2B6M21621

(Pages :3)

Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B11 – Numerical Methods

(2018 Admission onwards)

Time: 3 hours

Max. Marks: 120

Section A

Answer all *twelve* questions

(Each Question carries one mark)

1. Write the inequality for the number of iterations, $n$required to achieve an accuracy ε in bisection method for finding a root of $f(x) = 0$, which lies between **a** and **b**.
2. If $y(x_0) = y_0$, find the value of divided difference, $[x_0, x_0]$.
3. Define the *shift* operator, E.
4. Show that$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$, where δ is the central difference operator and E is the shift operator.
5. Write the formula for computing the value of$\frac{dy}{dx}$for tabular values of x using Newton's backward difference formula.
6. What is mean by partial pivoting.
7. State Adams-Bashforth formula.
8. State Newton's backward difference interpolation formula.
9. Write second order Runge-Kutta formula
10. Define spectral radius of a square matrix.
11. Define the central difference operator, δ.
12. State Guass' forward formula.

(12 x 1 = 12 Marks)

Section B

Answer any *ten* out of fourteen questions.

(Each question carries 4 marks.)

13 Construct a central difference table for (x_i, y_i); $i=0,1,2,\ldots\ldots 6$.

14 Find, from the following table, the area bounded by the curve and the x-axis from x=7.47 to x=7.52.

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

15. Find a double root of the equation $f(x) = x^3 - x^2 - x + 1 = 0$.

16. The function y = sin x is tabulated below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \sin x$	0	0.70711	1.0

Use Lagrange's interpolation formula , find the value of $\sin(\frac{\pi}{6})$.

17. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$, y(0)=0, use picard's method to obtain y(0.25).

18. Find a real root of equation $f(x) = x^3 - x - 1 = 0$ using bisection method.

19. Show that $\Delta = \nabla E = \delta E^{\frac{1}{2}}$.

20. Given $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) Where x_i are equally spaced, find $[x_0, x_1]$ and $[x_0, x_1, x_2]$.

21. Write Simpson's 1/3 formula.

22. Given $y' = -y, y(0) = 1$, find y(0.04) using Euler's method(Take h = 0.01).

23. Find the missing term in the following table:

x	0	1	2	3	4
y	1	3	9	--	81

24. Use the method of iteration to find a positive root, between 0 and 1, of the equation $xe^x = 1$.

25. Find a root of the equation $x\sin x + \cos x = 0$.

26. Given $\frac{dy}{dx} = 1 + y^2, y(0) = 0$, and y(0.2)=0.2027, y(0.4)=0.4228 and y(0.6)=0.6841; find y(0.8) using Adams-Moulton method.

Section C

Answer any *six* out of nine questions.

(Each question carries 7 rnarks.)

27. Find a real root of the equation $x^{2.2} = 69$ which lies between 5 and 8,using regula-falsi method, correct to two decimal places.

28. Using method of separation of symbols, show that $\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \quad ... + (-1)^n u_{x-n}$.

29. Show that the n^{th} difference of a polynomial of n^{th} degree is a constant.

30. Tabulate $y = x^3$ for x = 2,3,4 and 5 and calculate the cube root of 10 correct to three decimal places.

1. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

2. Use Guass elimination to solve

$2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$.

3. Find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $y' = x - y^2$, $y(0)=1$, using Taylor series method.

4. Given $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, Find $y(0.2)$ and $y(0.4)$ using fourth order Runge-Kutta formula.

5. Using the following table, find $f(x)$ as a polynomial in x using Newton's divided difference interpolation formula:

x	-1	0	3	6	7
y	3	-6	39	822	1611

Section D

Answer any *two* out of three questions.

(Each question carries 13 rnarks)

6. (a) Find a real root of the equation $xe^x=1$ correct to three decimal places using Ramanujan's method.

(b) If $y_1 = 4, y_3 = 12, y_4 = 19, and\ y_x = 7$, find x.

