1B5N23470

(Pages : 2)

Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

BMT5B05 – Abstract Algebra

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

PART - A

(All questions can be attended. Each questions carries 2 marks.)

1. True or false : Cancellation law holds in $\mathbf{Z_n}$·Justify your answer.
2. Find$\varphi(36)$.
3. Find the number of elements of $\mathbf{Z_{47}}$.
4. True or false : The product of two cycles is a cycle. Justify your answer.
5. Define the order of a permutation in $\mathbf{S_n}$.
6. Define the order of a group. Give an example of a finite group.
7. Find a subgroup of **Z**.
8. Explain the Klein- four group.
9. Find all the generators of $\mathbf{Z_{10}}$.
10. Define group isomorphism.
11. Give an example of a group homomorphism.
12. Define normal subgroup of a group G.
13. Write a subring of $\mathbf{Z_n}$.
14. Define commutative ring.
15. True or false : Every integral domain is a field.

(Ceiling : 25 Marks)

PART - B

(All questions can be attended. Each questions carries 5 marks.)

16. Show that the congruence class $[a]_n$ has a multiplicative inverse in $\mathbf{Z_n}$ if and only if $(a,n) = 1$.
17. Show that every non zero elements of $\mathbf{Z_n}$ which are relatively prime to n has a multiplicative inverse.
18. If $(a,n) = 1$, prove that $a^{\varphi(n)} \equiv 1 (\mathrm{mod}\ n)$.
19. If σ *and* are disjoint cycles, prove that $\sigma\tau = \tau\sigma$.
20. Prove that any permutation in S_n $(n \geq 2)$ can be written as a product of transpositions.
21. Prove that any group of prime order is cyclic.
22. If $\Phi: G_1 \rightarrow G_2$ is an isomorphism of groups and if G_1 is cyclic, then prove that G_2 is also cyclic.
23. Show that the multiplicative group $Z_8^{\,x}$ is isomorphic to $Z_2 x\ Z_2$.

(Ceiling : 35 Marks)

PART - C

(Answer any two questions. Each questions carries 10 marks.)

24. Prove that every permutation can be written as a product of disjoint cycles.

25. Let a be an element of the group G.Then prove that

    (a) If a has infinite order, then $a^k \neq b^m$ for all integers $k \neq m$.

    (b) If a has finite order, then $a^k = e$ if and only if order of a divides k.

    (c) If a has finite order $o(a) = n$, then for all integers k,m we have $a^k = a^m$ if and only if $k \equiv m \pmod{n}$.

26. Prove that every subgroup of a cyclic group is cyclic.

27. If G_1 and G_2 are groups and if $\Phi: G_1 \rightarrow G_2$ is a homomorphism with $K = \mathrm{Ker}(\Phi)$, then prove that $G_1/K \cong \Phi(G_1)$.

(2 x 10 = 20 Marks)

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1B5N23471 (Pages : 2) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

BMT5B06 – Basic Analysis

(2019 Admission onwards)

Time: 2 ½ hours

Max. Marks: 80

Section A

All Questions can be attended. Each question carries 2 marks. Ceiling 25 marks

1. Define an Uncountable set and give an example for uncountable set
2. If $a \neq o$ and b in R are such that a.b = 1, then prove that $b = \frac{1}{a}$
3. Let a,b,c be elements of R and if a > b and c >0, then prove that ca >cb
4. Write the set of real numbers x satisfying $x^2 > 3x + 4$
5. Let $a \in R$. If x belongs to the neighborhood $V_\varepsilon(a)$ for every $\varepsilon > 0$, then prove that x = a
6. If $S = \{ \frac{1}{n} - \frac{1}{m} : n, m \in N\}$, find Inf S and Sup S
7. Define Nested Intervals and give an example for Nested intervals
8. Using the definition of limit, prove that $\lim_{n \to \infty}(\frac{2n}{n+1}) = 2$
9. Define subsequence of a sequence. Give an example of unbounded sequence that has a convergent subsequence
10. Prove that Cauchy sequence of real numbers is bounded
11. Let the sequence $X = (x_n)$ converges to x. Then prove that the sequence ($|x_n|$) of absolute value converges to |x|
12. Find the modulus of a complex number $(1 - i)^2$
13. Find the real part of $\frac{(4+5i)+2i^3}{(2+i)^2}$
14. Define Open subset of the complex plane
15. Find the polar form of a complex number $z = -2+2i\sqrt{3}$

