

1B5N23493

(Pages : 2)

Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Statistics Degree Examination, November 2023

BST5B05 – Mathematical Methods in Statistics

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

**PART A**

Each question carries 2 marks.

1. Find the infimum and supremum of the following set  $\{\frac{(-1)^n}{n}; n \in \mathbb{N}\}$
2. State the order properties of  $R$ .
3. Prove that  $|x - y| \geq ||x| - |y||$
4. State Bernoulli inequality.
5. Define neighborhood of a point.
6. Define convergence of a sequence.
7. Find  $\lim_{n \rightarrow \infty} [\sqrt{n+1} - \sqrt{n}]$
8. State the necessary condition for the convergence of an infinite series.
9. Give an example of a continuous function which is not differentiable.
10. Define discontinuity of first kind.
11. State Taylor's theorem
12. Define Norm of a partition.
13. State Darboux's theorem.
14. Define Upper sum and Lower sum of a function  $f$  corresponding to the partition  $P$ .
15. State the second fundamental theorem of Integral calculus.

Maximum Mark = 25

**PART B**

Each question carries 5 marks.

16. State and prove nested interval property.
17. State and prove Cauchy's general principal of convergence.
18. Show that  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n^2+k} = 0$
19. Show that every convergent sequence is bounded.
20. Show that the function  $f(x) = |x| + |x - 1|$  is derivable at all points except 0 and 1.

21. Prove that a function  $f$  defined on an interval  $I$  is continuous at a point  $c \in I$  iff for every sequence  $\{c_n\}$  in  $I$  converging to  $c$ , we have  $\lim_{n \rightarrow \infty} f(c_n) = f(c)$
22. Show that the function  $f$  defined by  $f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$  is not integrable on any interval.
23. Suppose that  $f : [a, b] \rightarrow \mathbb{R}$  is bounded,  $P$  is a partition of  $[a, b]$ , and  $Q$  is refinement of  $P$ . Then show that  $U(f; Q) \leq U(f; P)$ .

**Maximum Mark = 35**

**PART C**

**Each question carries 10 marks (Answer any TWO Questions)**

24. State and prove principle of Mathematical Induction
25. Evaluate (a)  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1}\right)^n$   
(b)  $\lim_{n \rightarrow \infty} \sqrt[n]{a}, a > 0$ .
26. State and prove Rolle's theorem.
27. State and prove first fundamental theorem on integral Calculus.

**(2 x 10 = 20 Marks)**

## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

## Fifth Semester B.Sc Statistics Degree Examination, November 2023

## BST5B06 – Sample Surveys

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

## PART A

Each question carries 2 marks.

1. Distinguish between questionnaire and schedule.
2. What is purposive sampling?
3. Explain SRS.
4. Distinguish between stratum and cluster.
5. A population consists of 10 units. How many samples of size 3 can be taken from this population using simple random sampling without replacement?
6. Explain Lottery method in sample selection.
7. What are the different allocations in stratified sampling?
8. Explain circular systematic sampling.
9. Write the unbiased estimate of the population total.
10. Define principles of validity.
11. Explain prevalence study.
12. Find the relative efficiency of cluster sampling with the simple random sampling.
13. Explain equal allocation.
14. Explain two stage sampling. Give an example.
15. What are the advantages of stratified sampling?

(Maximum Mark = 25)

## PART B

Each question carries 5 marks.

16. Show that for a population with linear trend  $V_{st} : V_{sy} : V_{ran} = \frac{1}{n} : 1 : n$
17. Derive the sampling variance in SRSWR.
18. Explain the principles of sampling theory.

19. Write about sampling frame. Explain various defects associated with it
20. Explain census and sampling. Why sampling is preferred?
21. Explain Neyman allocation.
22. What are the principles of stratification?
23. Cluster sampling will be efficient only when the variation between clusters is as small as possible; Prove.

(Maximum Mark = 35)

### PART C

Answer any two question, each question carries 10 marks.

24. Show that in SRSWOR the sample mean  $\bar{y}$  is an unbiased estimate of the population mean. Also derive its variance.
25. Obtain the mean and its variance in equal cluster sampling.
26. Write about sampling and non sampling errors.
27. Explain the methods of allocation in stratified sampling and find efficiency of variances.