7. Evaluate $\int_0^1 \frac{1}{1+x}\, dx$ using

(a) trapezoidal rule with $h = 0.25$ and $h = 0.125$

(b) Simpson's rules with $h = 0.25$ and $h = 0.125$.

38. Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$.

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Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B12 – Number Theory & Linear Algebra

(2018 Admission onwards)

Time: 3 hours Max. Marks: 120

Part A: Answer all 12 questions. Each question carries 1 Mark.

1. State Division Algorithm of any two integers a and b.
2. Find the greatest common divisor of (49, 210, 350)
3. Illustrate by an example that the product of two integers, neither of which is congruent to 0, may turn out to be congruent to 0.
4. Define a multiplicative function
5. Find the value of $\phi(30)$
6. Evaluate the Euclidean number corresponding to the prime 11.
7. If $ca \equiv cb (\text{mod } n)$ and $gcd(c, n) = d$, then $\frac{n}{d}$ divides $a - b$. Is the statement true?
8. State Fermat's theorem.
9. Let $S = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2,2}\}$ such that $c = 1$. Is S a vector space?
10. Is union of a set of subspace of a vector space V a subspace?
11. Find the subspace of R^3 spanned by the singleton set $\{(1, 0, 0)\}$
12. If a linear map $f : V \to W$ is injective, then what is $Ker\ f$

(12×1=12 marks)

Part B: Answer any *TEN* questions. Each question carries 4 Marks.

13. Assuming that $gcd(a, b) = 1$, prove that $gcd(a + b, a - b) = 1$, 2.
14. Prove that $n^3 - n$ is divisible by 6.
15. If P_n is the n^{th} prime, then $P_n \leq 2^{2^{n-1}}$
16. Show that $2^{340} \equiv 1$ (mod 341)
17. Show that 6 is an integral root of $x^7 + x + 2 \equiv 0$ (mod 7).
18. Find all solutions of the congruence $12x \equiv 27(\text{mod } 8)$, if any exist
19. Using binary exponential algorithm calculate $5^{110}(\text{mod } 131)$.

20. Evaluate $\phi(p^k)$, where p is a prime, and any integer $k > 0$

21. Find the least positive integer x such that $x \equiv 5 \pmod 7$, $x \equiv 7 \pmod{11}$ and $x \equiv 3 \pmod{13}$

22. Define a vector space

23. Show that the mapping $f : R^2 \to R^3$ given by $f(a, b) = (a + b, a - b, b)$ is linear .

24. Let $f : V \to W$ be linear and if Y is a subspace of W, then show that $f^{\leftarrow}(Y)$ is a subspace of V.

25. Show that every linearly independent subset I of a finite dimennsional vector space V can be extended to form a basis

26. Show that no linearly independent subset of a vector space V can containing 0_V.

(10×4 = 40 Marks)

Part C: Answer any *SIX* questions. Each question carries 7 Marks.

27. Find least number which when divided by 9 gives the remainder 8, when divided by 8 gives the remainder 7, when divided by 7 gives the remainder 6, ... when divided by 3 gives the remainder 2, when divided by 2 gives the remainder 1.

28. Define the greatest integewr function. State any of the 4 basic properties.

29. Compute the remainder of 8^{103} when divided by 13.

30. If $(n-1)! \equiv$ -1(mod n), then show that n is a prime.

31. For $n > 1$, the sum of positive integers less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$

32. Determine whether set $\{[\alpha, 3\alpha, 5\alpha] | \alpha \in R\}$ is a subspace of R^3

33. Show that the set W of complex metrices $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is a real vector space of dimension 6.

34. Let $f : V \to W$ be linear, then show that
    (1) $f(0_V) = 0_W$
    (2) For all $x \in V$, $f(-x) = -f(x)$

35. Prove that span $S = < S >$

(6×7=42 Marks)

Part D: Answer any *TWO* questions. Each question carries 13 Marks.

