2M4A22558 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MST4E01 – Operations Research – I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A (Short Answer type)

(Answer any 4 questions. Weightage 2 for each question)

1. Find all the basic solutions of the following constraints of an LPP and identify which are feasible and which are degenerate solutions.

   $2x + 3y - z + 4u = 8;$

   $x - 2y + 6z - 7u = -3.$
2. Show that dual of the dual is primal.
3. Explain Vogel's Approximation Method (VAM).
4. Describe the steps in Hungarian Algorithm to solve an assignment problem.
5. What is parametric programming? When is it useful?
6. What are pure and mixed integer programming problems? Give one example each.
7. What are zero-sum games? Point out the importance of saddle point in game problems?

(4 x 2 = 8 weightage)

PART B (Short Essay type questions)

(Answer any 4 questions. Weightage 3 for each question)

8. Briefly explain the steps in the two-phase simplex method.
9. State and prove duality theorem in LPP.
10. Solve the following Transportation problem:

	Destination				
Origin	D1	D2	D3	D4	Supply
O1	3	5	7	6	50
O2	2	5	8	2	75
O3	3	6	9	2	25
Demand	20	20	50	50	

11. Describe Gomory's cutting plane method for solving an Integer Programming Problem.
12. There are n jobs to be processed on two machines, each job requiring the same sequence of operations and no passing is allowed. Stating clearly the assumptions (if any), provide an algorithm to determine the optimal sequencing of jobs.
13. Prove that every matrix game has a solution.
14. Explain the graphical method of solving game problems.

(4 x 3 = 12 weightage)

PART C (Long Essay type questions)

(Answer any 2 questions. Weightage 5 for each question)

15. Solve using dual simplex method:

Minimize $Z = 2x + y$

Subject to: $3x + y >= 3$

$4x + 3y >= 6$

$x + 2y >= 3$

$x, y >= 0$

16. Solve the Parametric LPP:

Maximize $z = (3 - 6\lambda)x + (2 - 2\lambda)y + (5 + 5\lambda)z$

Subject to : $x + 2y + z <= 40$

$3x + 2z <= 60$

$x + 4y <= 30$

$x, y, z >= 0.$

17. Solve using branch and bound technique:

Maximize $Z = -x + 4y$

Subject to $-10x + 20y <= 22$

$5x + 10y <= 49$

$x + 2y <= 5$

and $x, y >= 0$ and integers.

18. Solve the two person game with pay-off matrix using simplex method.

$$P = \begin{pmatrix} 1 & -3 & -5 \\ -4 & 2 & -3 \\ -7 & -8 & 0 \end{pmatrix}$$

(2 x 5 = 10 weightage)

2M4A22557 (Pages : 3) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MST4C14 – Multivariate Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part- A

Answer any 4 questions.

Each question carries 2 weightage.

1. If $\underset{\sim}{X}_1, \underset{\sim}{X}_2 \sim N_p(\mu, \Sigma)$, X_1 and X_2 are independent, derive the distribution of $X_1 + X_2$.

2. If $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$, what is the distribution of $(\underset{\sim}{X} - \mu)'\Sigma^{-1}(\underset{\sim}{X} - \mu)$? Justify your answer.

3. Given a sample of size N from $(X_1, X_2)' \sim N_2(\mu, \Sigma)$, how do you find the maximum likelihood estimator of $\rho = \text{corr}(X_1, X_2)$? Explain.

4. Describe how do you test the hypothesis $H_0 : \mu = \mu_0$, a given vector in $\Re^p$, given a random sample of size N from $N_p(\mu, \Sigma)$, when Σ is known.

5. Let $\underset{\sim}{X}$ be a p-component random vector and $\underset{\sim}{X}$ be partitioned into two sub vectors $\underset{\sim}{X}^{(1)}$, $\underset{\sim}{X}^{(2)}$ containing q $(q < p)$ and $p - q$ component of $\underset{\sim}{X}$. Prove that $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are independent if and only if $\text{cov}(\underset{\sim}{X}^{(1)}, \underset{\sim}{X}^{(2)}) = O_{q\times(p-q)}$.

6. Describe classification problem. Illustrate it through an example.

7. Describe orthogonal factor model.

(4×2= 8 weightage)

Part - B

Answer any 4 questions.

Each question carries 3 weightage.

8. Let $\underset{\sim}{X} = (X_1, X_2, X_3)'$ and $\underset{\sim}{X} \sim N_3(0, \Sigma)$ where $\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$. Derive the conditional distribution of $(X_1, X_2)'$ given $X_3 = x_3$.

9. Let $\underset{\sim}{X} \sim N_p(0, \Sigma)$. state and prove a necessary and sufficient condition for the independence of the quadratic forms $\underset{\sim}{X}'A\underset{\sim}{X}$ and $\underset{\sim}{X}'B\underset{\sim}{X}$.

10. Derive the maximum likelihood estimator of the parameters of multinormal distribution.

11. Prove that the statistic to be used for testing $H_0 : \mu = \mu_0$ based on a random sample of size N from $N_p(\mu, \Sigma)$ is the Hotelling's T^2 statistic when Σ is unknown.

12. Derive the sampling distribution of simple correlation coefficient in the null case for the multinormal distribution.

13. Derive the likelihood ratio test for testing the independence of subvectors of a multinormal distribution.

14. Describe the iterative procedure to calculate sample principal components.

(4×3= 12 weightage)

Part - C

Answer any 2 questions.

Each question carries 5 weightage.

15 a) Beginning with univariate normal distribution, derive the multivariate normal distribution.

b) Prove that a p-dimensional random vector $\underset{\sim}{X}$ is multivariate normal if and only if every linear combination of $\underset{\sim}{X}$ is univariate normal.

16. a) Check whether the maximum likelihood estimators of the parameters of $N_p(\mu, \Sigma)$ are unbiased and consistent.

b) Define Wishart distribution. What are the properties of Wishart distribution?

17. a) Derive the likelihood ratio test for testing the equality of two multinormal distributions.

b) What is spherical normal distribution? Briefly describe sphericity test.

Describe how do you classify an observation $\underset{\sim}{X}$ into one of two multivariate normal populations, when the parameters are known.

Define principal components. Prove that the problem of determining the principal components of $\underset{\sim}{X}$, reduces to the problem of finding the eigen values and eigen vectors of the dispersion matrix Σ of $\underset{\sim}{X}$ if Σ is known.

(2×5= 10 weightage)

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