QU 17

2B4A22529 (Pages : 2) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, April 2022

BMT4B04 – Linear Algebra

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

PART A

Answer all questions. Each question carries 2 marks.
Maximum mark from this section is 25.

1. Show that the empty set $\emptyset$ cannot be the subspace of any vector space.
2. Show that the product of two matrices is symmetric if and only if the matrices commute.
3. Find the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$.
4. Show that the matrices A and P^{-1}AP have the same determinant.
5. State the Plus/Minus theorem
6. Give an example for a system of equation with two equations and having no solution.
7. Find k if $\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$
8. Show by an example , a matrix as a transformation from R^4 to R^3
9. Define a) Row equivalent Matrices b) Elementary Matrices
10. Show that a finite set that contains the element zero is linearly dependent.
11. Use the arrow technique to evaluate the determinant $\begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix}$.
12. Find the wroskian of the functions $f_1(x) = 1$, $f_2(x) = e^x$, $f_3(x) = e^{2x}$.
13. Define the a) Basis of a vector space b) Dimension of a vector space.
14. Which is the standard basis of R^3. Give an example for an infinite dimensional vector space.
15. Write a basis for P_n, the vector space of polynomials of degree $\leq n$. What is its dimension.

PART B

Answer all questions. Each question carries 5 marks.
Maximum mark from this section is 35.

16. Check whether the vectors $u_1 = (1, 2, 3)$, $u_2 = (2, 5, 7)$ and $u_3 = (1, 3, 5)$ in R^3 are linearly independent or linearly dependent.

17. Consider the bases B = { (-3, 0, -3) , (-3, 2, -1) (1, 6, -1) } and B' = { (-6, -6, 0) , (-2, -6, 4) (-2, -3, 7) } for R^3 . Find the transition matrix from B to B'

18. Find the matrix P that diagonalises the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.

19. Show that, If A is an invertible matrix then $A^T A$ is also invertible

20. Use the inversion algorithm to find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

21. Define a Vector Space and give two examples.

22. Express the vector V = (3, 7, -4) in R^3 as a linear combination of the vectors $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 7)$ and $u_3 = (3, 5, 6)$

23. Show that the intersection of subspaces of a vectorspace is again a subspace.

PART C

Answer any TWO questions.
One question carries 10 marks.

24. Show that the set $S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ is a basis for $M_{2\times2}$, the vector space of all 2× 2 matrices. Find the coordinates of $A = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$ relative to this basis.

25. If W is a subspace of a finite dimensional vector space V then prove that
    a) W is finite dimensional
    b) dim(W) ≤ dim (V)
    c) W = V if and only if dim (W) = dim (V)

26. Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

27. State the Rank-Nullity theorem for matrices. Verify it for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$

(2 x 10 = 20 Marks)

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3B4A22530 (Pages : 2) Reg. No:.........................

Name:

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, April 2022

BMT4C04 – Mathematics – 4

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

PART A

All questions can be attended.
Each question carries 2 marks.

1. What is an ordinary differential equation (ODE)? Give one example.
2. What is mean by a trivial solution to a differential equation? Give an example for a differential equation having no trivial solution.
3. What are the values of m so that $y = x^m$ is a solution to the differential equation $xy'' + 2y' = 0$
4. State the superposition principle on homogeneous *n*th-order differential equations.
5. Convert the equation $6xydx + (9x^2 + 4y)dy = 0$ into an exact differential equation and verify that the new equation is exact.
6. Show that the set $\{y_1, y_2\}$ where $y_1 = e^{3x}$, $y_2 = e^{-3x}$ is a fundamental set of solutions to the differential equation $y'' - 9y = 0$ on $(-\infty, \infty)$.
7. Solve the differential equation $x^2y'' - 2xy' - 4y = 0$
8. Define the Laplace transform of a function f(t), $t \geq 0$. Using the definition find the Laplace transform of constant function.
9. Find the inverse Laplace transformation of $F(s) = \frac{s}{s^2+1}e^{\frac{-\pi s}{2}}$
10. State the first shifting theorem on Laplace transforms. Using the theorem write the Laplace transform of $e^{5t}t^3$
11. To what functional value the Fourier series of the function *f* defined by $f(x) = 0$ for $-\pi < x < 0$, $f(x) = \pi - x$ for $0 \leq x < \pi$ does converge at $x = 0$?
12. Give an example for (a) hyperbolic linear second-order PDE (b) elliptic linear second-order PDE. Justify your claims.

Ceiling 20 Marks

PART B

All questions can be attended.
Each question carries 5 marks.

13. What is the difference between a particular solution and a singular solution to a differential equation? Obtain a particular solution and a singular solution to the differential equation $dy/dx = xy^{1/2}$.
14. Solve the differential equation $x\,y' - (1 + x)y = xy^2$
15. Solve the differential equation $x\,y' - 4y = x^6e^x$. Give the largest interval over which the general solution is defined.
16. The function $y_1 = x^2$ is a solution of $x^2\,y'' - 3x\,y' + 4y = 0$. Find the general solution using the method of reduction of order.
17. Evaluate the inverse Laplace transform for the function $(2s + 5)/(s - 3)^2$.
18. Expand $f(x) = x, \quad -2 < x < 2$ in a Fourier Sine series.
19. Using the method of variation of parameters solve $y'' + y = \sec x$.

Ceiling 30 Marks

PART C

Answer any question . Each question carries 10 Marks

20. (a) Using the method of undetermined coefficients solve $y'' - y' + y = 2\sin 3x$

    (b) Solve using the Laplace transform: $y'' - 6\,y' + 9y = t^2e^{3t}, \quad y(0) = 2, \ y'(0) = 17$
21. Find the Fourier Series of the function f defined by

    $f(x) = 0$, for $-\pi < x < 0$, $f(x) = x^2$, for $0 \le x < \pi$.

    Deduce that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \ldots\ldots\ldots$

(1 x 10 = 10 Marks)