36. Find the complete solution of the linear diophantine equation $172x + 20y = 1000$.

37. The quadratic concruence $x^2 + 1 \equiv 0 \pmod p$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod 4$

38. Show that the linear map $f : R^3 \to R^3$ is given by $f(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ is neither injective nor surjective..

(2×13 = 26 Marks)

2B6M21623 (Pages :4) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B13 (E01)– Linear Programming

(2018 Admission onwards)

Time: 3 hours Max. Marks: 80

SECTION A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Show that $S = \{(x_1, x_2, x_3) \mid 2x_1 - x_2 + x_3 \leq 4\} \subset \mathbb{R}^3$ is a convex set.
2. Prove that intersection of two convex sets is a convex set.
3. Define feasible solution of a LPP.
4. Define surplus variables.
5. When does the simplex method indicate that the LPP has unbounded solution
6. In Big M method, what is the cost of artificial variables is called.
7. If the dual has no feasible solution, then the primal problem has an objective function that is ———.
8. The primal has 3 decision variables and 4 constraints. How many decision variables and constraints are in the dual.
9. Define a loop in a transportation problem.
10. What is an assignment problem.
11. Assignment problem is a special type of transportation problem. Justify your answer.
12. Define triangular basis of a transportation problem.

(12 x 1= 12 marks)

SECTION B

Answer any **nine** *out of twelve questions.*
Each question carries 2 mark.

13. Show that hyperplane in $\mathbb{R}^3$ is a convex set.

14. Plot the feasible region in the plane for the LPP constraints: $x_1 + 2x_2 \leq 10$, $x_1 + x_2 \leq 6$, $x_1 \leq 4$ and $x_1, x_2 \geq 0$.

15. What are the characteristics of the standard form of LPP.

16. Write a note on basic feasible solution and optimal basic feasible solution.

17. If k^{th} constraint of the primal problem is an equality, show that the k^{th} dual variable will be unrestricted in sign.

18. Verify dual of the dual is primal for the LPP. Maximize $z = 2x_1 + 5x_2$ subject to the constraints $5x_1 + 6x_2 \leq 3$, $-2x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$.

19. Define unrestricted variable in a LPP.

20. Suppose optimality criteria of a LPP satisfied. What is indicated by one or more artificial vectors are in the basis at zero level.

21. Write the mathematical form of the transportation problem.

22. Solve the transportation problem by North West Corner Rule

	I	II	II	IV	Supply
A	13	11	15	20	2000
B	17	14	12	13	6000
C	18	18	15	12	7000
Demand	3000	3000	4000	5000	

23. Write a note on unbalanced assignment problem.

24. A balanced transportation problem possesses a finite feasible solution and an optimal solution. Justify your answer.

(9 x 2= 18 marks)

SECTION C

Answer any six *out of nine questions.*
Each question carries 5 mark.

5. Prove that the set of all convex linear combinations of a finite number of vectors $x_1, x_2, \cdots, x_k$ in $\mathbb{R}^n$ is a convex set.

6. Solve graphically, Minimize $z = 6x_1 + x_2$ subject to the constraints: $2x_1 + x_2 \geq 3$, $x_1 - x_2 \geq 0$ and $x_1, x_2 \geq 0$.

7. State and prove Minimax theorem.

8. Explain the Big-M method.

9. Write the LPP in the standard form,

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 3x_2 - 4x_3 \\ \text{subject to:} \quad & 4x_1 - x_2 - 3x_3 \geq 2 \\ & 2x_1 + x_2 - 4x_3 \leq 6 \\ & x_1 - 3x_2 + 5x_3 = 4 \\ & x_1, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted} \end{aligned}$$

0. Write the dual of the LPP,

$$\begin{aligned} \text{Maximize} \quad & z = x_1 - 2x_2 + 3x_3 \\ \text{subject to:} \quad & -2x_1 + x_2 + 3x_3 = 2 \\ & 2x_1 + 3x_2 + 4x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

1. Explain Vogel's approximation method.

2. Find an initial basic feasible solution by column minima method:

	From			Availability
To	16	19	12	14
	22	13	19	16
	14	28	8	12
Requirements	10	15	17	

3. Solve the cost-minimizing assignment problem:

	I	II	II	IV	V
A	11	10	18	5	9
B	14	13	12	19	6
C	5	3	4	2	4
D	15	18	17	9	12
E	10	11	19	6	14

(6 x 5= 30 marks)

SECTION D

Answer any **two** *out of three questions.*
Each question carries 10 mark.