(2 x 10 = 20 Marks)

1B5N23495

(Pages : 2)

Reg. No: .....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Statistics Degree Examination, November 2023

BST5B07 – Linear Regression Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

**PART A**

**Each question carries 2 marks.**

1. Write any two uses of regression analysis.
2. Describe the role of the scatter diagram in analysing the relationship between two variables.
3. Prove that when the regression model contains an intercept, the sum of the residuals is always zero.
4. Distinguish between  $R^2$  and Adjusted  $R^2$ .
5. Write the confidence intervals corresponding to the regression coefficients of a simple linear regression model.
6. List the assumptions of a multiple linear regression model.
7. Prove that the hat matrix is symmetric and idempotent.
8. Consider a multiple linear regression model  $y = X\beta + \varepsilon$ . Write the mean and variance of the least square estimate of  $\beta$ .
9. Distinguish between standardized residual and studentized residual.
10. What is meant by PRESS residual?
11. What do you mean by model adequacy checking?
12. Write the regression model corresponding to a second order polynomial in two variables. Write its assumptions.
13. When do we use piecewise polynomial regression models?
14. Write the equation of a logistic regression model with a binary response function.
15. How do you interpret the regression coefficient of a binary logistic regression model when the linear predictor has only one predictor?

**Maximum Mark = 25**

### PART B

Each question carries 5 marks

16. Consider the simple linear regression model  $y = \beta_0 + \beta_1 x + \varepsilon$ . Under the usual assumptions of regression, derive (i)  $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)$  and (ii)  $\text{Cov}(\bar{y}, \hat{\beta}_1)$ .
17. Describe the  $t$  test for testing the significance of the intercept parameter of a simple linear regression model.
18. Explain procedure for testing the significance of regression in multiple linear regression model.
19. Derive the maximum likelihood estimates of the coefficients of a multiple linear regression model.
20. Explain PRESS statistic. Derive  $R^2$  for prediction based on PRESS statistic.
21. Consider a multiple linear regression model  $y = X\beta + \varepsilon$  with  $\text{Var}(\varepsilon) = \sigma^2 I$ , where  $I$  is the identity matrix. If  $H$  is the hat matrix and  $e$  is the residual, prove that  $\text{Var}(e) = \sigma^2(I - H)$ .
22. Discuss about various important considerations arise when fitting a polynomial regression model with one variable.
23. Compare simple linear regression model and binary logistic regression model.

Maximum Mark = 35

### PART C

Each question carries 10 marks (Answer any TWO Questions)

24. Derive the least square estimates of the coefficients of a simple linear regression model  $y = \beta_0 + \beta_1 x + \varepsilon$  and prove that, these estimates are unbiased.
25. State and prove Gauss- Markov theorem.
26. Describe the role of residuals in model adequacy checking?
27. Explain the maximum likelihood procedure for estimating the parameters of a binary logistic regression model.

(2 x 10 = 20 Marks)

## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

## Fifth Semester B.Sc Statistics Degree Examination, November 2023

## (Open Course)

## BST5D03 – Basic Statistics

(2019 Admission onwards)

Time: 2hours

Max. Marks: 60

## Section A

Each question carries two marks

- Describe judgment sampling and provide a scenario where it might be helpful.
- Give examples of sampling errors.
- What are the advantages of using a sample survey over a census?
- Find the mode(s) for the following dataset: 7, 9, 10, 7, 12, 15, 9, 10, 15, 9.
- Find the median of the following data  
18, 22, 24, 29, and 31
- When is the coefficient of variation more useful than the standard deviation?
- What is a scatter diagram, and how can it help to visualise relationships between variables?
- Define the concept of the limit of the correlation coefficient.
- Two dice are rolled. Calculate the probability of getting a sum of 7 or 11.
- Find the sample space of tossing two coins.
- Differentiate between classical and empirical definitions of probability.
- A standard deck of cards contains 52 cards. Calculate the probability of drawing a heart from the deck.

Maximum Marks = 20

## Section B

Each question carries five marks

- Explain the concept of non-sampling errors in a sample survey. Provide examples of non-sampling errors and discuss their potential impact on the survey results.
- Consider the following frequency distribution representing the scores of students in a mathematics test, Calculate the median score for this distribution.

| Score range | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|-------------|-------|-------|-------|-------|--------|
| Frequency   | 5     | 8     | 12    | 10    | 5      |

15. Three cards are drawn successively from a standard deck of cards without replacement. Calculate the probability that the first card is a heart, the second card is a diamond, and the third card is a club.
16. Explain the principle of least squares in the context of curve fitting. Suppose you have data points (1, 3), (2, 5), and (3, 7). Find the equation of the straight line that best fits these points.
17. State and prove the addition theorem of probability of two events
18. Explain the three definitions of probability: frequency, classical, and axiomatic.
19. Explain the absolute measures of dispersion.

Maximum Marks = 30

### Section C

Answer *any one* question. Each question carries ten marks

20. Fit a straight line to the following data.

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| X: | 5  | 10 | 15 | 20 | 25 | 30 |
| Y: | 20 | 24 | 28 | 30 | 34 | 36 |

21. Find the Karl Pearson Correlation coefficient for the following data

|    |   |   |   |   |   |   |   |   |   |    |
|----|---|---|---|---|---|---|---|---|---|----|
| X: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Y: | 8 | 7 | 6 | 6 | 5 | 6 | 4 | 3 | 2 | 1  |

Interpret the result.