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(Pages : 2)

Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Psychology Degree Examination April 2022

BST2C06 – Regression Analysis and Probability Theory

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

SECTION-A

Each question carries 2 Marks.

Maximum Marks that can be scored in this section is 20.

1. Define correlation. Give the implication for $\rho = -1$.
2. Define Spearman's rank correlation coefficient.
3. Give the formula for the two regression coefficients in terms of correlation coefficients.
4. Draw the scatter diagrams for positive, negative and zero correlations.
5. Give the normal equations for linear regression.
6. Find b_{xy} , if $3x + 2y + 4 = 0$ is the regression equation of x on y.
7. Write down the sample space corresponding to tossing a coin thrice.
8. State the statistical definition of probability.
9. Let A and B be independent events. If $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A \cap B)$.
10. State the addition theorem on probability.
11. If A and B are independent and exhaustive events and $P(A) = 0.6$. Find $P(B)$.
12. Check if the following p.m.f. or not

x	-1	0	1
$f(x)$	0.25	0.3	0.5

SECTION-B

Each question carries 5 Marks.

Maximum Marks that can be scored in this section is 30.

13. Why there are two regression lines. Where will they meet?
14. From the data given below, obtain the regression equation of x on y.

X:	2	3	7	8	10
Y:	10	9	11	8	12

15. Distinguish between partial and multiple correlation.

16. There are 17 balls numbered from 1 to 17 in a bag. If a person selects one at random, what is the probability that the number printed on the ball is an even number greater than 9.

17. Given A, B and C are independent events, $P(A) = 0.3$, $P(B) = 0.2$ and $P(C) = 0.4$. Find the probability for (i) all occurring, (ii) none occurring and (iii) exactly one occurring.

18. Given $P(A) = 0.5$, $P(A \cup B) = 0.7$ and $P(A|B) = 0.5$. Find $P(B)$.

19. Define p.m.f.

x	0	1	2	3
f(x)	0.2	0.35	0.2	k

If the table above represents a p.m.f. Obtain the value of k and $P(1 \leq X < 3)$.

SECTION-C

(Answer *any one* Question and carries 10 marks)

20. Find the point of intersection of the regression lines $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Also obtain the coefficient of correlation.

21. (i) Define conditional probability.

(ii)20% of all students are graduates and 80% are undergraduates. The probability that a graduate student is married is 0.5 and an undergraduate student is married is 0.1. One student is selected at random. What is the probability that the student selected is married.

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(Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination April 2022

BST2B02 – Bivariate Random Variables & Probability Distributions

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

PART A
Each question carries 2 marks

1. Show that $M_{ax+b}(t) = e^{bt}M_X(at)$.
2. State the properties of characteristic function.
3. Write the relationship between raw and central moments.
4. Find$E(X)$, where X denotes the number on a balanced die when thrown.
5. Define independence of two random variables.
6. Define the joint probability mass function of a bivariate random variable.
7. State and prove addition theorem of expectation.
8. Define conditional variance.
9. Let the moment generating function of a random variable X is$(0.6 + 0.4e^t)^8$, then write down the moment generating function of $Y = 3X + 2$ and also find $E(Y)$.
10. State Cauchy-Schwartz inequality.
11. Define covariance of (X, Y) in terms of expectations.
12. Define convergence in probability.
13. Write the binomial distribution for which mean is 6 and standard deviation is 2.
14. State Weak law of large numbers.
15. What is meant by a degenerated random variable?

Maximum Mark = 25

PART B
Each question carries 5 marks

16. Let X be a random variable with probability mass function

x	0	1	2	3
f(x)	1/3	1/2	1/24	1/8

Find $E(X-1)^2$.

17. Suppose that X is a random variable for which $E(X) = 10$ and $Var(X) = 25$. Find the values of a and b such that $Y = aX - b$ has expectation 0 and variance 1.

18. X and Y are discrete random variables having joint probability mass function $f(x,y) = \frac{2x+y}{27}, x = 0,1,2;\ y = 0,1,2$. Examine whether X and Y are independent.

