2B6A23035

(Pages : 2)

Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2023

BMT6B10 – Real Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A

All questions can be attended

Each question carries 2 marks

1. State Non Uniform Continuous function on a given set and give an example for a function which is not uniformly continuous.
2. If $f(x) = x^2$ for $x \in [0,1]$. Calculate the first Bernstein polynomial for f(x).
3. Let $f : [0, 5] \to R$ be defined by $f(x) = 4$. Show that f is Riemann integrable.
4. If f is Riemann integrable on [a, b] then prove that kf is Riemann integrable on [a, b] and for $k \in R$, $\int_a^b kf = k\int_a^b f$.
5. If $f \in R[a,b]$ and if $[c, d] \subseteq [a, b]$, then prove that the restriction of f to [c,d] is R[c, d].
6. If $f(x) = |x|$ for $x \in [-10, 10]$, then verify the First form of the Fundamental Theorem of Calculus.
7. Evaluate the integral $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\,dt$. Justify your steps.
8. Prove that the sequence $f_n(x) = \frac{1}{x+n}, n = 1,2,....$ converges point wise on $[0,\infty)$.
9. Show that the series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ is uniformly convergent on every interval [a, b].
10. Test the convergence of the improper integral $\int_{-\infty}^{0} \frac{dx}{(1-3x)^2}$.
11. Explain absolute convergent and conditionally convergent of improper integrals.
12. Test the convergence of $\int_3^6 \frac{\log x}{(x-3)^4}dx$.
13. Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}dx = \frac{1}{5}B(\frac{3}{5},\frac{1}{2})$.
14. Prove that $\Gamma n = (n-1)!$ where n is a positive integer.
15. Evaluate $\int_0^\infty e^{-x^2}dx$

Ceiling – 25 Marks

Section B

All questions can be attended

Each question carries 5 marks

16. State and prove Bolzano Intermediate Value Theorem.

17. Show that $g(x) = \sqrt{x}$ on $[0,\infty)$ is uniformly continous on $[0, \infty)$.

18. Let $h(x)=x^2$ for $x\in[0,k]$. Show that $h\in R[0,k]$ and evaluate its integral over the interval $[0,k]$.

19. If $f:[a, b] \to R$ is continuous on $[a, b]$, then prove that $f \in R[a,b]$.

20. Show that the sequence $S_n(x) = \dfrac{x}{nx+1}$, $x \geq 0$ is uniformly convergent on $[0,\infty)$.

21. Show that $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ is convergent.

22. Examine the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$

23. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

Ceiling – 35 Marks

Section C

Answer any two questions

Each question carries 10 marks

24. (a) State and prove Maximum Minimum Theorem.

    (b) Define Dirichlet function on $[0, 1]$ and prove that it is not Riemann integrable.

25. (a) Let (f_n) be a sequence of bounded functions on $A \subseteq R$, then prove that (f_n) converges uniformly on A to a bounded function f if and only if for each $\varepsilon > 0$ there is a number $H(\varepsilon)$ in N such that $\|f_m - f_n\|_A < \varepsilon$ for all $m, n \geq H(\varepsilon)$.

    (b) State and prove Cauchy's criterion for uniform convergence of series of functions.

26. (a) Test for the convergence of the improper integral $\int_1^{\infty} \frac{3}{e^x+5} dx$.

    (b) Evaluate the Cauchy Principal Value of $\int_{-\infty}^{\infty} \frac{1+x}{1+x^2} dx$.

27. (a) If $p > 0, q > 0$ prove that $\dfrac{B(p,q+1)}{q} = \dfrac{B(p+1,q)}{p}$

    (b) Prove that $\Gamma(1/2) = \sqrt{\pi}$.

$2\times10 = 20$ Marks

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Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2023

BMT6B11 – Complex Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

SECTION A

Answer the following questions. Each carries two marks

(Ceiling 25)