Section B

All Questions can be attended. Each question carries 5 marks. Ceiling 35 marks

16. Let S be a nonempty subset of R that is bounded above, and let a be any number in R. Define the $a + S = \{ a + s : s \in S\}$. Prove that $\text{Sup}\,(a + S) = a + \text{Sup}\, S$

17. State and Prove Cantor's theorem

18. Prove that the set of real numbers is not countable by using Nested Interval property

19. Let (x_n) be a sequence of positive real numbers such that $L = \lim \left(\frac{x_{n+1}}{x_n} \right)$ exists. Prove that if $L < 1$, then (x_n) converges and $\lim (x_n) = 0$

20. Prove that every Contractive sequence is a Cauchy sequence and therefore is convergent.

21. Find the two square roots of $\sqrt{3} + i$

22. Find an upper bound for $\left| \frac{-1}{z^4 - 5z + 1} \right|$ if $|z| = 2$

23. Describe the set of all points z in the complex plane that satisfy the equation $\left| \frac{z+1}{z-1} \right| = 4$

SECTION C

Answer any Two Questions. Each question carries 10 Marks. 20 Marks from this sectio

24. Prove that there exists a positive real number x such that $x^2 = 2$

25. State and prove Monotone Subsequence theorem

26. If $c > 0$, then prove that $\lim (c^{\frac{1}{n}}) = 1$

27. (a) Prove that $||z_1| - |z_2|| \le |z_1 + z_2|$

    (b) Describe the set of points z in the complex plane that satisfy the equation

    $|z - i| = |z - 1|$

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1B5N23472 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

BMT5B07 – Numerical Analysis

(2019 Admission onwards)

Time: 2 hours

Max. Marks: 60

Section A

All questions can be attended. Each question carries 2marks.

1. Use Bisection method to find p_4 for $f(x) = x^3 - x - 2 = 0$,on $[1, 2]$.
2. Determine all fixed points of the function$f(x) = \sqrt[3]{x}$.
3. Use Newton's method to find p_2 for $f(x) = x^2 - 6$ and $p_0 = 1$.
4. Let $f(x) = e^x - x - 1$, then show that f has a zero of multiplicity 2 at $x = 0$.
5. Determine the coefficient polynomials$L_0(x), L_1(x)$ and $L_2(x)$ through the nodes $x_0 = 2$, $x_1 = 2.75$, and $x_2 = 4$.
6. Determine the interpolating polynomial denoted $P_{1,2,4}(x)$ for $f(x) = e^x$ with $x_0 = 1,\ x_1 = 2, x_3 = 3, x_4 = 6$.
7. Write the Newton Forward-Difference formula and Backward-Difference formula.
8. Use the forward difference formula to approximate the derivative of $f(x) = x\ \ln x$ at $x_0 = 8.1$ using $h = 0.2$.
9. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule, with $n = 6$.
10. Show that $f(t, y) = t^2y + 1$ satisfies the Lipschitz condition on the interval $D = \{(t, y)\ /\ 0 \le t \le 1\text{and } -\infty < y < \infty\}$.
11. Show that the IVP $\frac{dy}{dt} = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$, is well posed on $D = \{(t, y) / 0 \le t \le 2 \text{ and } -\infty < y < \infty\}$.
12. Write the formulae for Adams-Bashforth Two-Step and Three-Step Explicit Methods.

(Ceiling ... 20 Marks)

Section B

All questions can be attended. Each question carries 5marks.

13. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ón $[1, 2]$. Use $p_0 = 1$.

14. Find the first three estimates for the equation $x^3 - 2x - 5 = 0$ by the secant method using $x_0 = 2$ and $x_1 = 3$.

15. Using Lagrange's interpolation formula, calculate $f(10)$ from the following table.

x	5	6	9	11
$f(x)$	12	13	14	16

16. Find $y(5)$ using Newton's forward differences interpolation formula from the following table.

x	0	10	20	30	40
y	7	18	32	48	85

17. Use the Composite Simpson's rule to approximate the integral $\int_1^2 x \ln x \, dx, \quad n = 4$.

18. Approximate $\int_2^4 (x^4 + 1) \, dx$ using Gaussian quadrature with $n = 3$.

19. Use Taylor's method of order two to approximate the solution for the initial-value problem$y' = 1 + (t - y)^2, 2 \leq t \leq 3, y(2) = 1$, with $h = 0.5$.

