1B6A22059

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Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2022

BMT6B14(E03) – Mathematical Programming with Python and Latex

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks.
Cieling 20 marks

1. Write the output of the python program

```
x = 12
y = 3
print(x = = 4*y)
```

2. What is slicing in python ? Give example of any two different types of slicing.

3. Explain **mutable** and **immutable** types in python.

4. Write the syntax of **while** loop in python.

5. Explain the output of the function **range(3,71,6)** in python.

6. Write the use of **break** statement in python.

7. Give example of two library functions in python and specify their uses.

8. How do you write comments in a python program ? What is its importance ?

9. Write the output of the python program:

```
a = 'abc.abc.abc'
aa = a.split('.')
print (aa)
mm = '+'.join(aa)
print (mm)
```

10. What is Mathplotlib ? Write a use of it.

11. Write three different document types in LaTeX.

12. Write the LaTeX code to get the output $\sum_{n=1}^{10}(2n+1)$.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 30 marks

13. Write a python program to input two numbers and to give the following output as print: The first number, the second number, the sum and the product.

14. Write a python program to get output as the multiplication tale of 7 starting from $1 \times 7 = 7$ upto $10 \times 7 = 70$.

15. Explain the **power** function used in python.
What is the default value of exponent in python ?

16. Give example of a python program to show the use of input and output of files.

17. Write a python program to set the order 2×3 for a matrix, input its entries and print the matrix.

18. Write a python program for a Pie Chart. *Assume your own data.*

19. Give one example each for non-numberised and numberised listings in LaTeX.

Section C

Answer any One Question. Each question carries 10 marks.
10 marks from this section

20. Write a python program to input the coefficients of a quadratic equation and to display the solution and nature of roots. Include proper comments, suitable messages for the inputs and formatted printing of the output.

21. Prepare the LaTeXcode to generate a question paper similar to the one that you are writing now. *It is enough to include only two questions in each section.*

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2022

BMT6B13-DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time: 2.5 hours Max. Markss : 80

PART A

***All* the questions can be attended.**
Each question carries 2 marks.

1. What is meant by a singular solution of a differential equation.
2. Define an exact differential equation. Check whether $(x+y)dy-(x-y)dx=0$ is exact ?
3. Solve the differential equation $y' + 2t^2y = 0$.
4. Find the interval in which the initial value problem:

   $ty' + 2y = 4t^2$, $y(1) = 2$ *has a unique solution.*
5. Find the order and degree of the differential equation $\frac{d^4y}{dx^4} + 5\left(\frac{d^2y}{dx^2}\right)^3 - \frac{dy}{dx} = y$.
6. Find the fundamental set of solutions of the differential equation $y'' + 6y' + 9y = 0$.
7. State Abel's theorem.
8. Check whether the functions $y_1 = t$ *and* $y_2 = t+3$ are linearly independent or *not*.
9. Define Unit step function. What is its Laplace transform?
10. Find the reduction formula for $L[t^n]$ for any positive integer n
11. Find the Laplace Transform of $e^{-3t}cos3ht$.
12. Find $L^{-1}\left[\frac{e^{-3s}}{(s-1)^4}\right]$
13. What is the fundamental period of $sin4t$?
14. Verify that $u(x,t) = x^3 + 3xt^2$ is a solution of the one-dimensional wave equation.
15. Find the Fourier Series expansion for $f(x) = x$ *in* $[-\pi,\pi]$ with $f(x) = f(x+2\pi)\ \forall x \in R$

(Ceiling 25 Marks)

PART B

All the questions can be attended.
Each question carries 5 marks.

16. Find an integrating factor for the differential equation $(y - x^2)dx + (x^2 siny - x)dy = 0$.
    And then solve the equation.
17. Solve the differential equation $t^2y' + 2ty = y^3$ where $t > 0$
18. Solve $x^2y'' - 3xy' + 4y = 0$ given $y(1) = 0$ *and* $y'(1) = 3$.
19. Solve the non homogeneous differential equation $y'' - 3y' + 2y = 4x + e^{3x}$
20. If $f(t) = t\ sinat$, then find $L[f(t)]$
21. Find $L^{-1}\left\{\frac{3s}{s^2-s-6}\right\}$.
22. Express the function $f(x) = x^2$ $when -1 < x < 1$ as a Fourier Series with period 2.
23. Find the half range sine series expansion of the function $f(x) = 1 - x,\ 0 \le x \le 1$

(ceiling 35 Marks)

PART C:

Answer any _two_ questions. Each question carries 10 marks.

