1B1N22290 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Mathematics Degree Examination, November 2022

BMT1B01 – Basic Logic and Calculus – I

(2022 Admission onwards)

Time : 2 ½ hours Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks

1. Write the contra positive of the implication:
   'If London is in France, the Paris is in England'
2. Verify $p \to q \equiv \sim p \vee q$
3. Negate the proposition $\forall x(x^2 = x)$ where UD is the set of integers.
4. Find $\lim_{x\to -1}(3x^2 + 2x + 1)^5$
5. Use the graph to find the limit: $\lim_{x\to 3^-}[x]$
6. Find $\lim_{x\to 0}\frac{\sin 2x}{3x}$
7. Given $\lim_{x\to 2} 3x = 6$. Find the value of δ that corresponds to $\epsilon = 0.01$
8. Verify that $f(x) = x^2 - 4x + 3$ satisfies the hypothesis of Rolle's Theorem on $[1, 3]$. Find the value of c.
9. Determine the intervals where $f(x) = x^{\frac{2}{3}}$ is concave upward and where it concave downward.
10. Find $\lim_{x\to -\infty}\frac{x^2+1}{x-2}$
11. Find dimensions of the rectangle of greatest area that has it's base on the x-axis and is inscribed in the parabola $y = 9 - x^2$.
12. Find $\int \frac{2x^2-1}{x^2}dx$
13. Evaluate $\sum_{k=1}^{10} 3k^2(2k + 1)$
14. Evaluate the definite integral by interpreting geometrically: $\int_0^4 \sqrt{16 - x^2}\, dx$
15. Suppose $\int_1^6 f(x)dx = 8$ and $\int_4^6 f(x)dx = 5$. Evaluate: $\int_1^4 f(x)dx$.

(ceiling 25 Marks)

Section B
All questions can be attended
Each question carries 5 marks

16. Prove directly that the product of two odd integers is an odd integer.

17. Let $P(x,y): y < x^2$ and x and y are real numbers. Determine the truth values of :

    i) $(\forall x)(\forall y)P(x,y)$ ii) $(\exists x)(\forall y)P(x,y)$

18. Let $f(x) = \begin{cases} 2x-4, if\ x<4 \\ x-2, if\ x \geq 4 \end{cases}$. Sketch the graph and evaluate

    $\lim_{x\to 4^-} f(x)$, $\lim_{x\to 4^+} f(x)$ *and* $\lim_{x\to 4} f(x)$

19. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$. Use the result to approximate $\sqrt{3.9}$

20. Find the extreme values of $f(x) = 3x^4 - 4x^3 - 8$ on $[-1,\ 2]$

21. Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using second derivative test.

22. The velocity function of a car moving along a straight road is given by $v(t) = t - 20,\ \ 0 \leq t \leq 40,$ where $v(t)$ is measured in feet per second and t in seconds. Find the position of the car at $t = 40$.

23. Find $\frac{dy}{dx}$ if $y = \int_0^{x^3} \cos t^2 dt$

(ceiling 35 Marks)

Section C
Answer any two questions
Each question carries 10 marks

24. Check the validity of the following argument:

    Babies are illogical.

    Nobody is despised who can manage a crocodile.

    Illogical persons are despised.

    ∴ Babies cannot manage crocodiles

25. a) Show that the function defined by $f(x) = \sqrt{4-x^2}$ is continuous on $[-2,\ 2]$

    b) Let $f(x) = \begin{cases} x+2, if\ x \leq 1 \\ kx^2, if\ x > 1 \end{cases}$, find the value of k that will make f continuous on $(-\infty, \infty)$.

26. Sketch the graph $f(x) = \frac{4x-4}{x^2}$

27. a) Show that $\int_a^b x\,dx = \frac{1}{2}(b^2 - a^2)$

    b) Compute Riemann sum for $f(x) = 4 - x^2$ on $[-1,3]$ using five subintervals and choosing evaluation points at the midpoints.

(2 x 10 = 20 Marks)

1B1N22291 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc (Chemistry, Physics & Statistics) Degree Examination, November 2022

BMT1C01– Mathematics – I

(2022 Admission onwards)

Time : 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks.

1. Find the limit $\lim_{x\to 2} \frac{x^4-16}{x-2}$.
2. Discuss the behaviour of $f(x) = \frac{1}{x^2}$ near $x = 0$.
3. Suppose that a function $f(x)$ is defined for all x in the interval $[-1, 1]$. Can anything be said about the existence of $\lim_{x\to 0} f(x)$? Give reasons.
4. If $\lim_{x\to -2} \frac{f(x)}{x^2} = 1$, then find $\lim_{x\to -2} f(x)$.
5. Find the absolute maximum and minimum values of $f(x) = -x - 4$ on the interval $[-4,1]$.
6. Find $\lim_{x\to\infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.
7. If $1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2\cos x} < 1$ for all x close to zero, find $\lim_{x\to 0} \frac{x \sin x}{2 - 2\cos x}$.
8. Find the slope of the curve $y = \frac{1}{x-1}$ at $x = 3$.
9. State intermediate value property of derivatives.
10. Suppose it costs $c(x) = x^3 - 6x^2 + 15x$ dollars to produce x radiators when 8 to 30 radiators are produced. A shop currently produces 10 radiators a day. About how much extra will it cost to produce one more radiator a day.
11. Find $\lim_{x\to 0} \frac{8x^2}{\cos x - 1}$.
12. State mean value theorem for definite integral.

(Ceiling: 20 Marks)

Section B

All questions can be attended.
Each question carries 5 marks.

13. Define $g(4)$ in a way that extends $g(x) = \frac{4x-x^2}{2-\sqrt{x}}$ to be continuous at $x = 4$.
14. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?
15. Find the derivative of $g(t) = \tan(5 - \sin 2t)$.
16. Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.
17. How rapidly will the fluid level inside a vertical cylindrical tank drop if we pump the fluid out at the rate of 3000 L/min.
18. Find $\frac{dy}{dx}$ if $y = \int_0^{\sqrt{x}} \sin(t^2)\, dt$.
19. If f is continuous on $[a, b]$, then prove that $F(x) = \int_a^x f(t)dt$ has a derivative at every point of $[a, b]$ and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x),\ a \le x \le b$.

(Ceiling: 30 Marks)

Section C

Answer any one question.
Question carries 10 marks.

20. i) The curve $y = ax^4 + bx + c$ passes through the point (1,2) and is tangent to the line $y = x$ at the origin. Find a, b and c.

    ii) Find the asymptotes of the graph of $f(x) = \frac{x^2-3}{2x-3}$.
21. i) Graph the function $y = x^3 - 3x + 3$ by including the coordinates of local extreme points and points of inflections.

    ii) Find the area of the region between the x-axis and the graph of $y = x^3 - 3x^2 + 2x, 0 \le x \le 2$.

($1 \times 10 = 10$ Marks)