19. Given $f(x,y) = 2, 0 < x < y < 1\ and\ f(x,y) = 0\ , elsewhere$. Find $E(Y/X = x)$.

20. Prove that $-1 \le r_{xy} \le 1$, where r_{xy} is Pearson's coefficient of correlation between any two random variables X and Y.

21. Establish the recurrence relation for the moments of Poisson distribution.

22. If $E(X) = 5, Var(X) = 3$ and if $P\ [|X - 5| < h] \ge 0.99$. Find the value of h.

23. If X and Y are independent Poisson variates, find the conditional distribution of X given $X + Y$. Identify the distribution.

Maximum Mark = 35

PART C

Each question carries 10 marks (Answer any TWO Questions)

24. (i)What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trail?

    (ii)A random variable X has the p.d.f. $f(x) = Ke^{-|x|}, -\infty < x < \infty$. Find the value of K. Also find the moment generating function.

25. Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} 2 - x - y; 0 \le x \le 1, 0 \le y \le 1 \\ 0, otherwise \end{cases}$

    Find (i) Marginal Probability density functions of X and Y.

    (ii)Conditional density functions.

26. If $f(x,y) = \begin{cases} \frac{x+3y}{24} \\ 0,\ otherwise. \end{cases}, x = 1,2; y = 1,2.$

    Compute $E(X|Y = 2), Var(X|Y = 2)$.

27. State and prove Chebycheff's inequality. A random variable X has mean 5 and variance 3. Find the lower bound for $P[|X - 5| < 3]$.

(2 x 10 = 20 Marks)

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Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination April 2022

BST2C02 –Probability Theory

(2019 Admission onwards)

Time: 2hours

Max. Marks : 60

Section-A

Each question carries 2 Marks

Maximum Marks that can be scored in this part is 20

1. Define probability mass function and state any two of its properties
2. Define the following with examples

   (a) Sample space

   (b) Mutually exclusive events
3. If the events $A = \{a,b,c,d,e\}$ and $B = \{d,f,g\}$ are exhaustive events, identify the events

   (i) $A \cap B^c$ (ii) $(A \cup B)^c$
4. If $P(A) = 0.5, P(B) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(A/B^c)$.
5. Define σ - field
6. Obtain the distribution function of X, with p.d.f $f(x) = 3x^2, 0 < x < 1$.
7. Find the p.m.f of $Y = X^2$, when the p.m.f of X is as follows

X	-2	-1	0	3
$f(x)$	1/4	1/4	1/4	1/4

8. If $E(X) = 3, E(X^2) = 25$. Find $V(3x-2)$
9. Show that $E(X^2) \geq [E(X)]^2$
10. Define characteristic function of a random variable and state its advantages over m.g.f
11. Find c if $f(x,y) = c(x+2y), x = 1,2, y = 0,1$. is the joint p.m.f of (x,y)
12. Define marginal probability density function

Section-B

Each question carries 5 Marks

Maximum Marks that can be scored in this part is 30

13. Explain the axiomatic definition of probability. Using this definition establish $0 \le P(A) \le 1$ for an event A.

14. In a box there are 8 white, 6 blue and 10 pink balls. If 3 balls are drawn at random from the box; what is the probability that

    (i) Two balls are white

    (ii) None of 3 is pink

    (iii) Three balls are blue

15. Given the p.d.f of X as $f(x) = 1$ for $0 < x < 1$. Find the p.d.f of $Y = -2\log_e X$

16. A continuous random variable has distribution function

$$F(x) = \begin{cases} 0, x < 0 \\ c(6x^2 + x^4), 0 \le x \le 2 \\ 1, x \ge 2 \end{cases}$$

Find (i) the p.d.f of X (ii)the value of c and (iii) $P(-1 < X < 1)$

17. State and prove interrelationship between raw moments and central moments

18. Define m.g.f of a random variable. Examine the effect of the shifting of the origin and change of scale in the m.g.f of a random variable