24. a) Solve the differential equation $x(y - x)dy = (y + x)ydx$
    (b) Using Picard's Iteration method solve the initial value problem $y' = x + y$ *with* $y(0) =$-
25. (a) Solve by using the method of variation of parameters $y'' + 4y = 3\ cosec\ t$
    (b) Solve the differential equation $y'' + y = cost$
26 (a) Using the convolution property find the inverse Laplace transform of $\frac{1}{s^2(s^2+9)}$
    (b) Using Laplace transform, solve the initial value problem
    $y'' - 3y' + 2y = 4\,e^{2t}$, $y(0) = -3$ *and* $y'(0) = 5$
27. (a) Find the half range cosine series expansions of the function

$$f(t) = \begin{cases} \frac{2k}{l}t\ ; & 0 \le t < \frac{l}{2} \\ \frac{2k}{l}(l - t)\ ; & \frac{l}{2} \le t < l \end{cases}$$

    (b)Solve using the method of separation of variables $u_x + u_y = 0$.

(2 × 10 = 20 Mark

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2022

BMT6B12 – Calculus of Multivariable – 2

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A

All questions can be attended. Each question carries 2 marks.
Cieling 25 marks

1. Write equation of normal to the curve $\frac{x^2}{16}-\frac{y^2}{9}=1$ at the point $(5,\frac{9}{4})$.
2. Locate the critical points for the function $f(x,y)=x^2+y^2-2x-4y$.
3. Write the method of Lagrange multiplier.
4. Evaluate $\int_1^2\int_0^1 x^2y\,dx\,dy$.
5. Write the integral $\int\int_R (x+y)dA$ using polar coordinates where R is the region in the first quadrant bounded by the circle $x^2+y^2=4$.
6. Write the formula to find the area of a surface $x=h(y,z)$.
7. Describe the method of converting a triple integral from rectangular coordinates to cylindrical coordinates.
8. Find the curl of the vector field $\mathbf{F}(x,y)=xy\hat{i}+(x+y^3)\hat{j}$.
9. Evaluate the line integral $\int_C (x+y)\,ds$ where C is $\mathbf{r}(t)=3t\hat{i}+4t\hat{j}, 0\le t\le 1$.
10. State the theorem connecting Independence of Path and Conservative Vector Fields.
11. If a line integral over any simple closed curve is zero, prove that the line integral is independent of path.
12. Test whether $\mathbf{F}(x,y)=2xy\hat{i}+x^2\hat{j}$ is a conservative vector field or not.
13. State the Green's theorem for simple regions.
    Explain the terms used in the statement.
14. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using line integral and Green's theorem.
15. State the Stoke's theorem.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 35 marks

16. Classify the critical points for the function $f(x,y) = x^2 + 3y^2 - 6xy - 2x + 4y$.

17. Reverse the order of integration and evaluate the integral $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$.

18. Find the area of the surface $z = \frac{1}{2}x^2 + y$ that lies above the triangular region with vertices $(0,0)$, $(1,0)$ and $(1,1)$.

19. Evaluate $\iiint_B (xy + x^2z + yz^2)\, dV$
where B is the cuboid $\{(x,y,z)) \mid -1 \le x \le 1, 0 \le y \le 2, 1 \le z \le 3\}$.

20. For a scalar function f and a vector function $\mathbf{F}$,
prove that $\text{curl}(f\mathbf{F}) = f\text{curl}\mathbf{F} + \nabla f \times \mathbf{F}$.

21. Find the work done by the force field $\mathbf{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ on a particle that moves along the curve $C\ :\ \mathbf{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k},\ 0 \le t \le 1$.

22. Find a parametrisation of the surface of the cone $z = \sqrt{x^2 + y^2}$.

23. Use divergence theorem to compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$ and $\mathbf{F}(x,y,z) = (x + \sin z)\hat{i} + (2y + \cos x)\hat{j} + (3z + \tan y)\hat{k}$.

