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3M4A22532 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MMT4E11– Graph Theory

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all the 8 questions.
Each question carries 1 weightage

1. Show that G is a forest if and only if every edge of G is a cut edge.
2. Draw all non-isomorphic spanning trees of K_5
3. Discuss the difference between cut vertex and vertex cut using an example.
4. Show that $r(3, 3) = 6$
5. Prove that $\alpha + \beta = \nu$
6. Define k-edge colourable graph with an example.
7. Prove that in a critical graph no vertex cut is a clique.
8. Define bridge in a graph. Give an example.

Part B

Answer any two questions from each unit.
Each question carries 2 weightage

Unit I

9. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.
10. Show that $\kappa \leq \kappa'$ where κ is the vertex connectivity and κ' edge connectivity of a graph.
11. Let G be a simple graph with degree sequence $(d_1, d_2, \ldots, d_\nu)$, where $d_1 \leq d_2 \leq \cdots \leq d_\nu$ and $\nu \geq 3$. Suppose that there is no value of m less than $\nu/2$ for which $d_m \leq m$ and $d_{\nu-m} < \nu - m$. Then show that G is Hamiltonian.

Unit II

12. Define (a) Maximum matching and (b) M-augmenting path in a graph. Prove that if G contains no M-augmenting path, then M is a maximum matching in G.

13. Every 3-regular graph without cut edges has a perfect matching.

14. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

Unit III

15. If G is simple, then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.

16. Prove that inner bridges avoid one another.

17. Prove that a loopless digraph D has an independent set S such that each vertex of D not in S is reachable from a vertex in S by a directed path of length atmost two.

Part C

Answer any six out of nine questions.
Each question carries 5 marks

18. Discuss Kruskal's algorithm. Show that any spanning tree $T^* = G[\{e_1, e_2, \ldots, e_{\nu-1}\}]$ constructed by Kruskal's algorithm is an optimal tree.

19. (a)Let G be a bipartite graph with bipartition (X, Y). Then show that G contains a matching that saturates every vertex in X if and only if

$$|N(S)| \geq |S| \ for \ all \ S \subseteq X$$

(b) If G is a k-regular bipartite graph with $k > 0$,then prove that G has a perfect matching.

20. If G is a simple graph then show that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

21. State and prove Kuratowski's theorem.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MMT4E09 –Differential Geometry

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PARTA

Answer ALL questions. Each question has1 weightage.

1. Sketch the level set at height c=2 for the function $f(x_1, x_2) = x_1^2 + x_2^2$.
2. Prove that$a_1x_1 + a_2x_2 \dots + a_{n+1}x_{n+1} = b$ where $0 \neq (a_1, a_2, a_3 \dots, a_{n+1}) \in R^{n+1}$ and $b \in R$, forms a n-surface in R^{n+1}.
3. Show that the two orientations on the n-sphere $x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = r^2$of radius $r > 0$are given by $N_1(p) = (p, \frac{p}{r})$ and $N_2(p) = (p, -\frac{p}{r})$.
4. Let X and Y be smooth vector fields along the parametrized curve $\alpha: I \to R^{n+1}$.Verify that $(\dot{X + Y}) = \dot{X} + \dot{Y}$.
5. Let X be parallel vector field along a parametrized curve $\alpha: I \to R^{n+1}$.Prove that Xhas constant length.
6. Define the circle of curvature of an oriented plane curve.
7. Let S be an oriented n-surface in $R^{n+1}$and let $p \in S$.Define the first and second fundamental forms of S at p.
8. Define a parametrized n-surface in R^{n+1}.

$(8\times 1 = 8\, weightage)$

PART B

Answer any two questions from each unit. Each questioncarries2 weightage.

UNIT I

9. Show that for each p $\in R^{n+1}$,the set $R_p^{n+1}$of all vectors at p is a vector space.
10. Show that at each regular point p on a level set $f^{-1}(c)$ of a smooth function there is a well defined tangent space consisting of all velocity vectors at p of all parametrized curves in $f^{-1}(c)$ passing through p.
11. Show that the maximum and minimum values of the function $g(x_1, x_2) = ax_1^2 + 2bx_1x_2 + cx_2^2$ where $a, b, c \in R$ on the unit circle $x_1^2 + x_2^2 = 1$are the eigen values of the matrix$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$.

