – I

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A: Short answer type questions
(Answer any four questions. weightage 2 for each question.)

1. Define a field and a sigma field. By an example show that a field need not be a sigma field.
2. Define limsup and liminf of a sequence of sets. How they are related? For a monotone sequence of sets show that limsup and liminf are equal.
3. Define distribution function of a random variable. Shaw that it is nondecreasing.State correspondence theorem.
4. Define MGF of a random variable. Explain any two uses of it.
5. State and prove CR inequality.
6. Discuss construction of a product space and state Fubini's theorem.
7. Show that convex combination of two distribution function is again a distribution function.

Part B: Short essays/problems
Answer any four questions.Weightage three for each question.

8. What do you mean by sigma field generated by a class of sets. Define Borel field. Is it the largest sigma field in the real line?
9. Define a random variable. Describe the probability space induced by a random variable.
10. Define a simple random variable. Show that any nonnegative random variable can be written as the limit of an increasing sequence of nonnegative simple random variables.
11. Define convergence in law and convergence in probability. Show that convergence in probability implies convergence in law.
12. If $\{A_n\}$ is a sequence of events show that $P[\liminf A_n] \leq \liminf P[A_n] \leq \limsup P[A_n] \leq P[\limsup A_n]$.
13. If X and Y are independent U(0,1) random variable find the distribution of their sum.
14. State and prove Jordan decomposition theorem on distribution function.

Part C: essays

Answer any two questions. Each question carries weightage 5.

15. State and prove Borel 0-1 laws.
16. State and prove basic inequality. Hence deduce Markov inequality and Chebyshev's inequality. Discuss an example in which Chebyshev's inequality attains equality.
17. State and prove Helly Bray theorems on distribution functions.
18. Show that almost sure convergence implies convergence in probability. If $\{X_n\}$ is a sequence of random variable that converges in probability to a random variable X. Show that there exists a sub sequence that converges almost surely to X.

61

1M1N24072 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2024

MST1C04 – Distribution Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A Answer any four (2 weightages each)

1. Explain the lack of memory property of geometric random variable.
2. Define probability generating function. Obtain the probability generating function of negative binomial random variable.
3. What is a mixture distribution? Explain with an example..
4. Define Cauchy distribution. If $X_1, X_2, \dots, X_n$ is a random sample from Cauchy distribution then obtain the distribution of the sample mean..
5. State and prove Cauchy – Schwartz inequality.
6. Obtain the pdf of the largest order statistics of a random sample of size n taken from uniform distribution over (-1,1).
7. Define noncentral student's t distribution. When will this reduces to central-t distribution

(2 x 4=8 weightages)

PART B Answer any four (3 weightages each)

8. If X and Y are independent gamma random variables then show that $U = X + Y$ and $V = \frac{X}{X+Y}$ are independent.
9. If $X \sim B(n,p)$, show that $E\left(\frac{X}{n} - p\right)^2 = \frac{pq}{n}$ and $Cov\left(\frac{X}{n}, \frac{n-X}{n}\right) = -\frac{pq}{n}$.
10. The trinomial distribution of two random variables X and Y s given by $f(x,y) = \frac{n!}{x!y!(n-x-y)!} p^x q^y (1-p-q)^{n-x-y}, x, y = 0,1,2,\dots,n,\ \ p, q \geq 0$ and $p + q \leq 1$.
    (i). Find the marginal distribution of X and Y.
    (ii). Find the conditional distribution of X and Y.
11. Let X & Y be jointly distributed with $f(x,y) = \begin{cases} \frac{1}{4}(1+xy), |x| < 1, |y| < 1 \\ 0, \ otherwise \end{cases}$ .Show that X & Y are not independent but $X^2 \& Y^2$ are independent.
12. Derive the MGF of normal distribution.
13. Describe the various types of distributions in the Pearson family.
14. Prove that the square of a t statistic follows F distribution.

(3x 4=12 weightages)

PART C Answer any two (5 weightages each)

15. Obtain the recurrence relation for cumulants of binomial distribution. Hence obtain the first four cumulants.

16. If $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean and variance then show that the sample mean is independent of the sample variance S^2.

17. Let $X_1, X_2, \ldots, X_n$ be iid random sample from a uniform distribution over $(0, \theta)$. Then find the following

    a) distribution of median

    b) $E(X_{n:n})$.

18. Define non-central F-distribution and derive its density.

(5x 2=10 weightages)

62

1M1N24073 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2024

MST1C05 – Sampling Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type Questions

Answer any four questions. (Weightage 2 for each question)

1. Define sampling. Write down the advantages of it.
2. Explain circular systematic sampling,
3. Explain stratified sampling.
4. What is the advantage of ratio method of estimation? Prove that the ratio estimator of population mean is not unbiased.
5. What is πps sampling?
6. Briefly explain PPSWR and PPSWOR.
7. Explain cluster sampling?

(4 x 2 = 8 weightage)

Part B

Short Essay Type / Problem solving type questions

Answer any four questions. (Weightage 3 for each question)

8. Prove that if N = nk, with usual notations, systematic sample mean $\bar{y}_{sy}$ is an unbiased estimator of population mean. Also find its variance
9. Explain proportional allocation in stratified sampling. Derive the variance of the unbiased estimator of population mean under this allocation.
10. Derive Heartley-Ross unbiased estimator of population men.
11. Explain Lahiri's method of selection in probability proportional to size sampling.
12. What is Des-Raj's ordered estimator? S.T. this estimator of population total is unbiased
13. Explain two-stage sampling with equal cluster sizes. Obtain the variance of unbiased estimator of population mean.
14. Derive the variance of estimator of population mean under cluster sampling in terms of intra cluster correlation coefficient.

(4 x3=12 weightage)

Part C

Long Essay Type questions

Answer any two questions. (Weightage 5 for each question)

15. Explain SRSWOR. Derive the variance of sample mean under this method and compare it with SRSWR. Also Set up the Confidence Interval for population mean.
16. Suggest a linear regression estimator of population mean. Find an approximate expression of its variance. Also derive the expression of minimum variance of it.
17. (a) Distinguish between ordered and unordered estimators. (b) Define Horvitz Thompson estimator for population total and check its unbiasedness. Also derive the variance of it.
18. Give an unbiased estimator of population Mean by Two-stage sampling with SRSWOR at both the stages. Derive the sampling variance.

(2 x5=10 weightage)