19. For two random variables X and Y, prove that

    $(i)\ V(X - Y) = V(X) + V(Y) - 2Cov(X,Y)$

    $(ii)\ Cov(X - a, Y - b) = Cov(x, y)$ *where a and b are any two consatnts*

Section-C

Answer any one question and carries 10 Marks

20. Let X and Y are two random variables with joint p.d.f $f(x,y) = 2,\ 0 < x < y < 1$. Find

    (a) correlation coefficient between X and Y and (b) $V(X/_{Y=y})$

21. (a) if A and B are two independent events. Prove that A^c and B^c are also independent

    (b)Define the mutually independence of three events A, B and C. Also illustrate that the pairwise independence of A, B and C need not imply their mutual independence.

(1 x 10 = 10 Marks)

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(Pages : 2)

Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination April 2022

BAS2C02 –Life Contingencies

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

PART A

(Each question carries *two* marks .Maximum 20 Marks)

1 Define select mortality

2. Write down an expression for $_tq_x$ in terms of the function l_x

3. State analytical laws of mortality

4. Define n-year tem assurance contract

5. Evaluate A_{40} and $\bar{A}_{40}$ based on AM92 Ultimate mortality at 4% *pa* interest.

6. Claire, aged exactly 30, buys a whole life assurance with a sum assured of £50,000 payable at the end of the year of her death. Calculate the expected present value of this benefit using AM92 Ultimate mortality and 6% *pa* interest.

7. Define whole life annuity payable in arrears

8. Calculate the value of $\ddot{a}_{[40]:20}$ using AM92 mortality and 4% *pa* interest.

9. Describe $_tP_{xy}$ and $_tq_{xy}$

10. Assuming that both lives are independently subject to AM92 mortality ,

    Calculate $_3P_{45:41}$ and $\mu_{38:30}$.

11. Define contingent assurances and reversionary annuities.

12. Calculate

    (a) $p_{\overline{62:65}}$

    (b) $_3q_{\overline{50:50}}$

PART B

(Each question carries *five* marks. Maximum30 Marks)

13.

(a) Calculate the value of $_{0.5}P_{58}$ using ELT15 (Females) mortality, assuming a uniform distribution of deaths between integer age.

(b) A population is subject to a constant force of mortality of 0.015 *pa*. Calculate probability that a life aged exactly 20 dies before age 21.25.

14. Explain critical illness assurance contracts.

15. A life insurance company issues a 3-year term assurance contract to a life aged exactly 42. The sum assured of 10,000 is payable at the end of the policy year of death. Calculate the expected present value of these benefits assuming AM92 Select mortality and an interest rate of 5% *pa* effective.

16. Define commutation function D_x and calculate a_{30}, $10|a_{30}$, and $10|a_{70}$, based on AM92 mortality and 4% *pa* interest.

17. Calculate the values of $\ddot{a}_{60:10}$ and $\bar{a}_{60:10}$ using AM92 mortality and 6% pa interest

18. prove that $n^{2}_{q_{xx}} = \frac{1}{2} {}_{n}q_{\overline{xx}}$

19. Ralph and Betty are both aged 65 exact. Upon Betty's death, Ralph will receive £20,000 *pa* payable annually in advance for the rest of his life starting from the end of the year of Betty's death, provided that Betty dies in the next 10 years. Ralph's mortality follows PMA92C20, Betty's mortality follows PFA92C20 and the interest rate for all future years is 4% *pa*. Calculate the EPV of this benefit to Ralph

PART C

(Each question carries *ten* marks .Maximum 10 Marks)

20. Given that p_{80}= 0.988 calculate ${}_{0.5}p_{80}$ assuming:

    i. A uniform distribution of deaths between integer ages

    ii. A constant force of mortality between integer ages

21. Derive the relationship between insurance payable at moment of death and the end of year of death.

(1 x 10 = 10 Marks)