UNIT II

12. Prove that for each pair of orthogonal unit vectors $\{e_1, e_2\}$ in R^3 and each $a \in R$, the great circle $\alpha(t) = \cos at\ e_1 + \sin at\ e_2$ are geodesics on the unitsphere $x_1^2 + x_2^2 + x_3^2 = 1$

13. Let S be a n-surface in R^{n+1}, let $p, q \in S$, and let α be a piecewise smooth parametrized curve from p to q. Prove that the parallel transport $P_\alpha: S_p \to S_q$ along α is a vector space isomorphism.

14. Let C be a connected oriented plane curve and let $\beta: I \to C$ be a unit speed parametrization of C. Prove that β is either one one or periodic.

Unit III

15. Prove that on each compact oriented n-surface S in R^{n+1} there exists a point p such that the second fundamental form at p is definite.

16. Describe a parametrized torus in R^4.

17. Let S be a compact, connected oriented n-surface in R^{n+1} whose Gauss Kronecker curvature is nowhere zero. Prove that the Gauss map $N: S \to S^n$ is a diffeomorphism.

$(6 \times 2 = 12\ weightage)$

PART C

Answer any two questions. Each question carries 5weightage.

18. a)Let X be a smooth vector field on an open set $U \subset R^{n+1}$ and let $p \in U$. Prove that there exists an open interval I containing 0 and an integral curve $\alpha: I \to U$ such that $\alpha(0) = p$.

    b) Find the integral curve through $p(0,1)$ of the vector field X on R^2 given by $X(p) = (p, X(p)$ where $X(x_1, x_2) = (-x_2, x_1)$

19. a)Find the spherical image of the hyperboloid $x_1^2 - x_2^2 - x_3^2 = 4, x_1 > 0$, oriented by $N = \frac{-\nabla f}{\|\nabla f\|}$ where $f(x_1, x_2, x_3) = x_1^2 - x_2^2 - x_3^2$.

    b) Let S be a compact, connected oriented n-surface in R^{n+1}. Prove that the Gauss map maps S onto the unit n-sphere S^n.

20. a)Show that the Weingarten map at each point p of an oriented n-surface in R^{n+1} is self adjoint.

    b)Show that the Weingarten map of the n-sphere $x_1^2 + x_2^2 + \cdots \ldots \ldots + x_{n+1}^2 = r^2$ of radius $r > 0$, oriented by inward unit normal is simply multiplication by $\frac{1}{r}$.

21. a) Let S be an n-surface in R^{n+1} and let $f: S \to R^k$. Then prove that f is smooth if and only if $f o \varphi: U \to R^k$ is smooth for each local parametrization $\varphi: U \to R^k$.

    b) State and prove the inverse function theorem for n-surfaces.

$(2 \times 5 = 10\ weightage)$

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MMT4E14 – Computer Oriented Numerical Analysis

(2019 Admission onwards)

Time: 1 ½ hours Max. Weightage : 15

Section A

Answer ALL questions. Each question carries 1 weight.

1. Define a "complex type" variable in Python. Give it a value.
   Write a command to print the sum of the real and imaginary parts ?

2. Write a Python programme, that will ask to input your name and will print it as the output.

3. Write a Python program to get the print out of multilication table of 7 upto $10 \times 7 = 70$ using a "FOR" loop.

4. What is the use of "BREAK" and "CONTINUE" in Python ? Explain using a suitable example.

$4 \times 1 = 4$ Weights.

Section B

Answer any THREE questions. Each question carries 2 weights.

5. Write a Python program to do the following. Asking the input of a natural number "n" and the number of natural numbers less than "n" and prime to "n".

6. Explain the problem and Simpson's 1/3 rule for numerical integration.

7. Write short notes on at least three different "Lists" in Python. (One example for each).

8. What is 'Data Structures' in Python ? Explain with suitable examples.

9. Write a Python proram to input the name, date of birth, and height of 10 persons and to show the print out in the format of a table, preferably with border and inner lines.