1. Define a differentiable complex function and show that $f(z) = |z|^2$ is no where differentiable.
2. Define analytic function and singular point of a function with an example
3. Express $\tan(\pi - 2i)$ in the form $a + ib$.
4. Let C denote the curve defined by $y = 2x + 1, -1 \leq x \leq 0$. Then evaluate $\int_C (3x^2 + 6y^2)ds$
5. Evaluate $\int_C xydx + x^2dy$ where C is the graph of $y = x^3, -1 \leq x \leq 2$
6. Prove, $\int_{-C} f(z)dz = -\int_C f(z)dz$ where C denote the contour extending from $z = z_1$ to $z = z_2$
7. Evaluate $\int_C \bar{z}dz$ wher C is the right half of the circle $|z| = 2$ from $z = -2i$ to $z = 2i$.
8. State Cauchy Goursat Theorem and solve $\int_C f(z)dz$ where $f(z) = \frac{z^2}{z^2+9}$ and $C : |z-1| = 1$
9. Prove that $\Sigma_{k=1}^{\infty} \frac{(3-4i)^k}{k!}$ converges
10. Show that the series $\Sigma_{k=1}^{\infty} \frac{(1+2i)^k}{5^k}$ connvergent and find its sum.
11. Obtain the Taylor series representation for $1/z$ about $z = i$
12. Find the Laurent series of $z^{-5} sinz$ with center 0.
13. Find the residues at the singular point (a) $\frac{4}{1-z}$ (b) $\frac{sinz}{z^4}$
14. Determin the zeros and their order of the function $f(z) = (z + 2 - i)^2$
15. Determine whether $z = 0$ is an essential singularity of $f(z) = e^{(z+1)/z}$

SECTION B

Answer the following questions. Each carries five marks

(Ceiling 35)

16. Find the real constants a and b , so that the complex function $f(z) = 1 + 5r\cos\theta + ar\sin\theta + i(3 + br\sin\theta + r\cos\theta)$ is entire

17. Let $u(x, y) = e^x(x\ cosy - y\ siny)$, show that u is harmonic function. Find its harmonic conjugate and hence the analytic function $f(z) = u + iv$

18. Let C be the arc of the circle $|\ z\ | = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating integral show that $|\int_C \frac{dz}{z^2-1}| \leq \frac{\pi}{3}$

19. Evaluate $\int_C f(z)dz$ where

    (a) $f(z) = x^2 + 3ixy$ and C is the line segment joining from $1 + i$ to $2 - i$.

    (b) $f(z) = z^2$ and C is the parabolla $y = x^2$ from $(0, 0)$ to $(2, 4)$.

20. Find all Laurent series expansion of $f(z) = \frac{1}{z^3 - z^4}$ with center $z = 0$.

21. Find all Maclaurin and Laurent series representation of the function $f(z) = \frac{-1}{(z-1)(z-2)}$ about $z = 0$.

22. Evaluate $\oint_C \frac{dz}{z sinz}$ where C is the unit circle in the positive direction

23. Evaluate $\int_0^{2\pi} \frac{dz}{13 - 5sinz}$

SECTION C

Answer any two questions ($2 \times 10 = 20$ Marks)

24. (a) The Principal branch f_1 of the complex logarithm defined by $f_1(z) = log_e r + i\theta, -\pi \leq \theta \leq \pi$. Prove that f_1 is analytic and its derivative is given by $f_1'(z) = \frac{1}{z}$

    (b) Show tha Lnz is not continuous on the negative real axis.

25. (a)State and prove Cauchy- goursat theorem for multiply connected domain.

    (b) Show that if C is any positively oriented closed contour surrounding the origin, then show that $\int_C (1/z)dz = 2\pi i$

26. State and Prove Taylor's Theorem

27. State and prove Cachy's Residue Theorem and Evaluate $\oint_C \frac{dz}{z^3(z-1)}$ where C is the circle$|\ z\ | = 2$ by using residue theorem

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2023

BMT6B12 – Calculus of Multivariable – 2

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A

All questions can be attended. Each question carries 2 marks.
Cieling 25 marks

1. Write equation of normal to the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ at the point $(2, \frac{2\sqrt{5}}{3})$.
2. Locate the critical points for the function $f(x,y)=2x^2+3y^2-4x+12y$.
3. Write the method of Lagrange multiplier.
4. Evaluate $\int_0^2\int_1^4 x^3y^2\,dy\,dx$.
5. Write the integral $\iint_R xy\,dA$ using polar coordinates where R is the region in the second quadrant bounded by the circle $x^2+y^2=1$.
6. Write the formula to find the area of a surface $y=g(x,z)$.
7. Describe the method of converting a triple integral from rectangular coordinates to spherical coordinates.
8. Find divergence of the vector field $\mathbf{F}(x,y,z)=x^2yz\hat{i}+(xy+y^2z^3)\hat{j}+(1+yz)\hat{k}$.
9. Evaluate the line integral $\int_C (x^2+2y)\,ds$ where C is $\mathbf{r}(t)=t\hat{i}+(t+1)\hat{j}, 0\leq t\leq 2$.
10. State the Fundamental Theorem for Line Integrals.
11. If a line integral is independent of path, prove that the line integral over any simple closed curve is zero.
12. Test whether $\mathbf{F}(x,y)=3x^2y\hat{i}+x^3\hat{j}$ is a conservative vector field or not.
13. Write the formula to find the area of a plane region bounded by a piecewise-smooth simple closed curve.
14. Evaluate $\int_C y\,dx-x\,dy$ where C is the boundary of the square bounded by the lines $x=0$, $x=1$, $y=0$ and $y=1$ oriented in the anticlockwise sense.
15. State the Divergence theorem.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 35 marks

