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2B5N22188

(Pages : 2)

Reg. No:..................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

(Open Course)

BMT5D03 – Linear Mathematical Models

(2019 Admission onwards)

Time: 2 hours

Max. Marks: 60

Section A

All questions can be attended.

Each question carries 2 marks.

1. Find slope of the line joining the points (3,2) and (4,5).
2. Define linear cost function.
3. Define least square line.
4. Find the value of x, y and z if $\begin{bmatrix} x+1 & y-3 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ z & 4 \end{bmatrix}$.
5. Check whether $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ are inverses of each other.
6. Define feasible region.
7. State corner point theorem.
8. Graph the inequality $2x - 6y > 12$.
9. State duality theorem
10. Write the solution that can be read from the table.

x_1	x_2	x_3	s_1	s_2	z	
1	0	4	5	1	0	8
3	1	1	2	0	0	4
-2	0	2	3	0	1	28

11. Write the standard form of maximization problem.
12. Find the transpose of the matrix $A = \begin{bmatrix} 3 & 9 & 5 \\ 0 & 2 & 4 \end{bmatrix}$.

(Ceiling 20 Marks)

Section B

All questions can be attended.

Each question carries 5 marks.

13. Find the equation of the line passing through the point $(1, -2)$ and parallel to the line having the equation $6x - 3y = 12$.

14. $A = \begin{bmatrix} 4 & 10 \\ -2 & -3 \\ 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ -1 & 3 \\ 4 & 7 \end{bmatrix}$, find $2A + 3B$.

15. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 0 \\ 3 & 0 & 4 \end{bmatrix}$.

16. Calculate correlation coefficient

x	1	1	2	2
y	1	2	1	2

17. Find all the corner points of the feasible region for the system of inequalities

$$x + 3y \leq 6$$

$$2x + 4y \geq 7$$

18. State dual problem for linear programming problem

Minimize $\quad w = 3y_1 + 6y_2 + 4y_3 + y_4$

Subject to: $\quad y_1 + y_2 + y_3 + y_4 \geq 50$

$$2y_1 + 2y_2 + 3y_3 + 4y_4 \geq 275$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0 .$$

19. Write the initial simplex tableau for the linear programming problem

Maximize $\quad z = 7x_1 + x_2$

Subject to: $\quad 4x_1 + 2x_2 \leq 5$

$$x_1 + 2x_2 \leq 4$$

$$x_1 \geq 0, x_2 \geq 0$$

(Ceiling 30 Marks)

Section C

Answer any one question.

20. Solve the following system of equations using Gauss Jordan method.

$$x + y + z = 3$$

$$2x + 3y + 7z = 0$$

$$x + 3y - 2z = 17$$

21. Solve the linear programming problem graphically.

Maximize $\quad z = 3x + 6y$

Subject to: $\quad 2x - 3y \leq 12$

$$x + y \leq 5$$

$$3x + 4y \geq 24$$

$$x \geq 0, y \geq 0$$

(1× 10 = 10 Marks)

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2B5N22183 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

BMT5B05 – Abstract Algebra

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks: 80

Part A
All questions can be attended.
Each questions carries2 marks.

1. Find the multiplicative inverse of [11] in Z_{16}.
2. Find $\phi(100)$.
3. Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 7 & 5 & 4 & 8 & 6 & 3 \end{pmatrix}$ in S_8.
4. True or false: Intersection of two subgroups is again a subgroup.
5. Give an example of non abelian group.
6. Give an example of a group G and elements $a,b \in G$ such that a and b have finite order but ab does not.
7. Find the order of $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ in $GL_2(R)$.
8. Find the order of $Z_{12}^{\times}$
9. Check whether $Z_2 \times Z_2$ is isomorphic to Z_4.
10. Is $Z_5^{\times}$ isomorphic to $Z_8^{\times}$
11. Draw the subgroup diagram of Z_{12}.
12. Define integral domain.
13. Give an example of commutative ring.
14. Find all the units in the ring of integers Z.
15. True or false: Every finite integral domain is a field.