$3 \times 2 = 6$ Weights.

Section C

Answer any ONE question. Each question carries 5 weights.

10. a) Write a Python Program to find the Largest of three positive integers you input.

    b) Explain the problem, the method, the algorithm and a Python program for the Bisection Method.

11. Explain the problem, the method, the algorithm and a Python program for the Numerical Solution of Ordinary Differential Equations.

$1 \times 5 = 5$ Weights.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, April 2022

MMT4C15 - Advanced Functional Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer *all* questions. Each carries 1 weightage

1. If A, B are linear operators on a normed space X, prove that A^{-1} and B^{-1} exists if and only if $(AB)^{-1}$ and $(BA)^{-1}$ exists.
2. Give an example of an operator whose spectrum is empty.
3. Define transpose of a linear map. If $F \in BL(X,Y)$, prove that $\| F \| = \| F' \|$.
4. If X is a reflexive normed space, show that X' is also reflexive.
5. Give an example to show that 0 can be a spectral value of a compact operator without being its eigen value.
6. If X is any inner product space and for $y \neq 0$ in X, define $f_y(x) = \langle x, y \rangle$, $x \in X$. Show that f is a bounded linear functional on X and $\| f \| = \| y \|$.
7. If $A \in BL(H)$, show that A is normal if and only if $\| A(x) \| = \| A^*(x) \|$ for all $x \in H$.
8. Define numerical range. If $A \in BL(H)$, show that $k \in \omega(A)$ if and only if $\bar{k} \in \omega(A^*)$.

$(8 \times 1 = 8$ weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. If X is a Banach space, prove that $A \in BL(X)$ is invertible iff A is bounded below and the range of A is dense in X.
10. If X is a Banach space, show that the set of all invertible operators is open in $BL(X)$ and the map $A \mapsto A^{-1}$ is continuous.
11. If X is a normed space and if X' is separable, show that X is also separable.

Unit 2

12. Show that, if Y is a Banach space, then $CL(X,Y)$ is a closed subspace of $BL(X,Y)$.

13. If X is an infinite dimensional normed space and if $A \in CL(X)$, show that 0 is an approximate eigen value of A.

14. State Riesz representation theorem. Give an example to show that this theorem need not hold for an incomplete inner product space.

Unit 3

15. Let H be a Hilbert space and $A \in BL(H)$. Then show that there is a unique $B \in BL(H)$ such that $\langle A(x), y\rangle = \langle x, B(y)\rangle$ for all $x, y \in H$.

16. If $H \neq \{0\}$ and $A \in BL(H)$ is self-adjoint, then show that $\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A]$.

17. Define Hilbert –Schmidt operator. Show that every Hilbert –Schmidt operator is compact.

(6 × 2 = 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage

18. (a) If X is a Banach space over K and if $A \in BL(X)$, prove that $\sigma(A)$ is a compact subset of K.

    (b) If $1 \leq p < \infty$ and if $\frac{1}{p} + \frac{1}{q} = 1$, show that the dual of ℓ^p is linearly isometric to ℓ^q.

19. (a) Define weak convergence. Give an example to show that weak convergence is weaker than norm convergence. Also, show that $x_n \xrightarrow{w} x$ in ℓ^1 iff $x_n \longrightarrow x$ in ℓ^1.

    (b) Show that every closed subspace of a reflexive normed space is reflexive.

20. (a) If A is a compact operator on a normed space X, prove that the eigen spectrum and the spectrum of A are countable sets and have 0 as the only possible limit point.

    (b) State and prove projection theorem.

21. (a) If H is a finite dimensional Hilbert space over C and if $A \in BL(H)$ is a normal operator, then prove that there is an orthonormal basis for H consisting of eigen vectors of A.

    (b) If $A \in BL(H)$, and if each A_n is a compact operator on the Hilbert space H and $\| A_n - A \| \to 0$, then show that A is compact.

(2 × 5 = 10 weightage)