34. Let $A \subset \mathbb{R}^n$ be any set. Prove that the convex hull of A is the set of all finite convex combinations of vectors in A.

35. Using simplex method to find the inverse of the matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 9 \end{pmatrix}$

36. Obtain an optimum basic feasible solution to the degenerate transportation problem:

	To			Available
From	7	3	4	2
	2	1	3	3
	3	4	6	5
Demand	4	1	5	10

(2 x 10= 20 marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B13 (E01)– Linear Programming

(2018 Admission onwards)

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24. A balanced transportation problem possesses a finite feasible solution and an optimal solution. Justify your answer.

(9 x 2= 18 marks)

SECTION C

Answer any six *out of nine questions.*
Each question carries 5 mark.

Prove that the set of all convex linear combinations of a finite number of vectors $x_1, x_2, \cdots, x_k$ in $\mathbb{R}^n$ is a convex set.

Solve graphically, Minimize $z = 6x_1 + x_2$ subject to the constraints: $2x_1 + x_2 \geq 3$, $x_1 - x_2 \geq 0$ and $x_1, x_2 \geq 0$.

State and prove Minimax theorem.

Explain the Big-M method.

Write the LPP in the standard form,

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Write the dual of the LPP,

$$\begin{aligned} \text{Maximize} \quad & z = x_1 - 2x_2 + 3x_3 \\ \text{subject to:} \quad & -2x_1 + x_2 + 3x_3 = 2 \\ & 2x_1 + 3x_2 + 4x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

. Explain Vogel's approximation method.

. Find an initial basic feasible solution by column minima method:

	From			Availability
To	16	19	12	14
	22	13	19	16
	14	28	8	12
Requirements	10	15	17	

. Solve the cost-minimizing assignment problem:

	I	II	II	IV	V
A	11	10	18	5	9
B	14	13	12	19	6
C	5	3	4	2	4
D	15	18	17	9	12
E	10	11	19	6	14

(6 x 5= 30 marks)

SECTION D

Answer any two *out of three questions.*
Each question carries 10 mark.

34. Let $A \subset \mathbb{R}^n$ be any set. Prove that the convex hull of A is the set of all finite convex combinations of vectors in A.

35. Using simplex method to find the inverse of the matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 9 \end{pmatrix}$

36. Obtain an optimum basic feasible solution to the degenerate transportation problem:

	To			Available
From	7	3	4	2
	2	1	3	3
	3	4	6	5
Demand	4	1	5	10

(2 x 10= 20 marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B09 – Real Analysis

(2018 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A Answer all questions. Each question carries 1 mark.

1. Is it true always that the image of a bounded interval by a continuous function is a bounded interval ? Give reason.
2. Give example of a continuous function with absolute maximum, but not with absolute minimum.
3. Prove that $f(x) = x^2, x \in [0,1]$ is a Lipschitz function.
4. Find $\| \mathscr{P} \|$, the norm of the partition $\mathscr{P} = \{0, 0.2, 0.45, 0.78, 0.87, 0.93, 0.98, 1\}$ of $[0, 1]$.
5. Identify the type of the improper integral $\int_1^\infty \frac{1}{1+x^2} dx$.
6. Define the radius of convergence of a power series.
7. Evaluate $\beta(2,1)$.
8. Write $\int_0^{\frac{\pi}{2}} \sin\theta \cos\theta d\theta$ as a Beta function.
9. What is the value of $\Gamma(\frac{1}{2})$?
10. If $f_n(x) = x^n, x \in [0, 1]$, what is the uniform norm of f_n ?
11. State the Weierstrass M-test for absolute convergence of a series of functions.
12. Define Cauchy's Pricipal Value of an improper integral.