(Ceiling 25 marks)

Part B

All questions can be attended.
Each questions carries5 marks.

16. Let S=R-{-1}. Define * on S by a*b = a+b+ab. Show that (S,*) is a group.
17. State and prove Euler theorem.
18. Let S be a set and let ~ be an equivalence relation on S. Prove that each element of S belongs to exactly one of the equivalence classes of S determined by the relation ~.
19. Let G_1 and G_2 be groups. If $a_1 \in G_1$ and $a_2 \in G_2$ have orders n and m respectively. Prove that order of (a_1,a_2) in $G_1 \times G_2$ is lcm[n,m].
20. Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}\ \forall\, a,b \in G$.
21. Let Hbe a finite nonempty subset of G. Prove thatH is a subgroup of G if and only if H is nonempty and $ab \in H\ \forall\, a,b \in H$.
22. Show that $T = \left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : ad \neq 0\right\}$ is a subgroup of $GL_2(R)$, where $GL_2(R)$. is the set of all invertible 2 x 2 matrices with entries in R..
23. Prove that every subgroup of cyclic group is cyclic.

(Ceiling 35 marks)

Answer any two questions.
Each questions carries10 marks.

24. (a) Prove that any permutation in S_n, $n \geq 2$ can be .written as a product of transpositions.
    (b) Prove that the set $GL_n(R)$ forms a group under matrix multiplication, where $GL_n(R)$ is the set of all invertible n x n matrices with entries in R.
25. (a) State and prove Lagranges theorem.
    (b) Prove that any cyclic group is abelian.
26. (a) Let S be any set. If σ and τ are disjoint cycles in Sym(S). Prove that $\sigma\tau = \tau\sigma$.
    (b)Prove that the set of all even permutations of S_n is a subgroup of S_n.
27. (a)Prove that infinite cyclic group G is isomorphic to Z.
    (b) Prove that every group is isomorphic to a permutation group.

(2 x 10 = 20 marks)

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2B5N22187 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

BMT5B09 – Calculus of Multivariable – I

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A
All questions can be attended
Each question carries 2 marks

1. State Area bounded by a Polar curve.
2. Sketch the curve described by the parametric equations $x = t^2 - 4$ and $y = 2t$ where $-1 \le t \le 2$
3. Find the representation of the point $\left(4, \frac{\pi}{6}\right)$ in rectangular coordinates.
4. Find an equation in rectangular coordinates for the surface with spherical equation $\rho = 4\cos\phi$.
5. Sketch the graph of $y = x^2 - 4$ in the cylinder form.
6. Find an equation of the plane containing $P(2, -1, 3)$ and parallel to the plane defined by $2x - 3y + 4z = 6$.
7. Find the domain of the vector function $\boldsymbol{r}(t) = \langle\sqrt{t}, \frac{1}{1-t}, \ln t\rangle$
8. Define Curvature.
9. Find the derivative of $\boldsymbol{r}(t) = (t^2+1)\boldsymbol{i} + e^{-t}\boldsymbol{j} - \sin 2t\, \boldsymbol{k}$.
10. Evaluate $\lim_{(x,y)\to(2,4)} \sqrt[3]{\frac{8xy}{2x+y}}$
11. Use the chain rule to find $\frac{dw}{dt}$, where $w = \ln(x + y^2), x = \tan t, y = \sec t$.
12. Show that the function $u(x, y) = e^x \cos y$ is harmonic in the $xy-$ plane.

(Ceiling 20 marks)

Section B

All questions can be attended

Each question carries 5 marks

13. Sketch the graph of the polar equation $r = 1 + \cos\theta$.

14. Find the rectangular coordinates of the point $\left(3, \frac{\pi}{3}, \frac{\pi}{4}\right)$.

15. State the rules of the differentiation.

16. Evaluate $\int_0^1 \boldsymbol{r}(t)dt$ if $\boldsymbol{r}(t) = t^2\boldsymbol{i} + \frac{1}{1+t}\boldsymbol{j} + e^{-t}\boldsymbol{k}$.

17. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

18. Find the velocity vector , acceleration vector and speed of a particle with position vector $\boldsymbol{r}(t) = t\boldsymbol{i} + t^2\boldsymbol{j} + \frac{1}{t}\boldsymbol{k},\ t \geq 0$

19. Let $w = x^2y + y^2z^3$, where $x = r\cos s, y = r\sin s$ and $z = re^s$. Find the value of $\frac{\partial w}{\partial s}$ when $r = 1$ and $s = 0$.

(Ceiling 30 Marks)

Section C

Answer any one question

20. Consider the cardioid $r = 1 + \cos\theta$.
    (a) Find the slope of the tangent line to the cardioid at the point where $\theta = \frac{\pi}{6}$.
    (b) Find the points on the cardioid where the tangent lines are horizontal and where the tangent lines are vertical.

21. (a) Find the curvature of the parabola $y = \frac{1}{4}x^2$ at the points where $x = 0$ and $x = 1$.
    (b) Find the points where the curvature is largest.

(1 x 10 = 10 Marks)

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2B5N22185 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

BMT5B07 – Numerical Analysis

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A
All questions can be attended.
Each question carries 2 marks.

1. Determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$ in bisection method.
2. Define fixed point of a function $f(x)$. Also find the fixed points of $f(x) = x^2 - 6$.
3. Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find p_2.
4. Write first, second and third divided difference formula of $f(x)$.
5. Approximate $f''(2.0)$ for the following data

x	1.9	2.0	2.1
f(x)	12.703199	14.778112	17.148957

6. Given $f(0.5) = 0.4794, f(0.6) = 0.5646$, and $f(0.7) = 0.6442$. Approximate $f'(0.5)$.
7. The quadrature formula $\int_{-1}^{1} f(x)\, dx = c_0f(-1) + c_1f(0) + c_2f(1)$ is exact for all polynomials of degree less than or equal to two. Determine c_0, c_1 and c_2.
8. Approximate $\int_0^{\frac{\pi}{4}} x \sin x \, dx$ using the Midpoint rule.
9. Show that the initial-value problem $y' = \frac{4t^3y}{1+t^4}$, $0 \le t \le 1$ $y(0) = 1$ has a unique solution
10. Use Euler's method to approximate the solution for the initial-value problem $y' = 1 + \frac{y}{t}, 1 \le t \le 2, y(1) = 2$ with $h = 0.5$.
11. Write Taylor's formula of order n to approximate the solution for the initial-value problem
12. Use open formula with $n = 1$ approximate $\int_0^{\frac{\pi}{4}} \sin x \, dx$.

(Ceiling ....20 Marks)

Section B

All questions can be attended.

Each question carries 5 marks.

13. Use the Bisection method to find p_4 for $f(x) = \sqrt{x} - \cos x = 0$ on $[0, 1]$.

14. Use false position method, find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ correct to three decimal places.

15. Determine the maximum error when the second Lagrange polynomial for $f(x) = \frac{1}{x}$ on $[2,4]$, using the nodes $x_0 = 2, x_1 = 2.75$, and $x_2 = 4$ is used to approximate $f(x)$ for $x \in [2,4]$.

16. Approximate $f(0.43)$ using the following data and the Stirling's formula

x	0.0	0.2	0.4	0.6	0.8
f(x)	1.00000	1.22140	1.49182	1.82212	2.22554

17. Use the most accurate three-point formula to determine each missing entry in the following table.

x	$f(x)$	$f'(x)$
2.0	**3.6887983**	
2.1	**3.6905701**	
2.2	**3.6688192**	
2.3	**3.6245909**	

18. Approximate $\int_0^1 x^2 e^{-x}\, dx$ using Trapezoidal rule and Simpson's rule. Also find a bound for the error in each method.