(Ceiling … 30 Marks)

Section C

Answer any ONEquestion.

20. (a) Use method of False Position to find the solution accurate to within $10^{-2}$for $f(x) = x^3 + \quad x - 1 = 0, \ [0,1]$.

(b)Use Neville's method to obtain the approximations for Lagrange interpolating polynomial of degree three to approximate $f(8.4)$ if

$f(8.1) = 16.94410, f(8.3) = \quad 17.56492, f(8.6) = 18.50515, f(8.7) = 18.82091$.

21. (a) What is $y(1.3)$ for the equation $\frac{dy}{dx} + 2xy^2 = 0$ with $y(1) = 1$ using Euler's method with step size $h = 0.1$.

(b) Use fourth order Runge-Kutta method to find $y(0.2)$, given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$. Choose $h = 0.2$.

(1 x 10 = 10 Marks)

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1B5N23473 (Pages : 4) Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

BMT5B08 – Linear Programming

(2019 Admission onwards)

Time: 2hours

Max. Marks: 60

Session A

All questions can be attended. Each question carries 2 marks.

1) Draw and shade the following subset of $\boldsymbol{R}^2$
   a) A bounded convex polyhedral subset
   b) A polyhedral convex subset having no extreme points

2) Represent the following linear programming problem in canonical slack form.

$$Maximize \quad P(x,y) = 20x + 15y$$

$$\text{Subject to} \quad 2x - y \leq 3,\ \ x + 2y \leq 5,\ \ x + y \geq 1\ \ and\, x, y \geq 0$$

3) Prove that in a maximum basic feasible tableau, the basic solution is a feasible solution.

4) Prove that the constrained set of the following LPP is unbounded.

$$Minimize \quad g(x,y,z) = 2x - y + z$$

$$Subject\ to \quad x + y + z \leq 5$$

$$2x - 3y - z \geq 3$$

$$x, y, z \geq 0$$

5) Construct the tucker tableau of the LPP.

$$\text{Maximize}\ f(x) = 3x + 2y$$

$$\text{Subject to} \quad 2x + y = -1, \quad x - 2y \geq 0 \quad x \geq 0$$

6) Define a minimum basic feasible tableau and give an example.

7) Find the basic solution of the tableau.

t_C	x_2	-1	
$-1/2$	$3/2$	$15/2$	$= -t_A$
-1	1	5	$= -t_B$
$1/2$	$1/2$	$25/2$	$= -x_1$
-100	50	-2500	$= P$

8) State the dual maximization problem of the following problem

$$\begin{aligned} &\textit{Maximize} \quad f(x_1, x_2, x_3) = x_1 + x_2 - x_3 \\ &\textit{Subject to} \quad x_1 - x_2 + x_3 = -1 \\ &\qquad -x_1 - x_2 + x_3 = 1 \\ &\qquad -x_1 + x_2 + x_3 \leq 1 \\ &\qquad x_1, x_3 \geq 0 \end{aligned}$$

9) Convert the following non canonical tableau into canonical form by applying pivot transformation.

$$\begin{array}{cc|c|l} x & \textcircled{y} & -1 & \\ \hline 1 & 2 & 10 & = -t_1 \\ -3 & -1 & -15 & = -t_2 \\ & & & \\ \hline 1 & 3 & 0 & = f \\ \hline \end{array}$$

10) State Von Neumann Minimax Theorem.

11) Write the Hungarian algorithm for solving assignment problem.

12) Using The Northwest-Corner Method, obtain an initial basic feasible solution of the transportation problem given below.

$$\begin{array}{|ccc|c} \hline 7 & 2 & 4 & 10 \\ 10 & 5 & 9 & 20 \\ 7 & 3 & 5 & 30 \\ \hline 20 & 10 & 30 & \end{array}$$

Session B

All questions can be attended. Each question carries 5 marks.