$12 \times 1 = 12$ marks.

Section B Answer any 10 questions. Each question carries 4 marks.

13. If $f : [0, 1] \longrightarrow [0, 1]$ is a continuous function, prove that $\exists \ c \in [0, 1]$ such that $f(c) = c$.
14. Give example of a uniformly continuous function on a suitable interval that is not a Lipschitz function. Explain your arguments.
15. Prove that $\sin x$ is uniformly continuous on $[0, \infty)$.
16. State the non-uniform continuity criteria.
17. For the function $f(x) = x^2$ on $[0, 2]$, taking $\mathscr{P} = \{0, .6, .8, 1.4, 1.8, 2\}$ as a partition and the mid point of each subinterval as the tags, write the Riemann sum.
18. State the Cauchy integrability condition. Give example of a function **not satisfying** the condition.
19. Explain the steps to prove that $\int_2^3 x^2 dx = \frac{19}{3}$. State the theorem used.
20. State the Taylor's theorem.
21. Give example of a convergent series which is not absolutely convergent.

22. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.

23. Find pointwise limit of the function $f(x) = (1-x)^n$ on $[0, 1]$.

24. State the theorem of interchange of limit and continuity.
Use it to prove that the convergence of $f_n(x) = x^n$ to 0 on $[0, 1]$ is not uniform.

25. If n is a natural number, prove that $\Gamma(n) = (n-1)\Gamma(n-1)$.

26. Evaluate $\int_0^1 \frac{x}{\sqrt{1-x}} dx$.

$10 \times 4 = 40$ **marks.**

Part C

Answer any Six questions. Each question carries 7 marks.

27. State and prove the Bolzano's Intermediate Value theorem.

28. Let $f : [a, b] \to \mathbf{R}$ be continuous and let $\varepsilon > 0$.
Prove that there exists a step function $s_\varepsilon : [a, b] \to \mathbf{R}$ such that $|f(x) - s_\varepsilon(x)| < \varepsilon$ for all $x \in [a, b]$.

29. Define $f \in \mathscr{R}[a, b]$. Prove the uniqueness of the Riemann integral.

30. State and prove the Squeeze theorem for Riemann Integrals.

31. Prove that $f : [0, 3] \longrightarrow \mathbf{R}$ is Riemann integrable where

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 2 & 1 < x \leq 2 \\ 3 & 2 < x \leq 3. \end{cases}$$

32. State and prove the Fundamental Theorem of Calculus (First Form).

33. Write the formulae to find radius of convergence of a power series $\sum_0^{\infty} a_n x^n$.
Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$.

34. State and prove the relation between Beta and Gamma functions.

$6 \times 7 = 42$ **marks.**

Part D

Answer any Two question. Each question carries 13 marks.

5. State and prove the Location of Roots theorem.

6. a) Prove that the sum of two Riemann Integrable functions is Riemann integrable.
   b) Prove that a monotone function is Riemann Integrable.

7. State and prove Integral testfor a series of functions.
Use it to discuss the convergence of the p-series $\sum_1^{\infty} \frac{1}{n^p}$.