Section C

Answer any Two Questions. Each question carries 10 marks.
20 marks from this section

24. Find the absolute maximum and the absolute minimum values of the function $f(x,y) = 2x^2 + y^2 - 4x - 2y + 3$ on the region $D = \{(x,y \mid 0 \le x \le 3, 0 \le y \le 2\}$.

25. Evaluate $\iiint_T (x^2y + yz^2)\, dV$ where T is the region bounded by the cylinder $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

26. Show that $\mathbf{F}(x,y,z) = 2xy^2z^3\hat{i} + 2x^2yz^3\hat{j} + 3x^2y^2z^2\hat{k}$ is a conservative field.
Find a scalar function $f(x,y,z)$ whose gradient is $\mathbf{F}(x,y,z)$.
Also evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from the point $(0,0,0)$ to the point $(1,1,1)$.

27. Find the surface area of a sphere of radius a using the idea of parametrised surface.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2022

BMT6B11 – Complex Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

SECTION A

Answer the following questions. Each carries two marks

(Ceiling 25)

1. Show that $f(z) = \bar{z}$ is no where differentiable.
2. If $f(z)$ has a derivative at z_0, then prove that $f(z)$ is continuous at z_0.
3. Suppose that $f(z)$ is analytic in a domain D, if $f'(z) = 0\ \forall z \in D$, prove that $f(z)$ is constant in D
4. If u and v are harmonic functions conjugate to each other in some domain, then prove that u and v must be constant there.
5. Let C denote the quarter circle defined by $x = 4\cos t, y = 4\sin t, 0 \leq t \leq 4$, Evaluate $\int_C xy^2 dy$
6. Evaluate $\int_C xy dx + x^2 dy$ where C is the graph of $y = x^3, -1 \leq x \leq 2$
7. Define complex valued functions and evaluate $\int_0^1 (1 + it)^2 dt$
8. If $| f(z) | \leq M$ every where on a contour C and L is the length of C then prove that $| \int_C f(z)dz | \leq ML$
9. State and prove Liouville's Theorem
10. Let $f(z) = 2z + 5i$ defined in $| z | \leq 2$. Find the points where $| f(z) |$ has its maximum and minimum in the region
11. True or false: Absolute convergence of a series implies convergence of that series. Is the converse true? Justify.
12. Prove that $\Sigma_{k=1}^{\infty} \frac{(3-4i)^k}{k!}$ converges
13. Identify t the type of singularity of (a) $\frac{\sin z}{z}$ (b) $\frac{1}{(2-z)^3}$
14. Identify the type of singularity of (a) $\frac{z^2}{1+z}$ (b) $ze^{1/z}$

15. Find the residues at the singular point (a) $\frac{4}{1-z}$ (b) $\frac{\sin z}{z^4}$

SECTION B

Answer the following questions. Each carries five marks

(Ceiling 35)

16. Explain $L'Hopital's$ Rule and evaluate $lim_{z \to 2-i} \frac{z^2-4z+5}{z^3-z-10i}$

17. Solve $\sin z = \cosh 4$

18. Evaluate $\int_C (6x^2 + 2y^2)dx + 4xydy$ where C is given by $x = \sqrt{t}, y = t, 4 \leq t \leq 9$

19. Let C be the arc of the circle $\mid z \mid= 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating integral show that $\mid \int_C \frac{dz}{z^2-1} \mid\leq \frac{\pi}{3}$

20. Find all Laurent series representation of the function $f(z) = \frac{1}{1-z^2}$ with center at $z = 1$

21. Find all Laurent series representation of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0.

22. Find the residue at at the singular point (a) $f(z) = \frac{z^3+2z}{(z-i)^3}$ (b) $g(z) = cot z$

23. Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}, a > b > 0$

SECTION C

Answer any two questions (2 × 10 = 20 Marks)

24. (a) Show that for the function

$$f(z) = \begin{cases} \frac{\bar{z}^2}{z} & when \quad z \neq 0 \\ 0 & otherwise \end{cases}$$

even though partial derivative of component function exists and satisfy C-R equations at $z = 0$, $f(z)$ is not differentiable at $z = 0$.

(b) Show that $f(z) = Re\ z$ is no where differentiable.