13) Solve the following linear programming problem by sketching the constraint set.

$$\begin{aligned} &\textit{Minimize}\ g(x,y) = 5x + 2y \\ &\quad \textit{subject to} \quad x + 3y \geq 14 \\ &\qquad 2x + y \geq 8 \\ &\qquad x, y \geq 0 \end{aligned}$$

14) Solve the following canonical linear programming problem by using simplex algorithm.

$$\begin{array}{c|cc|c|}
\cline{2-4}
x & 1 & 2 & 5 \\
y & 3 & 1 & 2 \\
\cline{2-4}
-1 & 14 & 8 & 0 \\
\cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{s_1} & \multicolumn{1}{c}{s_2} & \multicolumn{1}{c}{g}
\end{array}$$

15) Verify the duality equation for the following dual canonical tableau.

$$\begin{array}{c|cc|c|c}
 & x_1 & x_2 & -1 & \\
\cline{2-4}
y_1 & 1 & 2 & 3 & = -t_1 \\
y_2 & 4 & 5 & 6 & = -t_2 \\
\cline{2-4}
-1 & 7 & 8 & 9 & = f \\
\cline{2-4}
 & = s_1 & = s_2 & = g &
\end{array}$$

16) Solve the following tableau.

$$\begin{array}{|ccc|c|c}
x & y & \textcircled{z} & -1 & \\
\hline
1 & 1 & 1 & 3 & = -t_1 \\
0 & 1 & 2 & 2 & = 0 \\
1 & 1 & 0 & 1 & = -t_2 \\
\hline
1 & 1 & 1 & 0 & f \\
\hline
\end{array}$$

17) Using domination, reduce the following matrix games as far as possible and form the game tableau.

$$\begin{array}{cc}
 & \text{II} \\
\text{I} & \begin{bmatrix} 2 & 1 & 4 & 2 \\ 1 & 2 & 1 & 1 \\ -2 & 6 & 3 & -2 \\ 3 & -3 & 5 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}
\end{array}$$

18) By applying VAM, find a basic feasible solution of the following transportation problem

$$\begin{array}{|ccc|c}
\hline
7 & 2 & 4 & 10 \\
10 & 5 & 9 & 20 \\
7 & 3 & 5 & 30 \\
\hline
20 & 10 & 30 &
\end{array}$$

19) Solve the assignment problem.

	J_1	J_2	J_3	J_4
P_1	2	3	2	4
P_2	5	8	4	3
P_3	5	9	5	2
P_4	7	6	7	4

Session C

Answer any one. Question carries 10 marks

20) Solve the dual canonical linear programming problem

$$\begin{array}{c|ccc|c|l}
 & \textcircled{x_1} & x_2 & x_3 & -1 & \\
\hline
\textcircled{y_1} & 0 & -1 & -1 & -1 & = -0 \\
y_2 & -1 & -3 & 4 & 0 & = -t_1 \\
y_3 & -1 & 2 & -3 & 0 & = -t_2 \\
\hline
-1 & -1 & 0 & 0 & 0 & = f \\
\hline
 & = 0 & = s_1 & = s_2 & = g &
\end{array}$$

21) Write the transportation algorithm and using this solve the transportation problem.

5	12	8	50	26
11	4	10	8	20
14	50	1	9	30
12	20	26	10	

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1B5N23474 (Pages : 2) Reg. No:……………………

Name: ………………………

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

BMT5B09 – Calculus of Multivariable – I

(2019 Admission onwards)

Time: 2hours

Max. Marks: 60

Section A

All questions can be attended

Each question carries 2 marks

1. Sketch the curve represented by $x = t, y = t^2$.
2. Find $\frac{d^2y}{dx^2}$ if $x = t^2 - 4, y = t^3 - 3t$.
3. Find the representation of the point $(-1,1)$ in polar coordinates.
4. Find parametric equation for the line passing through the point $P_0(-2,1,3)$ and parallel to the vector $v = (1,2,-2)$.
5. Identify and sketch the surface $4x - 3y^2 - 12\,z^2 = 0$.
6. Find an equation in cylindrical coordinates for the surface $9x^2 + 9y^2 + 4z^2 = 36$
7. Find a vector function that describes the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x + y + 2z = 4$.
8. Find $\boldsymbol{r}(t)$ satisfying the condition $\boldsymbol{r}'(t) = 2\boldsymbol{i} + 4t\boldsymbol{j} - 6t^2\boldsymbol{k}; \boldsymbol{r}(0) = \boldsymbol{i} + \boldsymbol{k}$.
9. Define curvature and the curvature of the graph of a function.
10. Define level curve and sketch a contour map for the surface $f(x,y) = x^2 + y^2$.
11. Determine whether the function $f(x,y) = \frac{1}{y-x^2}$ is continuous.
12. Compute f_{xzy} and f_{yxz} if $f(x,y,z) = xe^{yz}$.