19. Use midpoint method to approximate the solution for the initial-value problem

$y' = -5y + 5t^2 + 2t, 0 \le t \le 1, y(0) = \frac{1}{3}$ with $h = 0.5$

$(\boldsymbol{Ceiling \ldots .30\ Marks})$

Section C

*Answer any **One** question.*

20. a) Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$.

b) The following data are given for a polynomial P(x) of unknown degree.

X	0	1	2
P(x)	2	-1	4

Determine the coefficient of x^2 in P(x) if all third-order forward differences are 1.

21. Approximate the solution for the initial-value problem

$y' = t^2 e^t + \frac{2y}{t}, 1 \le t \le 2, y(1) = 0$ with $h = 0.5$.

Using a) Euler's method b) Runge kutta method of order 4.

$(\boldsymbol{1 \times 10 = 10\ Marks})$

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(Pages : 2)

Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

BMT5B06 – Basic Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

Section A

All questions can be attended. Each question carries 2 marks.
Cieling 25 marks.

1. Define a denumerable set. Prove that the set of even natural numbers is a denumerable set.
2. State the Cantor's theorem.
3. If $a \in \mathbb{R}$ and $a \neq 0$ then prove that $a^2 \geq 0$.
4. Write the set of real numbers x satisfying $|x-1| < |x-5|$.
5. Using the density theorem, prove that there exists an irrational number between any two rational numbers.
6. Give an example of a nested sequence of intervals whose intersection is empty.
7. Using definition of limit, prove that $\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right) = 1$.
8. If $X = (x_n)$ and $Y = (y_n)$ are sequences which are convergent respectively to x and y, then prove that $X+Y$ is convergent to $x+y$.
9. Give example of two divergent sequences $X = (x_n)$ and $Y = (y_n)$ such that their product XY is a convergent sequence.
10. Define subsequence of a sequence.
    Give example of a sequence for which two subsequences converge to different limits.
11. Prove that a convergent sequence is a Cauchy sequence.
12. Define a contractive sequence. Give one example.
13. Write the real part of $\frac{(2+3i)(3-i)}{1+i}$.
14. If $z = \sqrt{3} - i$, then simplify z^6.
15. Write the modulus and principal argument of $z = -1 - i$.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 35 marks.

16. Prove that there does not exist a rational number whose square is 2.

17. State and prove Bernoulli's inequality.

18. Prove that for a bouned set $A \subseteq \mathbb{R}$, $\sup(-A) = -\inf(A)$.

19. If $\{I_n = [a_n, b_n] : n \in \mathbb{N}\}$ is a nested sequence of closed and bounded intervals, then prove that $\cap_{n=1}^{\infty} I_n$ is non-empty.

20. Prove that a monotone sequence is convergent if and only if it is bounded.

21. Let (x_n) be a sequence given by $x_1 = 1, x_2 = 2$ and $x_{n+2} = \frac{1}{2}(x_n + x_{n+1})$. Prove that (x_n) is a convergent sequence. *(No need to find the limit.)*

22. Find all cube roots of $-8i$.

23. Define the terms **Open Set**, **Domain** and **Region**. Describe the region given by the condition $|z-3+4i| < |z+3-4i|$.

Section C

Answer any Two Questions. Each question carries 10 marks.
20 marks from this section.

24. Prove that the set of sequences whose terms are either 0 or 1 only is an uncountale set.

25. Using Nested Intervals Property, prove that $\mathbb{R}$ is uncountable.

26. State and prove Bolzano Weierstrass Theorem.

27. Define the limit of a complex valued function at a point z_0 in a domain. Give example of a function having limit at a point in its domain. Prove that the function $f(z) = \frac{z}{\bar{z}}$ does not have a limit at 0.

