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(Pages : 2)

Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination April 2022

BMT2B02 – Calculus - 2

(2019 Admission onwards)

Time: 2.5 hours

Max. Marks : 80

Section A
All Questions can be attended
Each question carries 2 marks (Ceiling 25 marks)

1. Find the area of the region bounded by the curves $y = x^2$ and $y = x$
2. Find the volume of the solid obtained by revolving the region under the graph y=$\sqrt{x}$ on$0 \leq x \leq 2$ about x-axis
3. Find the arc length of the curve $y = 2x + 1,\ 0 \leq x \leq 3$
4. Find the derivative of $f(x) = ln\sqrt{1 + x^2}$
5. Solve $e^{2-3x} = 6$
6. Show that $e^{x_1}e^{x_2} = e^{x_1+x_2}$
7. Show that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
8. When you say that a real sequence $\{a_n\}$ is convergent to a point L, give an example for convergent sequence
9. Show that the series $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ is divergent
10. When you say that a sequence $\{a_n\}$ is monotonically increasing sequence, also give an example for it
11. What you mean by bounded sequence, give an example for bounded sequence which is convergent
12. Using Squeeze theorem, evaluate$\lim_{n\to\infty} \frac{(-1)^n}{n}$
13. Determine whether the series $\sum_{n=1}^{\infty} 5\left(\frac{4}{3}\right)^{n-1}$ is convergent or not, justify your answer
14. Let $y = x^x$, find the derivative of y
15. Find the integral $\int_0^3 2^x\, dx$

Section B

All Questions can be attended

Each question carries 5 marks (Ceiling 35 marks)

16. Find the area of the surface obtained by revolving the graph $x = y^3$ on $0 \le y \le 1$ about y-axis

17. Show that $\ln(ab) = \ln a + \ln b$

18. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ on $(-\infty, \infty)$

19. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

20. Find the value of p for which $\int_1^\infty \frac{1}{x^p} dx$ is convergent and divergent

21. Describe the improper integral of First kind, Second kind and Third kind, also give examples to each cases

22. State "The Alternate Series Test", hence prove that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is convergent

23. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}$

Section C

Answer any two questions

Each question carries 10 marks (2 X 10 = 20)

24. a) Evaluate the improper integral $\int_{-\infty}^{0} xe^x dx$

 b) Show that $\lim_{n \to \infty} \frac{n!}{n^n} = 0$

25. a) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

 b) Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent

26. a) Find the Maclarin series expansion of $f(x) = \frac{1}{1-x}$

 b) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$

27. a) Find the Maclarin series of $f(x) = x^2 \sin 2x$

 b) Find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$

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(Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Degree Examination April 2022

BMT2C02 – Mathematics 2

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended

Each question carries 2 marks

1. What is the polar coordinates of $(x, y) = (2, -4)$?
2. Find $\int \frac{dx}{\sqrt{4+x^2}}$.
3. Find the length of the graph of $f(x) = (x-1)^{\frac{3}{2}} + 2$ on $[1, 2]$.
4. Evaluate $\lim_{n\to\infty} \frac{n - 3n^2}{n^2+1}$.
5. Write down the Maclaurien series for $\sin x$.
6. Define absolutely convergent and conditionally convergent series.
7. Give the standered basis for the vector space R^n.
8. Define spanS, where $S \subset R^n$.
9. Give two properties of determinant.
10. State criterion for Orthogonal Diagonalizability.
11. Define LU- factorization of a square matrix A.
12. State Cayley Hamilton theorem.

(Ceiling: 20 Marks)

Turn over

Section B

All questions can be attended

Each question carries 5 marks

13. Find the slope of the line tangent to the graph of $r = cos3\theta$ at $(r, \theta) = (-1, \frac{\pi}{3})$.

14. Prove that $\tanh^{-1}x = \frac{1}{2}ln[\frac{(1+x)}{(1-x)}]$, $-1 < x < 1$.

15. Evaluate the area of the surface obtained by revolving the graph x^3 on $[0, 1]$ about the x axis.

16. Test for convergence (i)$\sum_{n=1}^{\infty} \frac{n^n}{n!}$ and (ii)$\sum_{n=1}^{\infty} \frac{3^n}{n^n}$

17. Find the Taylor series for $\frac{1}{1+x^2}$ at $x_0 = 1$.

18. Approximate $\int_0^{\frac{\pi}{2}} (x^2 + 1)dx$ with $n = 10$ using Trapezoidal Rule and Simpson's Rule.

19. Use Gaussian elimination method to solve

$$2x_1 + 6x_2 + x_3 = 7$$
$$x_1 + 2x_2 - x_3 = -1$$
$$5x_1 + 7x_2 - 4x_3 = 9.$$

(Ceiling: 30 Marks)

Section C

Answer any one question

Question carries 10 marks

20. (a) Find the length of the cardioid $r = 1 + cos\,\theta$ $(0 \leq \theta \leq 2\pi)$.

    (b) For which x does the series $\sum_{i=0}^{\infty} \frac{2}{\sqrt{i+1}} x^i$ converges?

21. (a) Use Gram-Schmidt orthogonalization process transform the basis B = {(1, 1, 0),(1, 2, 2), (2. 2, 1)} into an orthonormal basis.

    (b) Find the eigenvalues and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$.

(1×10 = 10 Marks)