16. Classify the critical points for the function $f(x,y) = 2x^2 + y^2 - 2xy - 8x - 2y + 2$.

17. Reverse the order of integration and evaluate the integral $\int_0^2 \int_{x^2}^4 x \cos y^2 \, dy \, dx$.

18. Find the area of the surface $z = 2 - x^2 + y$ that lies above the triangular region with vertices $(0,-1)$, $(1,0)$ and $(0,1)$.

19. Evaluate $\iiint_B (x^2y + yz^2) dV$
where B is the cuboid $\{(x,y,z)) \mid -1 \leq x \leq 2, 0 \leq y \leq 3, 1 \leq z \leq 2\}$.

20. Prove that the gradient of a scalar field is a conservative vector field.

21. Find the work done by the force field $\mathbf{F} = (x+2y)\hat{i} + 2z\hat{j} + (x-y)\hat{k}$ on a particle that moves along the straight line segment C from $(-1,3,2)$ to $(1,-2,4)$.

22. Find a parametrisation of the surface of the elliptical paraoloid $x^2 + 2y^2 - 3z = 4$.

23. Use Stoke's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = (y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$ and C is the boundary of the part of the plane $2x + 3y + z = 6$ in the first octant, oriented in a counterclockwise direction when viewed from above.

Section C

Answer any Two Questions. Each question carries 10 marks.
20 marks from this section

24. Find the absolute maximum and the absolute minimum values of the function $f(x,y) = 2x^2 + xy + y^2$ on the region $D = \{(x,y \mid -2 \leq x \leq 2, -1 \leq y \leq 1\}$.

25. Evaluate $\iiint_T (x^2y + yz^2) dV$ where T is the solid in he first octant bounded by the graphs of $z = 1 - x^2$ and $y = x$.

26. Show that $\mathbf{F}(x,y,z) = yz^2\hat{i} + xz^2\hat{j} + 2xyz\hat{k}$ is a conservative field. Find a scalar function $f(x,y,z)$ whose gradient is $\mathbf{F}(x,y,z)$. Also evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from the point $(0,0,1)$ to the point $(1,3,2)$.

27. Find the area of the surface represented by the equation $r(u,v) = u\cos v\hat{i} + u\sin v\hat{j} + v\hat{k}$ with parameter domain $D = \{(u,v) \mid 0 \leq u \leq 1, \quad 0 \leq v \leq 2\pi\}$.

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Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2023

BMT6B13 – Differential Equations

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

PART A

***All* the questions can be attended.**
Each question carries 2 marks.

1. What is the order of the differential equation $y'' - [1 + (y')^2]^{\frac{3}{2}} = 0$.
2. Solve the differential equation $y' - 2ty = 0$.
3. Find an integrating factor for the equation $\frac{dy}{dt} - y = 3e^t$.
4. Define an exact differential equation. Check whether $(1 + 4yx + 2y^2)dx + (1 + 4yx + 2x^2)dy = 0$ is exact?
5. Find the fundamental set or basis of solutions of the differential equation $y'' - 4\frac{dy}{dx} + 4y = 0$.
6. Find the general solution of the differential equation $y'' + 2y' + 5y = 0$.
7. Explain the ordinary and singular points of the differential equation $P(x)y'' + Q(x)y' + R(x)y = 0$.
8. Determine a lower bound for the radius of convergence of series solutions of the differential equation $(1 + x^2)y'' + 2xy' + 4x^2y = 0$ about the point $x = 0$.
9. Using definition of Laplace Transform, find $L[e^{at}]$.
10. Define Unit step function. What is its Laplace transform?
11. Find the Laplace Transform of $e^{2t}\sinh 7t$.
12. Find $L^{-1}\left[\frac{e^{-3s}}{(s-1)^4}\right]$.
13. What is the fundamental period of $\cos 5t$?
14. Determine whether the function $x^3 - 2x$ is even, odd, or neither.
15. What is the one-dimensional Heat Equation?