2B5N22186

(Pages :3) Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2022

BMT5B08 – Linear Programming

(2019 Admission onwards)

Time: 2 hours

Max. Marks: 60

Section A

All questions can be attended, Each question carries 2 marks

1. Explain the terms objective function, constraint set and feasible solution with an example
2. Convert the LPP in to canonical form

$$Minimize\ \ g(x,y) = x - y$$
$$subj\ to\ \ 2x - y \geq -1$$
$$0 \leq x \leq 2$$
$$y \geq 0$$

3. An oil company owns two refineries, say refinery A and refinery B. Refinery A is capable of producing 20 barrels of gasoline and 25 barrels of fuel oil per day; refinery B is capable of producing 40 barrels of gasoline and 20 barrels of fuel oil per day. The company requires at least 1000 barrels of gasoline and at least 800 barrels of fuel oil. If it costs $300 per day to operate refinery A and $500 per day to operate refinery B. Formulate as a mathematical problem
4. Discuss a convex set and extreme point of a convex set with an example.
5. Consider$(2,1), (4,-2) \in R^2$. Find the line segment between (2,1) and (4,–2) and identify any three points on it
6. Write the Pivote Transformation algorithm for maximization table
7. Trace the solution geometrically

$$Maxf = x + 3y$$
$$Subj\ to\ \ x + 2y \leq 10$$
$$-3x - y \leq -5$$
$$x, y\ unconstrained$$

8. For any pair of feasible solutions of dual canonical linear programming problems, prove that $g \geq f$.

9. State the canonical maximization linear programming problem represented by the tableau given below.

x	y	-1	
1	2	3	$= -t_1$
4	5	6	$= -t_2$
7	8	9	$= f$

10. Define balanced transportation problem with an example.

11. Find the basis using VAM method

	M_1	M_2	M_3	
W_1	2	1	2	40
W_2	9	4	7	60
W_3	1	2	9	10
	40	50	20	110

12. Explain the concept cycles in a transportation problem with an example.

(Ceiling 20marks)

Section B

All questions can be attended, each question carries 5marks

13. Write the next table using Pivot transformation

x_1	x_2	-1	
-20	-40	-1000	$= -t_1$
-25	-20*	-800	$= -t_2$
-300	-500	0	$= -g$

14. Define the negative transpose of the minimum tableau and write the simplex algorithm for minimum tableaus

15. Solve the noncanonical linear programming problem

$$Min\ f(x,y,z) = 3x + y + 2z$$

$$Subj\ to\ x + 2y + 3z \geq 24$$

$$2x + 4y + 3z = 36 \quad x, y, z \geq 0$$

16. State and prove Duality theorem

17. Write the dual simplex algorithm for Maximum tableaus

18. Solve

	M_1	M_2	M_3	M_4	
W_1	5	12	8	50	26
W_2	11	4	10	8	20
W_3	14	50	1	9	30
	15	20	26	15	

19. Solve

	J_1	J_2	J_3	
P_1	0.5	2	1	1
P_2	1.2	1/6	7	1
P_3	5/9	0	3.14	1
	1	1	1	

(Ceiling 30marks)

Section C

Answer any one question

20. Solve

	(x_1)	(x_2)	x_3	-1	
(y_1)	1	-1	2	1	= -0
y_2	2	0	2	-1	= $-t_1$
y_3	0	1	-1	-1	= $-t_2$
-1	1	-1	3	0	= f
	= 0	= 0	= s_1	= g	

21. An appliance company manufactures heaters and air conditioners. The production of one heater requires 2 hours in the parts division of the company and 1 hour in the assembly division of the company; the production of one air conditioner requires 1 hour in the parts division of the company and 2 hours in the assembly division of the company. The parts division is operated for at most 8 hours per day and the assembly division is operated for at most 10 hours per day. If the profit realized upon sale is \$30 per heater and \$50 per air conditioner, how many heaters and air conditioners should the company manufacture per day so as to maximize profits? Solve by simplex algorithm

(1X10= 10marks)