(Ceiling 20 Marks)

Section B

All questions can be attended

Each question carries 5 marks

13. Show that the surface area of a sphere of radius r is $4\pi r^2$.
14. Find the area of the region enclosed by the cardioid $r = 1 + \cos\theta$.
15. Find parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.
16. Find the antiderivative of $\boldsymbol{r}'(t) = \cos t\,\boldsymbol{i} + e^{-t}\,\boldsymbol{j} + \sqrt{t}\,\boldsymbol{k}$ satisfying the initial condition $\boldsymbol{r}(0) = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$.
17. Find the point on the graph of $y = e^{-x^2}$ at which the curvature is zero.

18. Find the tangential scalar and normal scalar components of acceleration of the particle $r(t) = ti + t^2j + t^3k$ at any time t.

19. If $w = f(x, y)$, where f has continuous second-order partial derivatives and let $x = r^2 + s^2$ and $y = 2rs$ then find $\frac{\partial^2 w}{\partial r^2}$.

(Ceiling 30 Marks)

Section C

Answer any one question

20. (a) Find the curvature of the parabola $y = \frac{1}{4}x^2$at the points where $x = 0$ and $x = 1$.

(b) Find the points where the curvature is largest.

21. The productivity of a country is given by the function $f(x, y) = 20\, x^{3/4}\, y^{1/4}$, where $x$units of labor and y units of capital are used.

(a) What are the marginal productivity of labor and the marginal productivity of capital when the amounts expended on labor and capital are 256 units and 16 units respectively?

(b) Should the government encourage capital investment rather than increased expenditure on labor at this time to increase the country's productivity?

(1 X 10 = 10 Marks)

1B5N23475

(Pages : 2)

Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2023

(Open Course)

BMT5D03 – Linear Mathematical Models

(2019 Admission onwards)

Time: 2hours

Max. Marks: 60

Section A

All questions can be attended.
Each question carries 2 marks.

1) Write the equation of line having slope m and that passes through (x_1, y_1).

2) Find x, y, p, q if $\begin{bmatrix} 2 & 1 \\ p & q \end{bmatrix} = \begin{bmatrix} x & y \\ -1 & 0 \end{bmatrix}$.

3) Find the transpose of the matrix $\begin{bmatrix} 1 & 5 & -1 \\ 5 & 8 & 1 \\ -4 & 6 & 2 \end{bmatrix}$.

4) Define slack variables.

5) Define a linear function.

6) Find the slope of a horizontal line.

7) Define least square line.

8) Find $A - B$ if $A = \begin{bmatrix} 3 & 6 & 1 \\ 5 & 1 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

9) Define basic feasible solution in a linear programming problem.

10) Graph the inequality $2x - 3y \leq 12$.

11) Find the order of the matrix $\begin{bmatrix} 6 & 5 & 0 \\ 3 & 0 & 1 \end{bmatrix}$.

12) Find $f(2)$ if $f(x) = 3x+7$.

(Ceiling 20 Marks)

Section B

All questions can be attended.

Each question carries 5 marks.

13) Find the equation of the line that passes through the point (3, 5) and is parallel to the line $2x + 5y = 4$.

14) Find AB given $A = \begin{bmatrix} 1 & -3 \\ 7 & 2 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \end{bmatrix}$.

15) Graph the feasible region for the system

$2x - 5y \leq 10$

$x + 2y \leq 8$

$x \geq 0$

$y \geq 0.$

16) Write the dual of the standard linear programming problem

Minimize $w = 7y_1 + 5y_2 + 8y_3$

subject to: $3y_1 + 2y_2 + y_3 \geq 10$

$4y_1 + 5y_2 \geq 25$

with $y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0.$

17) Solve the linear programming problem graphically

Maximize $z = 3x+4y.$

subject to : $2x+y \leq 4$

$-x+2y \leq 4$

$x \geq 0,\ y \geq 0.$

18) Describe Echelon method of solving a linear system.

19) Write the standard form of Maximization problem.

(Ceiling 30 Marks)

Section C

Answer any one question.

20) (a) Find A^{-1} if $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$.

(b) Graph the line $y = -3x$.

21) Use Gauss-Jordan method to solve the system

$$x + 5z = -6 + y$$
$$3x + 3y = 10 + z$$
$$x + 3y + 2z = 5.$$

(1 x 10 = 10 Marks)