(Ceiling 25 Marks)

PART B

***All* the questions can be attended.**
Each question carries 5 marks.

16. Solve the initial value problem $y' = y^2$ with $y(0) = 1$ and determine the interval in which the solution exists.
17. Solve the initial value problem $ty' + 2y = 4t^2, \quad y(1) = 2$.

18. If the Wronskian W of f and g is $3e^{4t}$ and if $f(t) = e^{2t}$, find $g(t)$.

19. State and Prove Abel's theorem.

20. Solve the non homogeneous differential equation $y'' + 4y = 3\cos 2t$.

21. Using Laplace transform, find the solution of the IVP, $y'' + 4y = 4t,\ y(0) = 1$ and $y'(0) = 5$.

22. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$.

23. Find the Fourier Series expansion for $f(x) = x^2\ \ in\ [-\pi, \pi]$ with $f(x) = f(x + 2\pi)\ \forall\, x \in \mathbb{R}$.

(Ceiling 35 Marks)

PART C

Answer any *two* questions. Each question carries 10 marks.

24. (a) Solve the differential equation $(x^2 - 2x + 2y^2)dx + 2xydy = 0$.

(b) Solve the differential equation $y' = \frac{y^2-x^2}{2xy}$.

25. (a) Determine the longest interval in which the solution of the initial value problem $t(t-4)y'' + 3ty' + 4y = 2,\ y(3) = 0, y'(3) = -1$, is certain to exist.

(b) Solve by using the method of variation of parameters $y'' + 4y' + 4y == \frac{e^{-2t}}{t^2}$.

26. (a) Using the convolution property find the inverse Laplace transform of $\frac{1}{s^2(s^2+9)}$.

(b) Using the method of Laplace transform, solve the differential equation

$$y'' + 2y' + 2y = \delta(t - \pi), \qquad y(0) = 1,\ y(0) = 0.$$

27. (a)Find the half range cosine series expansions of the function

$$f(t) = \begin{cases} \frac{2k}{l}t\ ; & 0 \le t < \frac{l}{2} \\ \frac{2k}{l}(l - t)\ ; & \frac{l}{2} \le t < l \end{cases}$$

(b)Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of $20^{\circ}C$ throughout and whose ends are maintained at $0^{\circ}C$ for all $t > 0$.

$(2 \times 10 = 20$ Marks)

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Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2023

BMT6B14(E03) – Mathematical Programming with Python and Latex

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks.
Cieling 20 marks

1. Explain the difference between the statements print(5/2) and print(5//2).
2. Consider a = 'PROGRAMMING' as a string in python.
   Write the output of the statements print(a[2:7]), print(a[2:]) and print(a[:7]).
3. What is **List** in python ?
4. Write the syntax of **for** loop in python.
5. What is output of the following python program ?
   ```
   a = [2, 5, 3, 4, 12]
   size = len(a)
   for k in range(size):
   a[k] = 0
   print a
   ```
6. Write the effect of **continue** statement in python.
7. What are python modules ? Give names of two of the modules.
8. How do you join two statement lines of a python program ?
9. Write the output of the python program:
   ```
   a = []
   a.append(3)
   a.insert(0,2.5)
   print (a, a[0])
   print (len(a))
   ```
10. Write the python statement to import numpy in a program.

11. Write the essential statements needed in all LaTeX codes.

12. Write the LaTeX code to get the output $\int_0^1 \frac{1}{1+x^2} dx$.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 30 marks

13. Write a python program to input two strings and to give the following output as print: The first string, the second string, their concatenated string.

14. Give example of the usage of **range** in a **for loop**.

15. Write a python program giving as output the factorial of a natural number which you input.

16. Give example of a python program to show the use of input and output of files.

17. Write a python program to set the order 2×2 for a matrix, input its entries and print its transpose.

18. Write a python program that plots the sine function.

19. Write a LaTeX program to get output as a two-way table to show the number of boys and girls studying in the Arts and Science disciplines.

Section C

Answer any One Question. Each question carries 10 marks.
10 marks from this section

20. Write a python program to input a natural number and giving the output as.
    a) All natural numbers less than and that are coprime to the number.
    b) Fionacci numbers in which the number of terms is the number input.

21. Prepare the LaTeX code to generate a question paper similar to the one that you are writing now. It is enough to nclude only two questions in each section.