25. (a)If f is a analytic in a simply connected domain D, then prove that f has an antiderivative in D, that is there exist a function F such that $F'(z) = f(z)$

(b) Evaluate $\int_0^{1+i} e^{\pi z} dz$

26. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for the following domains:

(a) $0 <\mid z \mid< 1$ (b) $\mid z \mid> 1$ (c) $0 <\mid z-1 \mid< 1$ (d) $\mid z-1 \mid> 1$

27. (a)A function analytic in a punctured disk $0 <\mid z - z_0 \mid\leq R$ has a pole of order n at $z = z_0$ if and only if $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z-z_0)^n}$ where ϕ is analytic at $z = z_0$ and $\phi(z_0) \neq 0$

(b) If f has a simple pole at $z = z_0$ then $Res(f(z), z_0) = lim_{z \to z_0}[(z - z_0)f(z)]$

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Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2022

BMT6B10 – Real Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks

1. Show that $f(x) = x^2$ is uniformly continuous on [-1, 1].
2. State and prove Lipschitz condition for uniform continuity.
3. State Bernstein Approximation Theorem.
4. Let $f : [0, 5] \to R$ be defined by $f(x) = 3$. Show that f is Riemann integrable.
5. If f is Riemann integrable then prove that the value of the integral is unique.
6. Prove that the step function is Riemann integrable.
7. State First form of the Fundamental Theorem of Calculus.
8. Evaluate the integral $\int_0^2 t^2\sqrt{1+t^3}\,dt$. Justify your steps.
9. Prove that the sequence $g_n(x) = x^n$ converges point wise on (-1, 1].
10. Show that $\sum_{n=1}^{\infty} \frac{1}{n^p + n^2x^2}$ is uniformly convergent for all values of x, if p>1.
11. Test the convergence of the improper integral $\int_0^{\infty} \sin x dx$.
12. If $\int_a^{\infty} f(x)dx$ converges absolutely then prove that $\int_a^{\infty} f(x)dx$ converges.
13. Test the convergence of $\int_1^5 \frac{dx}{\sqrt{x^4-1}}$.
14. Prove that the Beta function is symmetric.
15. Prove that $\Gamma n = (n-1)\Gamma(n-1)$.

Ceiling – 25 Marks

Section B
All questions can be attended
Each question carries 5 marks

16. State and prove Boundedness Theorem on Continuous functions.

17. Show that $g(x)=\sqrt{x}$ on [0,2] is not a Lipschitz function.

18. Let h(x)=x for $x\in[0,3]$. Show that $h\in R[0,3]$ and evaluate its integral over the interval [0,3].

19. If $f\in R[a, b]$ then prove that f is bounded on [a, b].

20. Show that the sequence $S_n(x)=\dfrac{n}{x+n}$ for every $x\geq 0$ is uniformly convergent on [0,m] for any m.

21. Show that $\int_a^{\infty}\frac{1}{x^p}dx$ is converges if p > 1 and diverges if $p\leq 1$, for a >0.

22. Examine the convergence of $\int_0^1\frac{dx}{\sqrt{1-x^2}}$

23. Evaluate $\int_0^{\pi/2}\sin^7 x dx$.

Ceiling – 35 Marks

Section C
Answer any two questions
Each question carries 10 marks

24. (a) State and prove Uniform Continuity Theorem.

(b) If $f\in R[a, b]$ and if $|f(x)|\leq M$ for all $x\in[a, b]$, then prove that

$$\left|\int_a^b f(x)dx\right|\leq M(b-a).$$

25. (a) Prove that a sequence (f_n) of bounded functions on $A\subseteq R$ converges uniformly on A to f if and only if $\|f_n-f\|_A\to 0$ as $n\to\infty$.

(b) Show that $h_n(x)=x^n(1-x)$ for $x\in[0,1]$ is converges uniformly on [0,1].

26. (a) Test for the convergence of the improper integral $\int_1^{\infty}\frac{dx}{x\sqrt{x^2+1}}$.

(b) Evaluate the Cauchy Principal Value of $\int_{-1}^{5}\frac{dx}{(x-1)^3}$.

27. (a) Prove that $B(m,n)=\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta$

(b) Prove that $B(m.n)=\dfrac{\Gamma m\Gamma n}{\Gamma(m+n)}$.

2×10 = 20 Marks