$2 \times 13 = 26$ **marks.**

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Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc. Degree Examination, March/April 2021

BMAT6B10 – Complex Analysis

(2018 Admission onwards)

Time: 3 hours

Max. Marks: 120

Section A

Answer all twelve questions

Each question carries 1mark

1. Show that $f(z) = Re\ z$ is no where differentiable.
2. Define analytic function
3. Derive $f'(z_0) = e^{-i\theta_0}[u_r + iv_r]$.
4. Write $\lceil \exp(2z + i) \rceil$ in terms of x and y.
5. State Cauchy Gousat theorem.
6. If u and v are harmonic functions conjugate to each other in some domain, then prove that u and v must be constant there.
7. Discuss the region of convergence of the series $1 - z + z^2 - z^3 + \cdots$.
8. State the extension of Cauchy's integral formula for multiply connected region.
9. Show that $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots |z| < \infty$.
10. Identify the singularity of $e^{\frac{1}{z}}$.
11. Define the concept of zeros of order m of an analytic function f at $z = z_0$
12. Discuss the nature of singularity of the function $f(z) = \frac{1}{(z-4)^5}$.

(12 X 1= 12 marks)

Section B

Answer any ten out of fourteen questions

Each question carries 4mark

13. Let $g(z) = (re^{i\theta})^{\frac{1}{2}}$, $r > 0, -\pi < \theta < \pi$ show that g(z) is analytic in its domain of definition and $g'(z) = \frac{1}{2[g(z)]}$.
14. Show that if a function $f(z) = u+iv$ is analytic at $z = x+iy$ then the component functions u and v have continuous partial derivatives of all orders at that point.
15. Find all values of z such that $e^z = 3 + 4i$.
16. Solve $sinz = cosh4$.
17. Suppose that a function $f(z)$ is analytic inside and on a positive oriented circle C_R, centered at z_0 with radius R and $\lceil f(z) \rceil \leq M_R$ the prove that $\lceil f^n(z) \rceil \leq \frac{n!\ M_R}{R^n}$.
18. Evaluate $\int_0^\infty \frac{dx}{x^2+1}$.
19. State and prove Morera's Theorem.

20. Integrate $\frac{z^2-1}{z^2+1}$ around $|z-i|=1$ in the counter clockwise direction.

21. Evaluate $\int_{|Z|=\frac{3}{2}} \frac{\tan z\, dz}{z^2-1}$.

22. Obtain the Taylor series representation for cosz at $z=\frac{\pi}{4}$.

23. Show that $\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}$.

24. Discuss the singularity of the function $\frac{1}{z^2(1+z)}$.

25. Find the Lauarents series of $z^2 exp\left(\frac{1}{z}\right)$.

26. Find the residue of the function $\exp\left(\frac{1}{z^2}\right)$.

(10 X 4 = 40 marks)

Section C

Answer any six out of nine questions

Each question carries 7 mark

27. Show that for the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ eventhough the partial derivatives of the component function satisfy C-R equations at *z=0* , *f(z)* is not differentiable at *z=0*.

28. If $u - v = (x-y)(x^2+4xy+y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x+iy$. find *f(z)* in terms of *z*.

29. State and prove Taylors theorem.

30. Show that $\int_C (z-z_0)^m dz = \begin{Bmatrix} 2\pi i \text{ if } m = -1 \\ 0 \quad \text{if } m \neq -1 \end{Bmatrix}$ where m is an integer and C is the circle with center z_0 and radius r in the counter clockwise sense.

31. State and prove Cauchy's integral formula.

32. State and prove Cauchy's residue theorem.

33. Find all Laurent series of $f(z) = -1/(z-1)(z-2)$about $z=0$.

34. Find the residue at the singular points of the function $\frac{1}{z(e^z-1)}$.

35. Evaluate $\int_0^{\infty} \frac{\cos ax\, dx}{x^2+b^2}$, $a > 0, b > 0$.

(6 X 7 =42 marks)

Section D

Answer any two out of three questions

Each question carries 13 mark

36. Suppose f(z) = u + iv and that $f'(z)$ exist at a point $z_0 = x_0 + iy_0$. Then the first order partial derivatives of u and v must exist at (x_0 , y_0) and they must satisfy the C.R. equations.

37. State and prove the Maximum Modulus Principle.

38. Discuss the problem of type $\int_0^{2\pi} F(\sin\theta, \cos\theta)d\theta$ and solve $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$, $a > b > 0$.

(2 X 13 = 26 marks)